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ADAPTIVE CONTROL FOR A CLASS OF HIGH-ORDER NONLINEAR UNCERTAIN SYSTEMS

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ABSTRACT

The adaptive stabilization problem of a class of high-order nonlinear systems with uncertainties is presented in this work. The difficulty of our design is that the Jacobian linearization of the system under consideration is uncontrollable. The total design of the controller was divided into two steps. First we study the nominal system without uncertainties. Then we adopt a Radial basis function neural networks (RBF NN) to approximate the system uncertainties, and design an robust adaptive control law to stabilize the system using the so-called integral sliding mode design approach based on the RBF NN. The reachability of the sliding surface and the convergence of the weight of the neural networks are showed by the Lyapunov theory. Finally, a simulation example is given to illustrate the effectiveness of the proposed method.

Keywords: *Nonlinear Systems, Integral Sliding Mode, Adaptive Control, Neural Networks*

1. INTRODUCTION

Over the past decades, the stabilization of nonlinear systems which Jacobian linearization at the equilibrium is uncontrollable has received a lot of attention (refer to [1-3], [6], [8] and references therein). Owing to the violation of the necessary condition for the existence of a smooth controller, this class of nonlinear systems should be treated by nonsmooth feedback[4]. However, to the best of the authors' knowledge the existing design for this class of systems mainly employ the recursive design [1-4], [14], which can be seen as variations of the well-known backstepping method[8]. More recently, Chen and Huang adopt a small gain approach to study the robust stabilization of a class of systems subject to input unmodeled dynamics with uncontrollable unstable linearization [7].

Sliding mode control (SMC) is an excellent robust control approach for uncertain systems [5]. It is well known for its ability to withstand model uncertainties satisfying the so-called matching condition and disturbances. In addition, the sliding mode controller is easy to implement which is very important in practical system. The conditional SMC has the problem of reaching phase, i.e. the trajectory of the designed solution is not robust even respect to matching uncertainties in the reaching phase preceding the sliding surface motion. V.Utkin, and J. Shi presented a new integral sliding mode control without reaching phase [9]. They define an integral-type sliding surface so that the system trajectories start in the surface right at the beginning of the process, which means the closed-loop system is robust since the first time instant [10], [11].

In order to cope with the highly uncertain nonlinear systems, approximator-based control approaches have been extensively studied [12],[13]. Theoretically, if the number of the neurons of a radial basis function(RBF) neural networks(NN) is chosen sufficiently large, then the RBF NN can approximate a continuous function to any desired degree of accuracy on any compact[14].

In this note, we propose a novel integral sliding mode design approach combined with RBF NN technique for the stabilization of a class of high-order nonlinear uncertain systems which Jacobian linearization is not stabilizable. Owing to the introducing of neural networks, the problem of chattering phenomenon of the conventional sliding mode control is decreased effectively.

The paper is organized as follows. In section 2, the problem statement is given. In section 3, a nonlinear integral sliding mode controller is constructed and the reaching condition of the sliding surface is guaranteed to be satisfied. In section 4, the validity of the proposed control scheme is illustrated by simulation examples. Finally, a short conclusion is given in section 5.

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2. PROBLEM STATEMENT

We consider the following high-order single input nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2^{p_1} \\ \dot{x}_2 &= x_3^{p_2} \\ &\cdots \\ \dot{x}_{n-1} &= x_n^{p_{n-1}} \\ \dot{x}_n &= u + d(x) \end{aligned} \quad (1)$$

where $x = [x_1,\ x_2,\ \cdots,\ x_n]^T \in R^n$ is the state vector with the initial value $x(t_0)$, $u \in R$ is the control input. $p_i, i = 1,\cdots,n-1$ are known positive odd integers. $d(x)$ is an unknown disturbance.

