Journal of Theoretical and Applied Information Technology
15th December 2012. Vol. 46 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

A TEXTURE IMAGE DENOISING MODEL USING THE COMBINATION OF TENSOR VOTING AND TOTAL VARIATION MINIMIZATION

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ABSTRACT

Combined with human vision principle, this paper firstly gives the definition of image frequency based on image local gradient and uses it to replace the image gradient in the traditional total variation (TV) model. And then tensor voting principle is introduced into the TV model and a novel texture image denoising method using the combination of tensor voting and total variation minimization is proposed. In the new model an image structure saliency function is given to replace the lagrangian multiplier λ in TV model, which can adjust the regularizing term and fidelity term according to the different areas of image structure features. Theoretical analysis and numerical experiment show that compared with other existing approaches the new model has an obvious anti-jamming capability and can accurately and subtly describe the sharp edges, feature structures and smooth areas, and can overcome staircase effect and over-smoothing generated by other TV models. Especially for the images with rich texture features and low signal to noise ratio (SNR), it can remove the noise while preserving significant image details and important characteristics and improve the image denoising effect.

Keywords: *Partial Differential Equation(PDE),Total Variation, Tensor Voting, Image Frequency, Texture Image Denoising*

1. INTRODUCTION

Image denoising is the fundamental problems in the field of digital image processing and computer vision. Most of the image information exists in the region of edges and in some small details, therefore, it is hoped that we can remove the image blurs and noises while keeping important features and sharp edges. And this is also the main task in image denoising and restoration. At present, the image diffusion method using partial differential equation (PDE) and total variation minimization becomes a main trend technology in the field of image denoising.

Image denoising methods can be divided into two classes according to the PDE derivation mode. One is procedure-oriented denoising method. Its denoising working principle is according to the evolution theory in physics which includes diffusion model、P-M model[1]、Catté model[2] and high order partial differential equation model[3],etc. It has been proved that P-M model is instability because its steady-state solution does not have the continuous dependency on the initial conditions, that is, its initial-value problem is ill-posed. Meanwhile, it is easy to be interfered by noise and the removal effect of noise with large gradient is not ideal. Compared with Catté model and P-M model, the former can effectively avoid the adverse effect of the large gradient noise on the diffusion and improve the denoising ability, to some extent, it can preferably protect the significant edges and detail information. But Catté model also has its disadvantages, that is, it is difficult to choose a proper scale factor which may tremendously influence Gaussian pretreatment filtering on the image smoothing degree. Secondly, because the edge has a larger gradient value which leads to the diffusion intensity close to zero, so there is no diffusion in the region of edges and it can not remove the noise of the image edges. High order partial differential equation model has the ability of noise removal, edge protection and effectively avoid the generation of staircase effect. But its restored image may generate many spots.

The other kind of methods is object-oriented denoising method, also called denoising method

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

based on variation theory. It gets an energy functional through the consideration of the properties of the image processing targets, and then solves the energy functional to obtain the restored image. Rudin, Osher and Fatemi proposed a ROF model based on total variation [4] which converted image denoising problem into a functional extreme value solution problem by introducing energy function so that achieved the purpose of noise removal. Later, many improved models based on ROF have been presented [5-12].

All in all, the above two kinds of denoising methods take image gradient as the only image characteristics description which easily makes large gradient noise to be kept as edge and affects the vision effect. As we know, on the one hand, the image local structure information can not only be manifested as gradient because gradient can not subtly describe the textures and corners; on the other hand, the derived direction information estimated according to gradient operation is inaccurate in the noise interference. So if the diffusion filtering method excessively depends on gradient, it is hard to avoid blurring the edges and details.

In order to better describe the texture image detail features, in this paper, firstly, we give a definition of image frequency based on image local gradient, and then combining with human vision principle, we use image frequency to take the place of image gradient and introduce tensor voting principle into the traditional ROF TV model, thus, we construct a new texture image denoising model using the combination of tensor voting and total variation minimization. Extensive experimental results show that the new model can adaptively adopt big denoising intensity to remove noise in the smooth area while small denoising intensity to preserve the edges; therefore, it obtains good noise suppression, edge and detail feature preservation effects.

