



## ON PRICE AND SERVICE COMPETITION WITH SUBSTITUTABLE GOODS

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### ABSTRACT

This research studies a case that two enterprises produce two different but substitutable goods. The consumer demand depends on two kinds of factors: prices and service levels of the product and substitutable product. This article is based on three scenarios: Nash Equilibrium, Enterprise Alliance and Stackelberg. Game-theoretic framework is applied to find the optimal solutions for every participant. This research has conclusions as follow. Firstly, if market base of one product increases or one product has some economic advantage in providing service, it benefits for itself but is bad for substitutable product. Secondly, enterprises will provide fewer services, gain fewer customers, but earn more profit in Enterprise Alliance than in Nash Equilibrium. Thirdly, when one enterprise is dominant, if substitutable goods influence its demand greatly, it will take the advantage to earn more profit; otherwise, it will give up the advantage to seek the Enterprise Alliance.

**Keywords:** *Substitutable Goods, Pricing Strategy, Service Level*

### 1. INTRODUCTION

As the development of economy, price is no longer the only competitive factor for enterprise. Some non-price factors, such as service, play more and more important role in enterprise competition. Many traditional manufacturing enterprises, such as IBM, GE, and DELL and so on, begin developing product services to enhance enterprise competitiveness. Their service industries provide more and more incomes and profits. So, it is very difficult to judge whether it is a manufacturing enterprise or a service enterprise.

There are many literatures about competition based on price and service. In [1], the authors find a conclusion that enterprises can keep old customers by service and attract new customers by price. In [2], the authors study the duopolistic interaction between two facilities which supply substitutable goods and make decisions on service and price, and compare the results to monopoly. In [3], the authors use conjoint analysis to investigate the relative importance of price and service in attracting consumer. In [4], the authors propose a theoretic model, in which passenger's choice behavior is used to investigate the role of competing service quality. In [5], the authors discuss a model for price and service competition of two participators. They research the optimal decisions of them under uncertainty demand. In [6], the author examines

how manufacturer coordinate channel distribution when two retailers compete with price and non-price factors. In [7], the authors provide a mechanism where the retailer adds value to the product, thereby differentiating it from the basic product being sold through direct sales.

Literatures are also rich in studies on game-analysis between enterprises. In [8], the authors use a game theory model to study the price competition between a manufacturer's direct channel and its traditional channel. In [9], the authors investigate the service competition in the dual-channel supply chain using the consumer choice model. They find that the manufacturer's optimal channel strategy depends on the channel environment. In [10], the author finds that the manufacturer and retailer can use revenue-sharing policies to achieve channel coordination effectively. In [11], the author study Retailer Stackelberg scenario where retailers have the initiative compared to their suppliers. The suppliers are mostly concerned with receiving orders from the retail giants.

However, there are little literature to study when two substitutable goods enterprises provide both products and services, especially they form a enterprise alliance. This research studies the case that two enterprises should determine the prices and service levels to influence demand dependently and obtain the optimum profits. We use game-theoretic approach to derive equilibrium solutions for prices,



service levels, demand quantities, and profits for each member. Three scenarios are considered: Nash Equilibrium, Enterprises Alliance and Stackelberg. The results of this research will find the role of service and price, which has not been focused by the existing literatures.

2. MODEL

In this case, two enterprises produce two different but substitutable goods and sell them to customer. We assume that there are only two enterprises in this area, which means there is no competition among them. We assume that all activities occur in a single period. We also assume that consumer demand for each product is sensitive to four factors: (1) product price, (2) product service level, (3) substitutable product price, (4) substitutable product service level. So there is a deterministic consumer demand that is influenced by prices  $p_1, p_2$  and service levels  $s'_1, s'_2$ .

2.1 Demand Function

In [12, 13], the authors set some basic characteristics for the demand of each product. So we make the assumptions for defining the demand function.

**ASSUMPTION 1.** The two products' demand structures are symmetric. Demand for one product is decreasing with its price increasing or substitutable product's price decreasing. At the same time, it is increasing with its own service increasing or substitutable product's service decreasing.

**ASSUMPTION 2.** There is a market base  $a_i$  which is used to measure the size of product  $i$ 's market. It equals to the demand of product  $i$  when both products are priced at zero and no service is offered.

From Assumption 1 and 2, the demand of product  $i$  can be expressed as:

$$Q_i = a_i - \alpha_p p_i + \beta_p p_j + \alpha_s s'_i - \beta_s s'_j \tag{1}$$

Where

$$a_i > 0, \alpha_p > \beta_p > 0, \alpha_s > \beta_s > 0.$$

$$i = 1, 2, j = 3 - i.$$

Parameters are defined as follows:

$a_i$  is a nonnegative constant. It can be thought of as a "market base", which is defined in Assumption 2. We assume that  $a_i$  is large enough so that  $Q_i$  will always be nonnegative.

$\alpha_p$  is the measure of the responsiveness of product's market demand to its price. If the price of product  $i$  is decreased by one unit, the product will gain  $\alpha_p$  more customers. While  $\beta_p$  is the measure of the responsiveness of product's market demand to substitutable product's price. If the price of product  $j$  is increased by one unit, the substitutable product  $i$  will gain  $\beta_p$  more customers. As described by C. Charoensiriwath and J. C. Lu [14],  $\alpha_p > \beta_p$ . The same explanation can also be used for parameters  $\alpha_s$  and  $\beta_s$ .