Let

$$f(x) = \left[x_2^{p_1}, x_3^{p_2}, \cdots, x_n^{p_{n-1}}, 0\right]^T,$$

$$B = [0,\ \ \cdots,\ \ 0,\ \ 1]_{1\times n}^T,$$

Then system (1) can be rewritten as

$$\dot{x} = f(x) + B(u + d(x)) \quad (2)$$

We use a RBF neural network to approximate the disturbance $d(x)$, then we get

$$\dot{x} = f(x) + B\left(u + \Phi W\right) \quad (3)$$

The column vector $W = [W_1, \cdots, W_m]^T$ is the weight vector of the NNs. $\Phi = [\phi_1(x) \ \ \cdots \ \ \phi_m(x)]$ is the basis functions vector, and the basis function $\phi_i(x)$ is chosen as the form of Gaussian functions

$$\phi_i(x) = \exp\left(-\frac{(x-c_i)^2}{2\delta_i^2}\right), \quad 1 \le i \le m \quad (4)$$

For disturbance $d(x)$, there exists an optimal weight vector W^* and a positive scalar ε^* such that

$$d(x) = \Phi W^* + \varepsilon(x), \quad |\varepsilon(x)| \le \varepsilon^* \quad (5)$$

where $\varepsilon(x)$ is the approximation error of the RBF networks. Let $\hat{W}$ be an estimate of W^*, the weight estimation error $\tilde{W} = W^* - \hat{W}$ .we have

$$d(x) = \Phi\hat{W} + \Phi\tilde{W} + \varepsilon, \quad |\varepsilon| \le \varepsilon^* \quad (6)$$

By choosing the number of the neurons of the networks sufficiently large, the upper bound of the approximation error ε^* can be reduced to be arbitrarily small.

Then equation (3) can be rewritten as

$$\dot{x} = f(x) + g(x)\left(u + \Phi W^* + \varepsilon(x)\right) \quad (7)$$

We first consider the stability of the following system

$$\dot{x} = f(x) + Bu \quad (8)$$

The above system (8) is feedback linearizable. Therefore, it is not difficult to develop a globally asymptotically stable control law $u_0(x) = \alpha(x)$ for its stabilization using feedback linearization method [15]. As for the detailed design method of feedback linearization, we can refer to [15,16] and references therein. For the reason of conciseness, in the next section, we assume that $u_0(x)$ is known.

3. NONLINEAR INTEGRAL SLIDING MODE CONTROL

A. *Integral sliding mode surface*

For system (7), we present a control law as

$$u = u_0(x) + u_1(x) \quad (9)$$

Where $u_1(x)$ is developed to cope with the uncertain term ΦW, which will be detailed in the following.

Assumption: The nominal system $\dot{x} = f(x) + Bu$ is globally asymptotically stabilizable by the control $u_0(x) = \alpha(x)$; i.e., there exist a Lyapunov function $V(x)$ satisfying

$$\gamma_1(\|x\|) \le V(x) \le \gamma_2(\|x\|)$$

$$\left(\frac{\partial V}{\partial x}\right)^T \left[f(x) + B\alpha(x)\right] \le -\gamma_3(\|x\|) \quad (10)$$

Where $\gamma_1(\cdot), \gamma_2(\cdot)$ are class κ_∞ functions, $\gamma_3(\cdot)$ is a class κ function.

Substituting $u = u_0(x) + u_1(x)$ into (2) we get

$$\dot{x} = f(x) + B\alpha(x) + Bu_1(x) + d(x) \quad (11)$$

Defining a nonlinear integral sliding surface

$$s(x) = G\left[x(t) - x(t_0) - \int_{t_0}^{t}\left(f(x) + B\alpha(x)\right)d\tau\right] \quad (12)$$

where $G = [g_1,\ \cdots,\ g_n] \in R^{1\times n}$ is the coefficient vector which must satisfy $g_i \neq 0, i = 1,\cdots,n$ and $GB \neq 0$. Noticing that $s(x) = 0$ at the instant $t = t_0$, which means the system always starts at the sliding manifold.

The derivative of s along time t is

$$\begin{aligned} \dot{s}(x) &= G\left[f(x) + Bu + d(x) - \left(f(x) + Bu_0\right)\right] \\ &= G\left[Bu_1 + \Phi W\right] \end{aligned} \quad (13)$$

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When the state trajectories of the system enter the sliding surface s, there are $s=0$ and $\dot{s}=0$. The system dynamics in the ideal sliding surface s is exactly equivalent to $\dot{x}=f(x)+g(x)u_0$, as a result, the system stability is guaranted.