2. LOCAL STRUCTURE FEATURES REPRESENTATION

2.1 Definition of Image Frequency

In ref.[13],the author gives the other form of function frequency. We let periodic function $f(x)=\alpha\sin(Tx)$, $x\in[0,2\pi]$, where positive integer T is the frequency of periodic function $f(x)$, and $\alpha\neq 0$.

The frequency of $f(x)$ can also be denoted

$$T=\frac{\left(\int_0^{2\pi}(\nabla f(x))^2dx\right)^{\frac{1}{2}}}{\left(\int_0^{2\pi}f^2(x)dx\right)^{\frac{1}{2}}} \tag{1}$$

where $\nabla f(x)=\frac{\partial f(x)}{\partial x}=\alpha T\cos(Tx)$.

Suppose that $(L^2(\Omega),\langle\cdot,\cdot\rangle)$ is the Hilbert space of continuous function with n dimensions. For $f(x),g(x)\in L^2(\Omega)$ we can define the inner product as

$$\langle f(x),g(x)\rangle=\int_\Omega f(x)g(x)dx \qquad \forall f(x),g(x)\in L^2(\Omega) \tag{2}$$

Here, we define norm as

$$\|f(x)\|=\sqrt{\langle f(x),f(x)\rangle} \tag{3}$$

and the gradient in the point of x can be defined as

$$\nabla f(x)=\frac{\partial f(x)}{\partial x}=\left(\frac{\partial f(x)}{\partial x_1},\frac{\partial f(x)}{\partial x_2},\cdots,\frac{\partial f(x)}{\partial x_n}\right) \tag{4}$$

Therefore, function frequency based on image gradient can be denoted as

$$F(f)=\frac{\|\nabla f(x)\|}{\|f(x)\|} \tag{5}$$

According to the Weber-Fechner's law, the minimum perceptible brightness difference human vision can distinguish in the different background luminance u is $\delta_u=k\cdot u$ (k is a constant)[14]. Based on this principle, so if an image $u(x,y)$ is regarded as the function in (5), it minimizes the total variation model subjected to the known noise characteristics, but it is known that the same gradient module has different vision effect in different background luminance. Therefore, the image frequency based on gradient can better describe the image vision effect than simple gradient. Combined with Weber-Fechner's law, we introduce image frequency into TV model to obtain the better restoration effect by the minimization of TV model.

2.2 Matrix of Image Local Structure Features

The traditional diffusion behavior based on PDE or total variation usually controls by image gradient module while image gradient can not fully express the image local structure information, such as edge, corner and other details. Using the matrix of local features of image $u(x,y)$ as follow, we

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

can get more abundant local structure information [15].

$$T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} u_x^2 & u_{xy} \\ u_{xy} & u_y^2 \end{pmatrix} * G_t \qquad (6)$$

where

$a = u_x^2 = \left(\frac{\partial u(x,y)}{\partial x}\right)^2$, $c = u_y^2 = \left(\frac{\partial u(x,y)}{\partial y}\right)^2$, $b = u_{xy} = \frac{\partial^2 u(x,y)}{\partial x \partial y}$, G_t is the Gaussian kernel function.

Through the calculation, we can get the two eigenvalues of T

$$\mu_{1,2} = \frac{1}{2}\left(a + c \pm \sqrt{(a-c)^2 + 4b^2}\right) \qquad (7)$$

As we know, the convolution of a 2-dimentional function and a 2-dimentional Gaussian kernel function is equivalent to the weighted average in the neighborhood of each point of scale t . Therefore, the eigenvalues and the image structure features have the following relations:

A. When the two eigenvalues are large and approximately equal, that is $\mu_1 \approx \mu_2 >> 0$, it means that the gray values have faster change in the two orthogonal directions; so we can say it is image corner or T shape local structure features.

B. When one eigenvalue is large while the other is small, that is $\mu_1 >> \mu_2$ or $\mu_1 << \mu_2$, it means that the gray value change rate in one direction is larger while the other direction is smaller, and this indicates that the image has obviously sharp edges and texture structure features.