A. Cost Structure and Profit Function

In this model, each firm has the same goal: to maximize its own profit. So we make the following assumptions for cost structure:

**ASSUMPTION 3.** Both members have perfect information of the demands and the cost structures of each member. Based on this, they try to maximize their own profit.

To specify enterprises' profit functions, we note that they carry two types of cost: product cost and service cost. In order to simplify calculation, we assume that product cost is 0, which do not affect the results of analysis. The service cost means the cost of providing service to customer. So we assume diminishing returns of service. Just as described by A. A. Tsay and N. Agrawal[13], we specify it in Assumption 4.

**ASSUMPTION 4.** Cost of service has a decreasing return property: the next dollar invested produce less unit of service than the last dollar. It means that it becomes more expensive to provide the next unit of service.

This diminishing return of service can be described by the quadratic form of service cost. We assume that the cost of providing  $s'_i$  units of service is  $\frac{1}{2} \eta'_i s'^2_i$ . The enterprise's profit function can be described as:

$$\Pi_i = p_i Q_i - \frac{1}{2} \eta'_i s'^2_i \tag{2}$$

As substitutable products, the relative importance of service and price to demand are probably equal, so we make the Assumptions 5:

**ASSUMPTION 5.** The relative importance of service and price to demand are equal to product,

which means  $\frac{\alpha_p}{\alpha_s} = \frac{\beta_p}{\beta_s}$ .



We define  $\alpha_s = k\alpha_p, \beta_s = k\beta_p$ ,  $k$  is a constant and  $k > 0$ .

We can also define  $\alpha_p = \alpha, \beta_p = \beta, s_i = ks_i'$ ,

$$s_j = ks_j', \eta_i = \frac{\eta_i'}{k^2}$$

So the equal (1) (2) are change as follow:

$$Q_i = a_i - \alpha p_i + \beta p_j + \alpha s_i - \beta s_j \quad (3)$$

$$\Pi_i = p_i Q_i - \frac{1}{2} \eta_i s_i^2 \quad (4)$$

As  $k$  is a constant and  $k > 0$ , so  $s_i, s_j, \eta_i$  have the same meanings with  $s_i', s_j', \eta_i'$ .

### 2.2 Strategic Interactions

Strategic interaction means how the game is solved. In this research, three scenarios take place:

(1) Nash Equilibrium: The two firms in the system has equal bargaining power, and try to maximize their own profit.

(2) Enterprise Alliance: The two enterprises adopt a cooperative strategy, which seek to maximize their total profit.

(3) Stackelberg: One firm is dominant comparing to the other and thus is the Stackelberg Leader.

## 3. NASH EQUILIBRIUM

### 3.1 Reaction Function

In this model, every firm has equal bargaining power and makes its decision simultaneously. The two enterprises try to maximize their profit function. The first order condition can be shown as:

$$\frac{\partial \Pi_i}{\partial p_i} = a_i - 2\alpha p_i + \beta p_j + \alpha s_i - \beta s_j = 0$$

$$\frac{\partial \Pi_i}{\partial s_i} = \alpha p_i - \eta_i s_i = 0$$

Where  $i = 1, 2, j = 3 - i$ .

Then we should check the Hessian for optimality. We have

$$\frac{\partial^2 \Pi_i}{\partial p_i^2} = -2\alpha, \frac{\partial^2 \Pi_i}{\partial s_i^2} = -\eta_i, \frac{\partial^2 \Pi_i}{\partial p_i \partial s_i} = \alpha.$$

When  $\eta_i > \frac{\alpha}{2}$ , the Hessian is a negative definite matrix and the second order condition is satisfied. Therefore, the  $p_1, p_2$  and  $s_1, s_2$  calculated are the optimal reaction functions for the enterprises.

Using the first and second order optimality conditions above, we have the expression for the firm's reaction functions as follow:

$$s_i^{N*} = \alpha T_i \quad (7)$$

$$p_i^{N*} = \eta_i T_i \quad (8)$$

$$Q_i^{N*} = \alpha \eta_i T \quad (9)$$

$$\Pi_i^{N*} = (\eta_i - \frac{1}{2}\alpha) \eta_i \alpha T_i^2 \quad (10)$$

Where  $T_i$  are constants defined as follow:

$$T_i = \frac{\alpha(2\eta_j - \alpha)a_i - \beta(\alpha - \eta_j)a_j}{\alpha^2(2\eta_i - \alpha)(2\eta_j - \alpha) - \beta^2(\alpha - \eta_i)(\alpha - \eta_j)}$$

### 3.2 Numerical Studies

Because production technology could improve or consumption environment may change sometimes, so this section, we use numerical approach to study the behavior of firm when environment change. We explore how profit is affected by the changes of some parameters. Thus, it can help us to understand the robustness of our results to the changes.

#### A. Numerical $a_i$

OBSERVATION. 3.1 The increase of market base of product  $i$  brings in more revenue for product  $i$ . Namely,  $\frac{\partial \Pi_i}{\partial a_i} > 0$ .

Proof:

Because  $\eta_j > \frac{\alpha}{2}$ , which means  $\alpha(2\eta_j - \alpha) > 0$ ,

so  $\frac{\partial \Pi_i}{\partial a_i} > 0$ .