B. *The compensator* u_1

In this work, the control u_1 is constructed as

$$u_1=-\frac{\rho s}{GB}-\Phi\hat{W}-\varepsilon*\mathrm{sgn}(s),\ \ \rho>0 \qquad (14)$$

Where $\mathrm{sgn}(\cdot)$ is the signum function, $\hat{W}$ is the estimate of W^*, define the estimate error $\tilde{W}=W^*-\hat{W}$. Then, the updating law of $\hat{W}$ can be chosen as

$$\dot{\hat{W}}=sGB\Phi\Gamma \qquad (15)$$

Where gain matrix $\Gamma>0$ and satisfying $\Gamma\in R^{m\times m}$.

Result: For nonlinear systems described by the equation (1), the sate trajectories of the closed-loop system should enter the sliding surface defined by the equation (12) with the control $u=u_0+u_1$, where u_1 is designed as the equations (14),(15).

Proof: Consider the Lyapunov function candidate

$$V=\frac{1}{2}s^2+\frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W} \qquad (16)$$

The time derivation of this function V is

$$\dot{V}=s\dot{s}+\dot{\tilde{W}}^T\Gamma^{-1}\tilde{W} \qquad (17)$$

Substitute (13) ~ (15) into (17), it follows that

$$\begin{aligned}\dot{V}&=s\dot{s}+\dot{\tilde{W}}^T\Gamma^{-1}\tilde{W}\\&=sG\left[B\left(u_1+\Phi W^*+D\right)\right]-\dot{\hat{W}}^T\Gamma^{-1}\tilde{W}\\&=sGBu_1+sGB\left(\Phi\tilde{W}+\Phi\hat{W}+\varepsilon(x)\right)-\Phi\Gamma\Gamma^{-1}\tilde{W}\\&=-\rho s^2-sDB\varepsilon*\mathrm{sgn}(s)+sGB\varepsilon\\&\le-\rho s^2\end{aligned} \qquad (18)$$

Obviously, $\dot{V}<0$ when $s\neq0$. Therefore, the sliding mode surface s must be reachable and $\tilde{W}$ reaches a small neighborhood of the manifold

$$GB\left(u_1+\Phi W\right)=0 \qquad (19)$$

Substitute (13) into (18), we get

$$GB\left(-\frac{\rho s}{GB}-\Phi\hat{W}-\varepsilon*\mathrm{sgn}(s)+\Phi W\right)=0 \qquad (20)$$

i.e.

$$-\rho s+GB\left(-\Phi\tilde{W}-\varepsilon*\mathrm{sgn}(s)\right)=0 \qquad (21)$$

In the following, we analyze the closed-loop system's dynamics in the integral sliding surface. As for (2), the closed-loop dynamics in the integral sliding surface s ($s=\dot{s}=0$) is

$$\begin{aligned}\dot{x}_s&=f(x_s)+B\alpha(x_s)\\&\quad+B\left(-\frac{\rho s}{GB}-\Phi\hat{W}-\varepsilon*\mathrm{sgn}(s)+d(x)\right)\\&=f(x_s)+B\alpha(x_s)+B\left(-\Phi\tilde{W}-\varepsilon*\mathrm{sgn}(s)\right)\end{aligned} \qquad (22)$$

From (22), it can be concluded that the stabilization of the closed-loop system depends on the stability result of the system $\dot{x}=f(x)+B\alpha(x)$ and on the approximated effects of the NNs.

According to the assumption (10), we get

$$\begin{aligned}\dot{V}_s&=\left(\frac{\partial V}{\partial x_s}\right)^T\left[f(x_s)+B\alpha(x_s)\right.\\&\qquad\left.+B\left(-\Phi\tilde{W}-\varepsilon*\mathrm{sgn}(s)\right)\right]\\&\le-\gamma_3(\|x_s\|)+\left\|\frac{\partial V}{\partial x_s}\right\|\beta'(x_s)\end{aligned} \qquad (23)$$

where $\beta'(x)=\left\|B\left(-\Phi\tilde{W}-\varepsilon*\mathrm{sgn}(s)\right)\right\|$.