C. When the two eigenvalues are small and approximately equal, that is $\mu_1 \approx \mu_2 \approx 0$,it means that the image gray value change rate in this point is small in its neighborhood of any direction, so we can think it is the smooth area.

Through the analysis of local feature matrix T ,we can obtain more abundant image local structure information, and we can use it to control the diffusion process in order to implement noise removal and edge and detail structure features protection simultaneously.

3. A TEXTURE IMAGE DENOISING MODEL USING THE COMBINATION OF TENSOR VOTING AND TOTAL VARIATION MINIMIZATION

3.1 Image Denoising Model Based on Energy Minimization

The classical variation denoising algorithm is Total Variation (TV) model, which is proposed by Rudin, Osher and Fatemi, also called ROF TV model [4]. This algorithm seeks an equilibrium state (minimal energy) of an energy functional comprised of the TV norm of the image. They put forward the energy minimization equation in two dimensional continuous frameworks:

$$\min E(u) = \iint_\Omega |\nabla u|\, dxdy \qquad (8)$$

where u subjects to:

$$\begin{cases} \iint_\Omega u(x,y)dxdy = \iint_\Omega u_0(x,y)dxdy \\ \frac{1}{|\Omega|}\iint_\Omega (u(x,y) - u_0(x,y))^2 dxdy = \sigma^2 \end{cases} \qquad (9)$$

Suppose the noise is approximated by an additive white Gaussian process of zero mean and standard deviation σ^2 .By introducing the Lagrange multiplier λ , the problem can be converted to a corresponding minimization problem without constraints:

$$\min E(u) = \iint_\Omega |\nabla u|\, dxdy + \frac{\lambda}{2}\iint_\Omega |u(x,y) - u_0(x,y)|^2\, dxdy \qquad (10)$$

where the Lagrange multiplier $\lambda > 0$ is given.

On the right side of equation (10), the first term is a regularizing (smooth) item, to remove noise or small details in the minimization process. The second term is fidelity (approximation) item, to keep important features and sharp edges and reduce the image distortion degree. When the value of λ is small, the correspondence smooth item $\iint_\Omega |\nabla u| dxdy$ plays a more important role which makes the desired image u more smooth. However, when λ gets bigger, the correspondence fidelity item $\iint_\Omega |u(x,y) - u_0(x,y)|^2\, dxdy$ becomes the dominant part which means smaller diffusion and makes the desired image u closer to its initial value, thus the image edges are protected.

The corresponding Euler-Lagrange equation of model (10) is:

$$-\nabla \cdot \left(\frac{1}{|\nabla u|}\nabla u\right) + \lambda(u - u_0) = 0 \qquad (11)$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

From (11), we note that this approach in essence is an anisotropic diffusion and its diffusion coefficient is equal to $1/\nabla u$ .Moreover, from the local coordinate expression we notice that the diffusion operator in TV model only proceeds along the vertical direction (the edge tangent direction) of the image gradient, while without diffusion along the gradient direction. Therefore, the noise can be suppressed and meanwhile image edges and other feature information can be preserved well. However, the edge tangent direction derived from image smooth area does not actually exist. If the diffusion behavior only diffuses along the edge tangent direction in the smooth area, the noise can not be suppressed sufficiently, and it may be easy to cause fake edges and staircase effect. On the other hand, the value of Lagrangian multiplier λ depends on noise intensity and its gradient module, so it has a bad anti-jamming capability. In order to conquer these problems, we introduce tensor voting into TV model and construct an image structure saliency function $c(s)$ included image detail features to replace the Lagrangian multiplier λ in TV model for the purpose of balancing the effect of regularizing term and fidelity item.