OBSERVATION. 3.2 If  $\eta_j > \alpha$ , The increase of market base of product  $i$  also brings in more revenue for substitutable product  $j$ . Namely,  $\frac{\partial \Pi_j}{\partial a_i} > 0$ . While, if  $\eta_j < \alpha$ , it will damage

substitutable product  $j$  profit. Namely,  $\frac{\partial \Pi_j}{\partial a_i} < 0$

Proof:

When  $\eta_j > \alpha$ , which means  $\alpha - \eta_j < 0$ , so

$\frac{\partial \Pi_j}{\partial a_i} > 0$

When  $\eta_j < \alpha$ , which means  $\alpha - \eta_j > 0$ , so

$\frac{\partial \Pi_j}{\partial a_i} < 0$

The conclusions show that when  $\eta_j > \alpha$ , substitutable products have spillover effect. The market share expansion of one product, not only increases its own profit, but also indirectly

improves substitutable product's benefit. While, if  $\eta_j < \alpha$ , the results is contrast.

**B. Numerical  $\eta_i$**

OBSERVATION.3.3 The decrease of service cost  $\eta_i$  brings in more revenue for product  $i$  but reduces substitutable goods  $j$  profit.

Namely,  $\frac{\partial \Pi_i}{\partial \eta_i} < 0, \frac{\partial \Pi_j}{\partial \eta_i} > 0$ .

Proof: ellipsis.

Due to the results, there is a strong competition relationship between the enterprises. They try to enlarge their own market share and lower their own service costs, but at the same time, they would not like to help substitutable goods enterprise.

**C. Numerical Simulation Experiment**

In this section, we use numerical simulation experiment to check OBSERVATION.3.1, 3.2 and 3.3. All of the following results are derived by MATLAB.

We set  $a_i = a_j = 100, \eta_i = \eta_j = 6, \alpha = 10, \beta = 1$  as the initial data. We can get  $s_i^{N^*} = 41.67, p_i^{N^*} = 25, Q_i^{N^*} = 250, \Pi_i^{N^*} = 1041.67$ .

When the market base of product  $i$  increases form 100 to 110, we can get the changes of  $\Pi_i^{N^*}$  and  $\Pi_j^{N^*}$  as Figure 1.

From it, we can observe that increase of  $a_i$  benefits to product  $i$ , but is bad for substitutable product  $j$ , because  $\alpha - \eta_j > 0$ .

When the service cost  $\eta_i$  decreases form 6 to 5.5, we can get the changes of  $\Pi_i^{N^*}$  and  $\Pi_j^{N^*}$  as Figure 2.

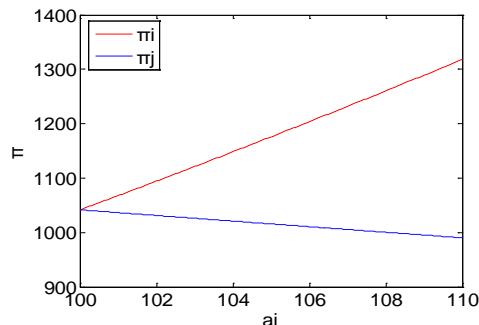


Figure 1: Changes Of  $\Pi_i^{N^*}$  And  $\Pi_j^{N^*}$  When  $a_i$  Increases

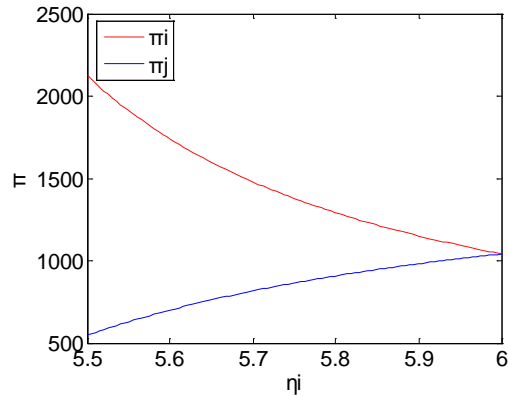


Figure 2: Changes Of  $\Pi_i^{N^*}$  And  $\Pi_j^{N^*}$  When  $\eta_i$  Decreases

The decrease of  $\eta_i$  brings profit to product  $i$  but is bad for substitutable product  $j$ .

**4. ENTERPRISE ALLIANCE**

**4.1 Reaction Function**

In this model, the two enterprises adopt a cooperative strategy, which seek to maximize their total profit. So the profit function is:

$$\Pi = p_i Q_i - \frac{1}{2} \eta_i s_i^2 + p_j Q_j - \frac{1}{2} \eta_j s_j^2 \tag{11}$$

The first order condition can be shown as:

$$\frac{\partial \Pi}{\partial s_i} = \alpha p_i - \eta_i s_i - \beta p_j = 0 \tag{12}$$

$$\frac{\partial \Pi}{\partial p_i} = a_i - 2\alpha p_i + 2\beta p_j + \alpha s_i - \beta s_j = 0 \tag{13}$$

Where  $i = 1, 2, j = 3 - i$ .

Then we check the Hessian for optimality. We

have  $\frac{\partial^2 \Pi}{\partial s_i^2} = -\eta_i, \frac{\partial^2 \Pi}{\partial s_i \partial s_j} = 0, \frac{\partial^2 \Pi}{\partial s_i \partial p_i} = \alpha$ ,

$\frac{\partial^2 \Pi}{\partial s_i \partial p_j} = -\beta, \frac{\partial^2 \Pi}{\partial p_i \partial p_j} = 2\beta, \frac{\partial^2 \Pi}{\partial p_i^2} = -2\alpha$ .