Remark: from (23), it can be conclude that

(1) if

$$\beta'(x_s)\le\gamma_3(\|x_s\|)\Big/\left\|\frac{\partial V}{\partial x_s}\right\|\beta'(x_s),$$
$$\forall x_s\in R^n, x_s\neq0$$

the closed-loop dynamics in the sliding surface is globally asymptotically stable.

(2) If

$$\beta'(x_s)\le-\gamma_3(\|x_s\|)\Big/\left\|\frac{\partial V}{\partial x_s}\right\|\beta'(x_s),$$
$$\forall x_s\in R^n, \text{and } 0<d_1<\|x_s\|<d_2$$

the closed-loop dynamics in the sliding surface is locally uniformly ultimately bounded.

In the next section, a numerical simulation is presented to validate the effectiveness of the design.

4. SIMULATION RESULTS

Consider the following system

$$\begin{aligned}\dot{x}_1&=x_2^{\,3}\\\dot{x}_2&=x_3^{\,5}\\\dot{x}_3&=u+d(x)\end{aligned} \qquad (24)$$

Suppose that $d(x)=x_2^{\,2}\sin(x_1)$ which is unknown. The initial state value is $x(t_0)=[0.9,-0.9,0.9]^T$.

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We seek for a control $u(x_1, x_2, x_3)$ for system (24) to drive the closed-loop system state at the above initial state $x(t_0)$ to converge to the origin.

For the reduced system $\dot{x}_1 = x_2^{\ 3}$, $\dot{x}_2 = x_3^{\ 5}$, $\dot{x}_3 = u$ We firstly develop the following globally asymptotically stable control u_0.

$$u_0 = -(4 + 9x_2^{\ 2})x_3 - 10x_2 - 90x_1 - 12x_2^{\ 3} \quad (25)$$

Then choose proper coefficient vector G and positive coefficient k_0 to design the compensator $u_1(x)$.

For example, we choose $G = [1 \quad 1 \quad 1]$, $k_0 = 50$. After simulation, the results are demonstrated by figure 1 (the control u) and figure 2 (The state response curves). From the figure 2, it can be seen that the values of the system state converge to zero asymptotically. Which illustrate the correctness of the theoretical result

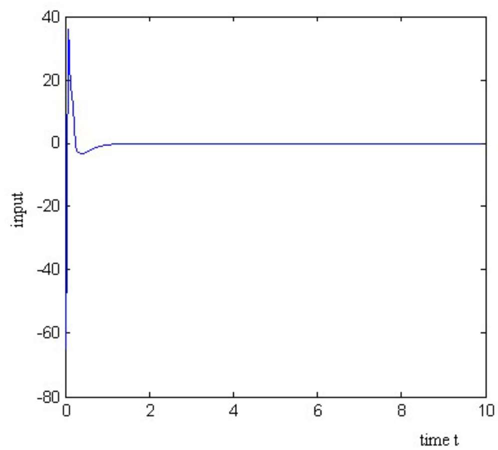

Figure 1: Curve Of Control U.

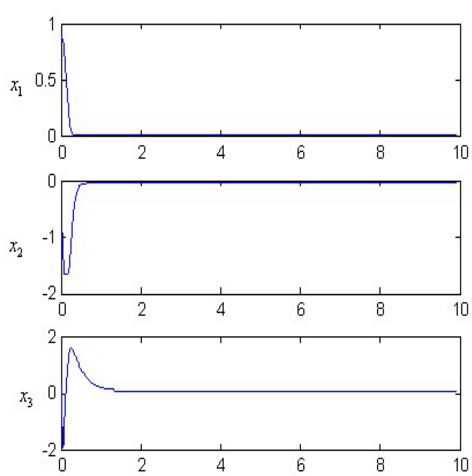

Figure 2: Response Of The System State.

5. CONCLUSION

A novel nonlinear design approach based on an integral sliding mode surface is proposed for a class of high-order nonlinear systems. By choosing the design parameters properly according to the design method, the sliding surface can be reached from the initial state, therefore, the stability of the closed-loop system is obtained. Finally, numerical examples validate the effectiveness of the presented design.

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