3.2 Tensor Voting Theory

Tensor voting is method to extract significant geometric features such as regions, curves, surfaces, and the intersection between them [16,17]. The spectrum theorem states that any tensor T can be expressed as a linear combination of its eigenvalues and eigenvectors

$$T = (\mu_1 - \mu_2)\upsilon_1\upsilon_1^T + \mu_2(\upsilon_1\upsilon_1^T + \upsilon_2\upsilon_2^T) \quad (12)$$

where μ_1, μ_2 are the eigenvalues, and υ_1, υ_2 are the eigenvectors corresponding to μ_1, μ_2 respectively. $\upsilon_1\upsilon_1^T$ describes a stick tensor which shows its curve characteristics, $\upsilon_1\upsilon_1^T + \upsilon_2\upsilon_2^T$ describes a ball tensor which shows its node characteristics. Intuitively, equation (12) decomposes a second order symmetric definite tensor into a linear combination of stick tensor and ball tensor, and its geometric construction is shown in Figure 1.

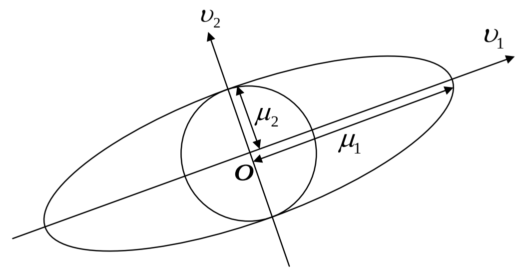


Figure 1: The Geometric Construction Of Second Order Symmetric Definite Tensor

From equation (12), we note that the first item (stick tensor) plays a main role in the above equation when $\mu_1 >> \mu_2$, on the contrary, the second item (ball tensor) plays a main role when μ_2 has a large value.

Here, we define a coherence function to represent image local structure features

$$c(\mu) = |\mu_1\mu_2| + |\mu_1 - \mu_2|^2 \quad (13)$$

$c(\mu) \to 0$ indicates it's in the smooth area, therefore, we mainly hope it to do diffusion, so the regularizing (smooth) item in TV model plays a more important role; In case $c(\mu) >> 0$,it means the point lies in the edge or corner or T shape local structure area, therefore, we hope to weaken the diffusion behavior in order to keep edges and detail features, now the fidelity item in TV model plays a important role.

Therefore, an image structure saliency function $c(s)$ is introduced to control the diffusion behaviors by choosing a proper value of regularizing item and fidelity item according to the area the point lies in.

$$c(s) = k + \exp(-[|\mu_1\mu_2| + (\mu_1 - \mu_2)^2]) \quad (14)$$

where k is a control parameter related to noise intensity. In this paper the value of k is 2 times of the noise standard deviation.

Equation(14) states clearly: when $|\mu_1\mu_2| + |\mu_1 - \mu_2|^2$ has a small value, it indicates the value of image structure feature saliency function is small and it is the smooth area, while $c(s)$ has a relatively large value which strengthens the effect of regularizing item. If $|\mu_1\mu_2| + |\mu_1 - \mu_2|^2$ has a large value, it indicates the value of feature saliency function is large and it is the non-smooth area, while $c(s)$ has a relatively small value which strengthens the effect of fidelity item. So the introduction of image structure feature saliency function can adaptively balance the effect of regularizing item and fidelity item in TV model.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

We now propose the TV denoising model based on image frequency and tensor voting as follows:

$$\min E(u) = \iint_{\Omega} c(s)\cdot\Phi\left(\frac{|\nabla u|}{u}\right)dxdy + \frac{1}{2}\iint_{\Omega}|u(x,y)-u_0(x,y)|^2\,dxdy \qquad (15)$$

The corresponding Euler-Lagrange function of (15) is

$$-\frac{1}{u}\nabla\cdot\left(\Phi'\left(\frac{|\nabla u|}{u}\right)\frac{\nabla u}{|\nabla u|}\right)\cdot c(s) - \frac{|\nabla u|}{u^2}\Phi'\left(\frac{|\nabla u|}{u}\right)\cdot c(s) + (u-u_0) = 0 \qquad (16)$$

and its corresponding gradient decent flow is

$$\frac{\partial u}{\partial t} = \frac{1}{u}\nabla\cdot\left(\Phi'\left(\frac{|\nabla u|}{u}\right)\frac{\nabla u}{|\nabla u|}\right)\cdot c(s) + \frac{|\nabla u|}{u^2}\Phi'\left(\frac{|\nabla u|}{u}\right)\cdot c(s) - (u-u_0) \qquad (17)$$