When  $2\alpha\eta_i\eta_j - \alpha^2\eta_i - \beta^2\eta_j > 0$  and  $(\alpha^2 - \beta^2) - 2\alpha(\eta_i + \eta_j) + 4\eta_i\eta_j > 0$ , the second order condition is satisfied and the Hessian is a negative definite matrix. Therefore, the  $p_i, p_j$  and  $s_i, s_j$  calculated are the optimal reaction functions for the enterprises.

Using the first and second order optimality conditions above, we have the following expression for the firm's reaction functions:

$$p_i^{A*} = \frac{1}{(\alpha^2 - \beta^2)} * \frac{(2\alpha\eta_i\eta_j - \alpha^2\eta_i - \beta^2\eta_j)a_i - \beta[\alpha(\eta_i + \eta_j) - 2\eta_i\eta_j]a_j}{(\alpha^2 - \beta^2) - 2\alpha(\eta_i + \eta_j) + 4\eta_i\eta_j} \quad (14)$$

$$s_i^{A*} = \frac{(2\eta_j - \alpha)a_i - \beta a_j}{(\alpha^2 - \beta^2) - 2\alpha(\eta_i + \eta_j) + 4\eta_i\eta_j} \quad (15)$$

$$Q_i^{A*} = \eta_i \frac{(2\eta_j - \alpha)a_i - \beta a_j}{(\alpha^2 - \beta^2) - 2\alpha(\eta_i + \eta_j) + 4\eta_i\eta_j} \quad (16)$$

$$\Pi_i^{A*} = \frac{\eta_i[\alpha(2\eta_j - \alpha)a_i^2 - 2\beta(\alpha - \eta_j)a_i a_j - \beta^2 a_j^2]}{2(\alpha^2 - \beta^2)[(\alpha^2 - \beta^2) - 2\alpha(\eta_i + \eta_j) + 4\eta_i\eta_j]} \quad (17)$$

#### 4.2 Numerical Studies

##### A. Numerical $a_i$

OBSERVATION. 4.1 The increase of market base of product  $i$  brings in more revenue for product  $i$  but reduces substitutable product  $j$  profit. Namely,  $\frac{\partial \Pi_i}{\partial a_i} > 0$   $\frac{\partial \Pi_j}{\partial a_i} < 0$ .

Proof: ellipsis.

The conclusion shows that there is a competition between the substitutable products.

##### B. Numerical $\eta_i$

OBSERVATION.4.2 The decrease of service cost  $\eta_i$  brings in more revenue for product  $i$  but is bad for substitutable goods  $j$ . Namely  $\frac{\partial \Pi_i}{\partial \eta_i} < 0$ ,

$$\frac{\partial \Pi_j}{\partial \eta_i} > 0.$$

Proof: ellipsis.

The conclusion also shows that there is a competition relationship between the enterprises. The result is similar with OBSERVATION. 3.3

##### C. Numerical Simulation Experiment

In this section, we use numerical simulation experiment to check OBSERVATION.4.1 and 4.2.

We also set  $a_i = a_j = 100$ ,  $\eta_i = \eta_j = 6$ ,  $\alpha = 10$ ,  $\beta = 1$  as the initial data. We can

get  $s_i^{N*} = 33.33$ ,  $p_i^{N*} = 22.22$ ,  $Q_i^{N*} = 200$ ,  $\Pi_i^{N*} = 1111.11$ .

When the market base of product  $i$  increases from 100 to 110, we can get the changes of  $\Pi_i^{A*}$  and  $\Pi_j^{A*}$  as Figure 3.

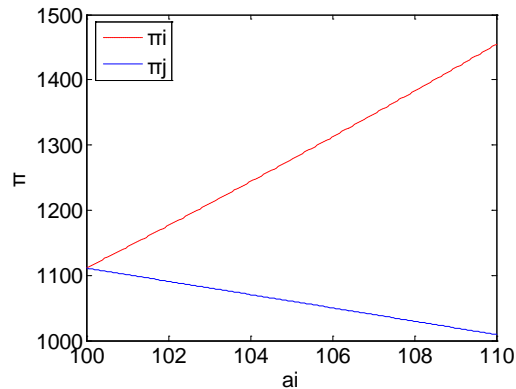


Figure 3: Changes Of  $\Pi_i^{A*}$  And  $\Pi_j^{A*}$  When  $a_i$  Increases

From it, we can observe that the increase of  $a_i$  benefits to itself but reduces the substitutable product  $j$  profit.

When the service cost  $\eta_i$  decreases from 6 to 5.5, we can get the changes of  $\Pi_i^{A*}$  and  $\Pi_j^{A*}$  as Figure 4.

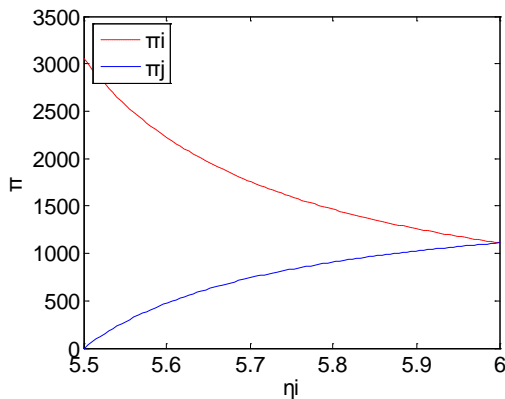


Figure 4: Changes Of  $\Pi_i^{A*}$  And  $\Pi_j^{A*}$  When  $\eta_i$  Decreases

The decrease of  $\eta_i$  brings profit to product  $i$  but is bad for substitutable product  $j$ .



5. STACKELBERG

5.1 Reaction Function

The Stackelberg scenario arises in markets where firm  $j$  have the initiative compared to firm  $i$ . The problem is solved backwards. First, the firm  $i$ ' problem is solved to derive the response function conditional on the price and service level chosen by the firm  $j$ . Then the firm  $j$ ' problem is solved given that the firm  $j$  knows how the firm  $i$  would react to the price and service level he sets.

A. Firm  $i$  Reaction Function

First, firm  $j$  set price  $p_j$  service level  $s_j$  as the earlier decision. Then, firm  $i$  in this game choose  $p_i, s_i$  to maximize his equilibrium profit.

The first order conditions are

$$\frac{\partial \Pi_i}{\partial p_i} = a_i - 2\alpha p_i + \beta p_j + \alpha s_i - \beta s_j = 0$$

$$\frac{\partial \Pi_i}{\partial s_i} = \alpha p_i - \eta_i s_i = 0$$

Then we check the Hessian for optimality. We have  $\frac{\partial^2 \Pi_i}{\partial p_i^2} = -2\alpha, \frac{\partial^2 \Pi_i}{\partial s_i^2} = -\eta_i, \frac{\partial^2 \Pi_i}{\partial p_i \partial s_i} = \alpha$ . So

when  $\eta_i > \frac{\alpha}{2}$ , the Hessian is a negative definite matrix and the second order condition is satisfied.

Using the first and second order conditions above, the response price and service level for firm  $i$  can be derived

$$s_i^{S*} = \frac{a_i + \beta p_j^{S*} - \beta s_j^{S*}}{2\eta_i - \alpha} \tag{20}$$

$$p_i^{S*} = \frac{\eta_i a_i + \beta p_j^{S*} - \beta s_j^{S*}}{\alpha} \tag{21}$$

B. Firm  $j$  Reaction Function

Having the information about the reaction functions of firm  $i$ , the firm  $j$  would then use them to maximize its profit.

$$\begin{aligned} \Pi_j &= p_j Q_j(p_j, p_i(p_j, s_j), s_j, s_i(p_j, s_j)) - \frac{1}{2} \eta_j s_j^2 \\ &= p_j Q_j(p_j, s_j) - \frac{1}{2} \eta_j s_j^2 \end{aligned} \tag{22}$$

The firm  $j$  in this game choose price  $p_j$  and service level  $s_j$  to maximize his equilibrium profit.

The first order condition can be shown as

$$\frac{\partial \Pi_j}{\partial p_j} = Q_j(p_j, s_j) + p_j \frac{\partial Q_j(p_j, s_j)}{\partial p_j} = 0 \tag{23}$$

$$\frac{\partial \Pi_j}{\partial s_j} = p_j \frac{\partial Q_j(p_j, s_j)}{\partial s_j} - \eta_j s_j = 0 \tag{24}$$

Then we check the Hessian for optimality. We have

$$\frac{\partial^2 \Pi_j}{\partial p_j^2} = -2 \frac{\alpha \beta^2 + \alpha^2 (2\eta_i - \alpha) - \beta^2 \eta_i}{\alpha (2\eta_i - \alpha)}$$

$$\frac{\partial^2 \Pi_j}{\partial s_j^2} = -\eta_j,$$

$$\frac{\partial^2 \Pi_j}{\partial p_j \partial s_j} = \frac{\alpha \beta^2 + \alpha^2 (2\eta_i - \alpha) - \beta^2 \eta_i}{\alpha (2\eta_i - \alpha)}$$

When  $\alpha(2\eta_i - \alpha)(2\eta_j - \alpha) - \beta^2 \alpha + \beta^2 \eta_i > 0$ , the Hessian is a negative definite matrix and the second order condition is satisfied.

Using the first and second order conditions above, the response price and service level for firm  $j$  can be derived

$$s_j^{S*} = \frac{\alpha(2\eta_i - \alpha)a_j + \beta(\eta_i - \alpha)a_i}{\alpha(2\eta_i - \alpha)(2\eta_j - \alpha) - \beta^2 \alpha + \beta^2 \eta_i} \tag{25}$$

$$p_j^{S*} = \frac{\alpha \eta_j (2\eta_i - \alpha)}{\alpha^2 (2\eta_i - \alpha) + \beta^2 \alpha - \beta^2 \eta_i} * \frac{\alpha(2\eta_i - \alpha)a_j + \beta(\eta_i - \alpha)a_i}{\alpha(2\eta_i - \alpha)(2\eta_j - \alpha) - \beta^2 \alpha + \beta^2 \eta_i} \tag{26}$$

$$Q_j^{S*} = \eta_j \frac{\alpha(2\eta_i - \alpha)a_j + \beta(\eta_i - \alpha)a_i}{\alpha(2\eta_i - \alpha)(2\eta_j - \alpha) - \beta^2 \alpha + \beta^2 \eta_i} \tag{27}$$

$$\Pi_j^{S*} = \frac{\eta_j}{2[\alpha(2\eta_i - \alpha)(2\eta_j - \alpha) - \beta^2 \alpha + \beta^2 \eta_i]} * \frac{[\alpha(2\eta_i - \alpha)a_j + \beta(\eta_i - \alpha)a_i]^2}{\alpha^2 (2\eta_i - \alpha) + \beta^2 \alpha - \beta^2 \eta_i} \tag{28}$$