and the corresponding Euler-Lagrange function in (η,ξ) coordinate system is

$$c(s)\cdot\left(\frac{1}{|\nabla u|}\Phi'\left(\frac{|\nabla u|}{u}\right)u_{\xi\xi} + \frac{1}{u}\Phi''\left(\frac{|\nabla u|}{u}\right)u_{\eta\eta} - \frac{(\nabla u)^2}{u^2}\Phi''\left(\frac{|\nabla u|}{u}\right) + \frac{|\nabla u|}{u}\Phi'\left(\frac{|\nabla u|}{u}\right)\right) - u(u-u_0) = 0 \qquad (18)$$

where $\Phi(\cdot)$ is called potential function, generally it is a non-negative increasing function. According to equation(18),if we want to keep edges and details, it is hoped that when the value of image frequency is small it tends to Gaussian smoothing, when the value of image frequency is big it is almost no smoothing in vertical edge direction while smooth quantity along the edge direction is bigger than it in vertical direction. So the potential function should meet the equation (19-a) and (19-b).

$$\lim_{s\to+\infty}\Phi''(s) = 0 \qquad (19\text{-a})$$

$$\lim_{s\to+\infty}[s\cdot\Phi''(s)/\Phi'(s)] = 0 \qquad (19\text{-b})$$

Here we choose the potential function as

$$\Phi(s) = (1+s^2)^{1/2} - 1 \qquad (20)$$

The curve of potential function and its first derivative and second derivative are shown in figure 2.

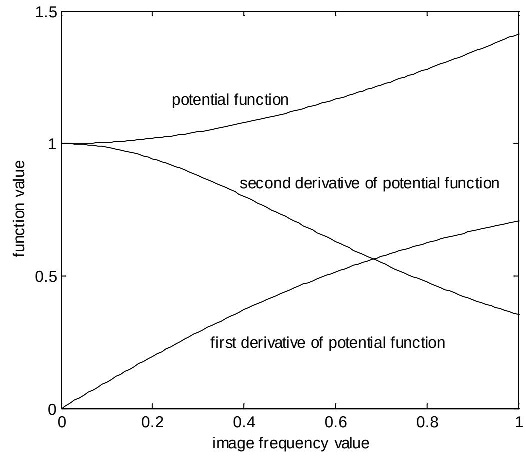


Figure 2: Curves Of Potential Function $\Phi(s)$ And $\Phi'(s),\Phi''(s)$

From (18) we note that it has different diffusion coefficients in normal direction and tangent direction，and fig.2 shows that the first derivative and second derivative of potential function have different change rules. In the area of image edge, that is, the image frequency has a larger value; the value of diffusion coefficient in tangent direction is greater than that in normal direction. Therefore, the noise can be suppressed while image edges and other feature information can be preserved well. when has a smaller value, relative to tangent direction, the value of diffusion coefficient in normal direction begins to increase, that means it obtains a stronger diffusion ability.

4. RESULTS AND ANALYSIS

In order to verify the effectiveness of the method proposed in this paper, we performed lots of simulation experiments. We used Baboon and Barb grayscale images as examples to explain and analysis (See Figure 3). First, we added different levels of zero-mean Gaussian noise to the original image as input. The size of images was 256×256 and grayscale was 256. In the simulation experiments, we took peak signal to noise (PSNR) and mean absolute error (MAE) as an objective criterion to the restored image evaluation, and compared the three denoising models: ROF TV model [4], generalized TV model(BS TV) [12] and the proposed model in this paper.