Using the results, we can also obtain the response price and service level of firm  $i$ .

$$s_i^{S*} = \alpha \frac{M_i}{N_i} \tag{29}$$

$$p_i^{S*} = \eta_i \frac{M_i}{N_i} \tag{30}$$

$$Q_i^{S*} = \alpha \eta_i \frac{M_i}{N_i} \tag{31}$$

$$\Pi_i^{S*} = \frac{1}{2} \alpha \eta_i (2\eta_i - \alpha) \left( \frac{M_i}{N_i} \right)^2 \quad (32)$$

Where  $M_i, N_i$  are constants defined as follow:

$$M_i = [\alpha^2 (2\eta_i - \alpha)(2\eta_j - \alpha) - \beta^2 (\eta_i - \alpha)(\eta_j - \alpha)] a_i + [\alpha \beta (2\eta_i - \alpha)(\eta_j - \alpha) - \beta^2 \alpha + \beta^2 \eta_i] a_j$$

$$N_i = [\alpha (2\eta_i - \alpha)(2\eta_j - \alpha) - \beta^2 \alpha + \beta^2 \eta_i] * [\alpha^2 (2\eta_i - \alpha) + \beta^2 \alpha - \beta^2 \eta_i]$$

We also set  $a_i = a_j = 100, \eta_i = \eta_j = 6, \alpha = 10, \beta = 1$  as the initial data. We can get

$$s_i^{S*} = 40.84, p_i^{S*} = 24.51, Q_i^{S*} = 245.10, \Pi_i^{S*} = 1001.22, s_j^{S*} = 44.44, p_j^{S*} = 26.14, Q_j^{S*} = 266.67, \Pi_j^{S*} = 1045.75.$$

When the market base of product  $i$  increases from 100 to 110, we can get the changes of  $\Pi_i^{S*}$  and  $\Pi_j^{S*}$  as Figure 5.

## 5.2 Numerical Studies

### A. Numerical $a_i$ and $a_j$

OBSERVATION. 5.1 The increase of market base of product  $i$  brings in more revenue for product  $i$ . Namely  $\frac{\partial \Pi_i}{\partial a_i} > 0$ .

Proof: ellipsis.

OBSERVATION. 5.2 When  $\eta_j > \alpha$ , The increase in market base of product  $i$  also brings in more revenue for substitutable product  $j$ . Namely,  $\frac{\partial \Pi_j}{\partial a_i} > 0$ . While, if  $\eta_j < \alpha$ , it will reduce

substitutable product  $j$  profits. Namely,  $\frac{\partial \Pi_j}{\partial a_i} < 0$ .

Proof: ellipsis

The influences of change of  $a_j$  to  $\Pi_j$  and  $\Pi_i$  are the same with  $a_i$ . The result is similar with OBSERVATION. 3.1 and OBSERVATION. 3.2

### B. Numerical $\eta_i$

OBSERVATION.5.3 The decrease in service cost  $\eta_i$  brings in more revenue for product  $i$  but is bad for substitutable goods  $j$ . Namely  $\frac{\partial \Pi_i}{\partial \eta_i} < 0$ ,

$$\frac{\partial \Pi_j}{\partial \eta_i} > 0.$$

The influences of change of  $a_j$  to  $\Pi_j$  and  $\Pi_i$  are the same with  $a_i$ . The result is similar with OBSERVATION. 3.3 and OBSERVATION. 4.2.

### C. Numerical Simulation Experiment

In this section, we use numerical simulation experiment to check OBSERVATION.5.1 and 5.2.

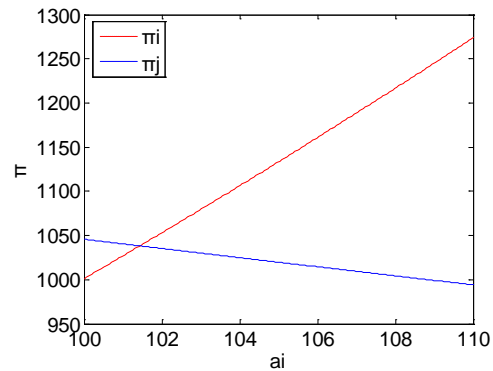


Figure 5: Changes Of  $\Pi_i^{S*}$  And  $\Pi_j^{S*}$  When  $a_i$  Increases

From it, we can observe that increase of  $a_i$  benefits to product  $i$ , but is bad for substitutable product  $j$ , because  $\alpha - \eta_j > 0$ .

When the market base of product  $j$  increases from 100 to 110, we can get the changes of  $\Pi_i^{S*}$  and  $\Pi_j^{S*}$  as Figure 6.

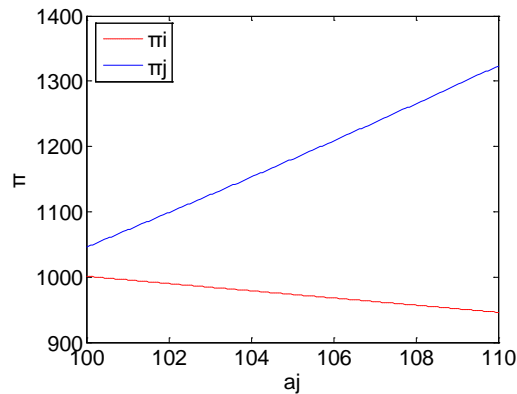


Figure 6: Changes Of  $\Pi_i^{S*}$  And  $\Pi_j^{S*}$  When  $a_j$  Increases

From it, we can observe that increase of  $a_j$  benefits to product  $j$ , but is bad for substitutable product  $i$ , because  $\alpha - \eta_i > 0$ .

When the service cost  $\eta_i$  decreases form 6 to 5.5, we can get the changes of  $\Pi_i^{S*}$  and  $\Pi_j^{S*}$  as Figure 7.

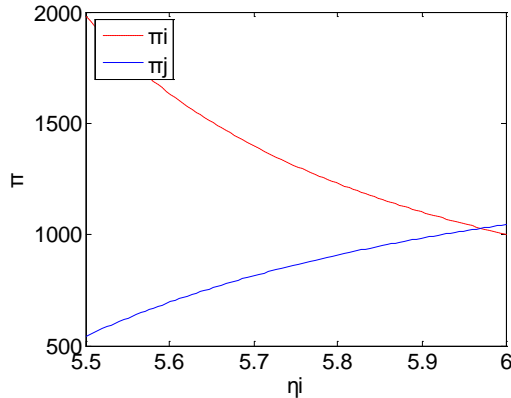


Figure 7: Changes Of  $\Pi_i^{S*}$  And  $\Pi_j^{S*}$  When  $\eta_i$  Decreases

The decrease of  $\eta_i$  brings profit to product  $i$  but is bad for substitutable product  $j$ .

When the service cost  $\eta_j$  decreases form 6 to 5.5, we can get the changes of  $\Pi_i^{S*}$  and  $\Pi_j^{S*}$  as Figure 8.

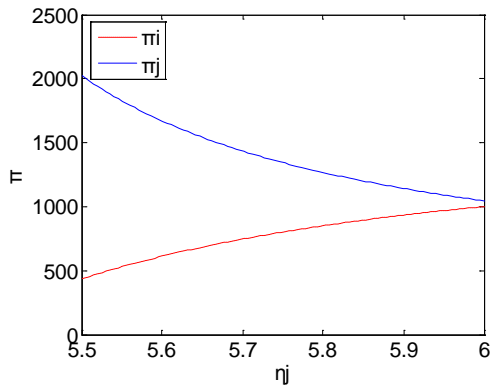


Figure 8 Changes Of  $\Pi_i^{S*}$  And  $\Pi_j^{S*}$  When  $\eta_j$  Decreases

The decrease of  $\eta_j$  brings profit to product  $j$  but is bad for substitutable product  $i$ .

## 6. COMPARISON OF RESULTS

In this section, we compare the results from the three different scenarios to observe the effect of competition strategy on prices, service levels, and

profits of each member. In order to separate the effects of different competition strategy from the effects of the differences of parameters, we assume enterprises have same parameters (same market base and service cost). This assumption simplifies the results by setting  $a_i = a_j = a$ ,  $\eta_i = \eta_j = \eta$ .

**OBSERVATION.6.1** When the two manufacturers are identical,  $s_i^{N*} > s_i^{A*}$ ,  $Q_i^{N*} > Q_i^{A*}$ . If  $\alpha - \beta - \eta > 0$ ,  $p_i^{N*} > p_i^{A*}$ ; otherwise,  $p_i^{N*} < p_i^{A*}$ .

**Proof:**

According to the equations (7), (15)

$$s_i^{N*} = \frac{a}{2\eta - \alpha + \beta - \frac{\beta\eta}{\alpha}}, s_i^{A*} = \frac{a}{2\eta - \alpha + \beta}$$

So  $s_i^{N*} > s_i^{A*}$

According to the equations (9), (16)

$$Q_i^{N*} = \frac{\eta a}{2\eta - \alpha + \beta - \frac{\beta\eta}{\alpha}}, Q_i^{A*} = \frac{\eta a}{2\eta - \alpha + \beta}$$

So  $Q_i^{N*} > Q_i^{A*}$

According to the equations (8), (14)

$$p_i^{N*} = \frac{\eta a}{2\eta\alpha - \alpha^2 + \alpha\beta - \beta\eta}$$

$$p_i^{A*} = \frac{\eta a}{2\eta\alpha - \alpha^2 + \alpha\beta - \beta\eta + \beta(\alpha - \beta - \eta)}$$

So if  $\alpha - \beta - \eta > 0$ ,  $p_i^{N*} > p_i^{A*}$ ; otherwise,  $p_i^{N*} < p_i^{A*}$ .

This proposition states that when the enterprises form a alliance, consumers can receive lower services than in Nash Equilibrium. And in this case, the demands also decrease, which means there will be less consumers to buy the products.

**OBSERVATION.6.2** When the two manufacturers are identical,  $\Pi_i^{N*} < \Pi_i^{A*}$

**Proof:**

According to the equations (10), (17)

$$\Pi_i^{N*} = \frac{1}{2} \frac{\eta a^2}{2\eta\alpha - \alpha^2 + 2\alpha\beta - 2\eta\beta + \frac{\beta^2(\alpha - \eta)^2}{\alpha(2\eta - \alpha)}}$$

$$\Pi_i^{A*} = \frac{1}{2} \frac{\eta a^2}{2\eta\alpha - \alpha^2 + 2\alpha\beta - 2\eta\beta - \beta^2}$$

So  $\Pi_i^{N*} < \Pi_i^{A*}$

This proposition means when the two firms in the system has equal bargaining power, if the two enterprises form a alliance, they can earn more profit, which means they can achieve pareto improvement. So the two enterprises are willing to





cooperate. But as discuss in OBSERVATION. 4.1 and 4.2, their cooperation is only focus on price and service level, but not includes enlarging market share or reducing service costs.