Peak signal to noise (PSNR) is defined by:

$$PSNR = 10\lg\frac{255^2}{\frac{1}{m\times n}\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{u}_{ij}-u\ )^2} \qquad (21)$$

Mean absolute error (MAE) is defined by:

Journal of Theoretical and Applied Information Technology
15th December 2012. Vol. 46 No.1

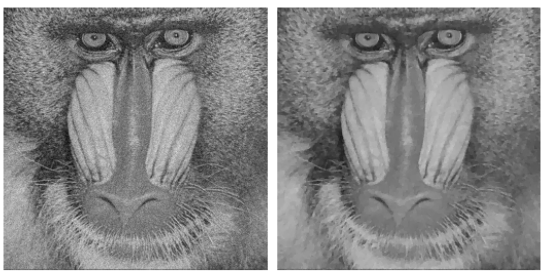
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$MAE = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} | \widetilde{u}_{ij} - u | \qquad (22)$$

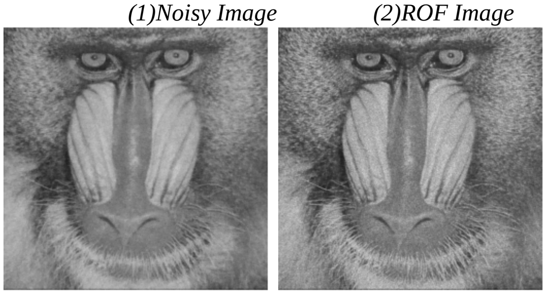
where $\widetilde{u}_{ij}$ is the gray value of the restored image and u is the gray value of original image without pollution. $m \times n$ is the picture size.

(1) Baboon (2)Barb

Figure 3: Original Images

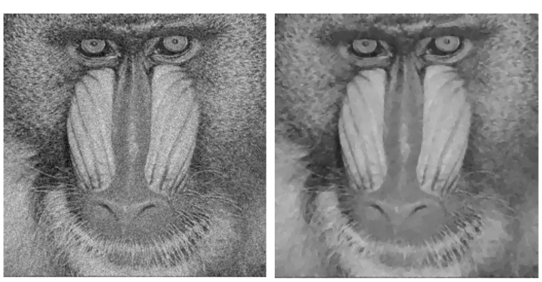


(1)Noisy Image (2)ROF Image

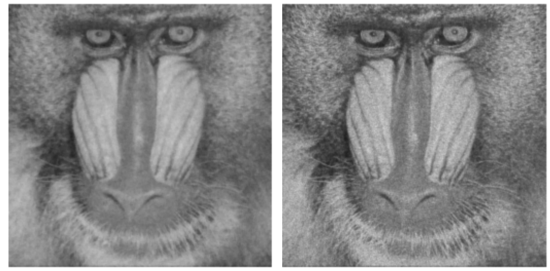


(3) BS Image (4) The Proposed Method

Figure 4: Noise Intensity $\sigma^2 = 10$,Denoising Effect Of Baboon Image Using Three Approaches

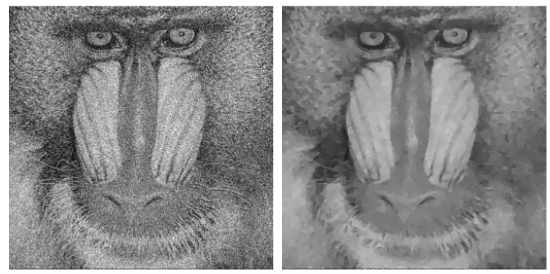


(1)Noisy Image (2)ROF Image

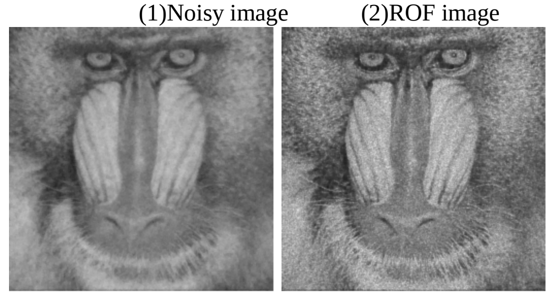


(3) BS Image (4) The Proposed Method

Figure 5: Noise Intensity $\sigma^2 = 15$,Denoising Effect Of Baboon Image Using Three Approaches



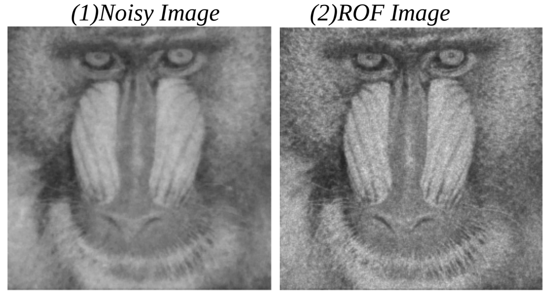
(1)Noisy image (2)ROF image



(3) BS Image (4) The Proposed Method

Figure 6: Noise Intensity $\sigma^2 = 20$,Denoising Effect Of Baboon Image Using Three Approaches