OBSERVATION.6.3 When the two manufacturers are identical,  $\Pi_i^{S*} < \Pi_i^{A*}$ .

Proof: ellipsis.

This proposition means when firm  $j$  have the initiative compared to  $i$ , firm  $i$ ' profit will reduce comparing to Nash Equilibrium because of its disadvantage.

OBSERVATION.6.4 When the two manufacturers are identical, if

$$\beta > \frac{\alpha(2\eta - \alpha)(2\alpha - \eta)}{2\eta^2}, \quad \Pi_j^{A*} < \Pi_j^{S*}; \quad \text{if}$$

$$\beta < \frac{\alpha(2\eta - \alpha)(2\alpha - \eta)}{2\eta^2}, \quad \Pi_j^{A*} > \Pi_j^{S*}.$$

Proof: ellipsis.

This proposition means when the firm have the initiative, if  $\beta$  is big enough, it will use the advantage to obtain more profits, conversely, it will give up the advantage and turn to seek the cooperation.

## 7. CONCLUSION

The primary objective of this article is to highlight the importance of service with two substitutable products, which face end consumers who are sensitive to both prices and services. We have explored the problem through three different scenarios. Using game theoretic approach, our analyses have found a number of insights into economic behavior of firms, which could be used as the basis for theoretical research in the future.

In this paper, we obtain expressions for optimal prices, service levels, demand quantities and profits for each product. Then we have analyzed the results and discuss the influence of each parameter. Our results show that as one product increases its market base, or has economic advantage in providing service, its profits is increasing while the other one suffer a loss. The results also show that the consumer can receive less service in Enterprise Alliance. Firms also can earn more profit in this scenario. We also find that when the firm possesses more bargaining power, it will use it to obtain more profits if  $\beta$  is big enough, conversely, it will give up and turn to seek cooperation.

In this paper we only study where there are two product enterprises. Other possibilities may include the situation where there are more enterprises. Another approach is to extend the model over

multiple periods to study temporal dynamics. In this paper, both the members have perfect information of the demands and the cost structures of each member. We also can research on the reaction and countermeasure of enterprises where information is asymmetric.

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## REFERENCES

- [1] A. M. McGahan, P. Ghemawat, "Competition to Retain Customers", *Marketing Science*, Vol. 13, No. 2, 1994, pp. 165-176.
- [2] B. D. Borger, K. V. Dender, "Prices, Capacities and Service Levels in a Congestible Bertrand Duopoly", *Journal of Urban Economics*, Vol. 60, No. 2, 2006, pp. 264-283.
- [3] J. C. Darian, A. R. Wiman, L. A. Tucc, "Retail Patronage Intentions: The Relative Importance of Perceived Prices and Salesperson Service Attributes", *Journal of Retailing and Consumer Services*, Vol. 12, No. 1, 2005, pp. 15-23.
- [4] R. C. Jou, S. H. Lam, D. A. Hensher, C. C. Chen, C. W. Ku, "The Effect of Service Quality and Price on International Airline Competition", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 44, No. 4, 2008, pp. 580-592.
- [5] T. J. Xiao, D. Q. Yang, "Price And Service Competition of Supply Chains with Risk-averse Retailers under Demand Uncertainty", *International Journal of Production Economics*, Vol. 114, No. 1, pp. 187-200, 2008.
- [6] G. Iyer, "Coordinating Channels Under Price and Nonprice Competition, *Marketing Science*", Vol. 17, No. 4, 1998, pp. 338-355.
- [7] S. Mukhopadhyay, D. Yao, X. Yue, "Information Sharing of Value-adding Retailer in a Mixed Channel Hi-tech Supply Chain", *Journal of Business Research*, Vol. 61, No. 9, 2008, pp. 950-958.
- [8] K. D. Cattani, W. G. Gilland, J. M. Swaminathan, "Boiling Frogs: Pricing Strategies for a Manufacturer Adding a Direct Channel That Competes With the Traditional Channel", *Production and Operations Management*, Vol. 15, No. 1, 2006, pp. 40-57.
- [9] K. Y. Chen, M. Kaya, Ö. Özer, "Dual Sales Channel Management with Service



- Competition”, *Manufacturing & Service Operations Management*, Vol. 10, No. 4, 2008, pp. 654-675.
- [10] C. Koulamas, A Newsvendor Problem with Revenue Sharing and Channel Coordination. *Decision Sciences*, Vol. 37, No. 1, 2006, pp. 91-100.
- [11] P. Kumar, “The Competitive Impact of Service Process Improvement: Examining Customers’ Waiting Experiences in Retail Markets”, *Journal of Retailing*, Vol. 81, No. 3, 2005, pp. 171-180.
- [12] H. Zhang, “Vertical Information Exchange in a Supply Chain with Duopoly Retailers”, *Production and Operations Management*, Vol. 11, No. 4, 2002, pp. 531-546.
- [13] A. A. Tsay, N. Agrawal, “Channel Dynamics under Price and Service Competition”, *Manufacturing & Service Operations Management*, Vol. 2, No. 4, 2000, pp.372-391.
- [14] C. Charoensiriwath, J. C. Lu, “Competition Under Retail Price and Manufacturer Service”, *Economic Modelling*, Vol. 28, no. 3, 2011, pp. 1256-1264.