(1)Noisy Image (2)ROF Image



(3) BS Image (4) The Proposed Method

Figure 7: Noise Intensity $\sigma^2 = 25$,Denoising Effect Of Baboon Image Using Three Approaches

Journal of Theoretical and Applied Information Technology
15th December 2012. Vol. 46 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

(1)Noisy Image *(2)ROF Image*

(3) BS Image *(4) The Proposed Method*

Figure 8: Noise Intensity $\sigma^2 = 10$ *,Denoising Effect Of Barb Image Using Three Approaches*

(1)Noisy Image *(2)ROF Image*

(3) BS Image *(4) The Proposed Method*

Figure 9: Noise Intensity $\sigma^2 = 15$ *,Denoising Effect Of Barb Image Using Three Approaches*

(1)Noisy Image *(2)ROF Image*

(3) BS Image *(4) The Proposed Method*

Figure 10: Noise Intensity $\sigma^2 = 20$ *,Denoising Effect Of Barb Image Using Three Approaches*

(1)Noisy Image *(2)ROF Image*

(3) BS Image *(4) The Proposed Method*

Figure 11: Noise Intensity $\sigma^2 = 25$ *,Denoising Effect Of Barb Image Using Three Approaches*

From tab.1 and tab.2, compared with ROF TV model and Generalized TV model, the proposed model in this paper has achieved better denoising effect under different noise levels. Moreover, from figure 4 to 11,it has appeared that the proposed method combining with tensor voting and TV energy minimization can accurately describe the image sharp edges and smooth areas ,overcome staircase effect and edge blurring generated from ROF model and other TV models ,preserve the edges and important detail features of the restored image well, in addition, the results has appeared to be state-of-the-art for those images with rich texture features and low SNR , human eyes can perceive the improvement of image quality intuitively from the proposed model reconstruction.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Table 1 : The Comparing Three Different Models Of PSNR(Db)

	Baboon			Barb		
σ^2	ROF TV	BS TV	The propsed method	ROF TV	BS TV	The propsed method
5	33.39	33.20	34.48	34.61	34.06	35.33
10	29.34	29.08	31.74	31.35	30.30	32.72
15	27.39	27.16	28.72	26.60	28.46	29.05
20	26.24	26.01	26.26	25.33	25.65	27.60
25	25.42	25.25	25.74	25.14	25.31	26.96

Table 2: The Comparing Three Different Models Of MAE

	Baboon			Barb		
σ^2	ROF TV	BS TV	The propsed method	ROF TV	BS TV	The propsed method
5	4.06	4.26	3.79	3.39	3.67	3.37
10	6.51	6.79	6.34	4.86	5.65	5.60
15	8.17	8.45	8.10	6.16	7.40	7.33
20	9.32	9.65	9.27	6.97	8.05	8.76
25	10.23	10.55	10.61	7.64	8.87	8.95

5. CONCLUSION

In this paper, firstly, we analyze the image total variation denoising methods. And then combining with human vision principle, we use image frequency to instead of the gradient and introduce tensor voting into the traditional TV model, and an image structure saliency function included image detail features is given to replace the Lagrangian multiplier λ in the traditional TV model for the purpose of balancing the effect of regularizing term and fidelity item, thus we propose a novel texture image denoising method using the combination of tensor voting and total variation minimization. The propose method in this paper has a strong ability of denoising and can accurately and subtly describe the image edge area and smooth area. The analysis of experimental results have indicated that, compared with other existing approaches, the proposed method can overcome staircase effect and over-smoothing, especially for those images with rich texture features and low SNR, it can remove the noise while preserving significant image details and important characteristics. Further more, the restored images are more in line with the visual experience.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 61170161) and Natural Science Foundation of Shandong Province (ZR2012FQ029).

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