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AN IMPROVED METHOD TO ESTIMATE THE FUNDAMENTAL MATRIX BASED ON 7-POINT ALGORITHM

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ABSTRACT

The fundamental matrix plays an important role in the field of computer vision. How to estimate it accurately has become a major subject. In this paper, an improved approach based on the 7-point algorithm is presented. In this method, initial estimations of the fundamental matrix are provided by the 7-point algorithm. Then an optimal fundamental matrix will be selected which offers relatively low mean, low maximum and low standard deviation of the discrepancy between points and epipolar lines. Experimental results on synthetic data and real images show that the proposed algorithm is superior over the 7-point algorithm in terms of accuracy and anti-noise ability.

Keywords: *Fundamental Matrix, Epipolar Geometry, 7-Point Algorithm*

1. INTRODUCTION

An important task of computer vision is to obtain the estimation of three-dimensional information. In the case that the camera parameters and the motion between them are unknown, an intrinsic projective geometry can be extracted from images taken by two cameras focusing on a single scene. This is the so called epipolar geometry. Epipolar geometry is independent from the structure of the scene and it relates only to the internal parameters and relative pose of the two camera systems. All the geometric information contained in the two images can be described by a 3×3 singular matrix called the fundamental matrix. In the past, almost all the work on motion and stereo was under the condition that the intrinsic parameters of cameras are known. However, these parameters change greatly under different scenes. The fundamental matrix can be estimated from the point correspondences of the two images without knowing the internal information of the cameras in advance. So the computation of the fundamental matrix is crucial in the field of computer vision.

Present researches have proposed many algorithms which are classified into linear methods, iterative methods and robust methods. The linear methods include 7-point algorithm, 8-point algorithm, least-squares technique, orthogonal least-squares technique and analytic method with rank-2 constraint [2]. The iterative methods optimize the linear methods. They are classified into two groups: those that minimize the distance between points and epipolar lines and those that are based on the gradient [1]. The iterative methods have improved the accuracy compared with the linear methods. But they consume more time and cannot deal with outliers. The robust methods can cope with outliers, bad locations and false matching. The general used robust methods include M-Estimators, Least-Median-Squares (LMedS), Random Sampling (RANSAC), MLESAC and MAPSAC [7]. The robust methods have developed greatly these days.

The linear methods are the foundation of other methods and usually provide initial results for them. The 7-point algorithm needs only 7 point correspondences and gives a rank-2 fundamental matrix [1]. The 8-point algorithm minimizes the error in the estimation when there are redundant points. The equation of minimization is the residual of Eq. (2.1) [10]. Another method to solve the equation is the orthogonal least-squares technique [7]. The analytic method with rank-2 constraint imposes the rank-2 constraint during the minimization [1]. The linear methods are time-saving but sensitive to noise.

In this paper, I look into the linear methods because they have a connection with the method proposed. The 7-point algorithm has been used in 1994 by Huang and Netravali. In the same year, Hartley, Torr, Beardsley and Murray also used it to estimate the fundamental matrix [1].The advantage of the 7-point algorithm is that only 7 point matches are required to estimate the fundamental matrix.

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However, when there are false locations and bad matches of points, the advantage becomes a drawback. Furthermore, when more point correspondences are provided, the 7-point algorithm cannot be applied. Another problem of the linear methods is that the quantity minimized is not physically meaningful. As the coordinates of points are extracted from the images, a physically meaningful quantity should be something that can be measured in the image plane.

This article is divided as follows. First, an introduction of epipolar geometry and the fundamental matrix is presented. Secondly, an improved method to estimate the fundamental matrix is described. Steps and a flow chart are provided to make the method understandable. Section 4 deals with synthetic data and real images to examine the proposed methods with comparison to the 7-point algorithm. Finally, a conclusion is drawn in section 5.

2. EPIPOLAR GEOMETRY AND THE FUNDAMENTAL MATRIX

Epipolar geometry exists between the two camera systems looking at the same scene. Consider C and $C^{'}$ are the optical centers of two cameras shown in Fig.1. Given a point m in the first image, its corresponding point $m^{'}$ on the second image lies on a line called the epipolar line of m, denoted by $l^{'}$. This is known as the epipolar constraint. In order to match m and $m^{'}$, the following equations must be satisfied:

$$m^{'T}Fm = 0 \qquad (2.1)$$

Given homogeneous coordinates of points m_i and $m_i^{'}$. All the methods of estimating the fundamental matrix is based on solving a homogeneous system of equations: Eq. (2.1). The equations can be rewritten in the following way:

$$U_n f = 0 \qquad (2.2)$$

Where

$$f = (F_{11,}F_{21}, F_{31}, F_{12}, F_{22}, F_{32}, F_{13}, F_{23}, F_{33})^T \qquad (2.3)$$

$$U_n = \begin{bmatrix} u_1u_1^{'} & u_1v_1^{'} & u_1 & v_1u_1^{'} & v_1v_1^{'} & v_1 & u_1^{'} & v_1^{'} & 1 \\ u_2u_2^{'} & u_2v_2^{'} & u_2 & v_2u_2^{'} & v_2v_2^{'} & v_2 & u_2^{'} & v_2^{'} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n^{'} & u_nv_n^{'} & u_n & v_nu_n^{'} & v_nv_n^{'} & v_n & u_n^{'} & v_n^{'} & 1 \end{bmatrix}_{n\times 9} \qquad (2.4)$$

The fundamental matrix has only 7 degrees of freedom. Thus 7 is the minimum number of point matches needed to solve the equations. Through singular value decomposition, vectors f_1 and f_2 are obtained, which span the null space of U_7 [4] [5] [6]. Reshape the vectors f_1 and f_2 into 3×3 matrices F_1 and F_2. The solution of the above equations is a set of matrices of the form:

$$F = \alpha F_1 + (1-\alpha)F_2 \qquad (2.5)$$

In order to make the rank of the matrix to be 2, the expression $\det(\alpha F_1 + (1-\alpha)F_2)$ is used and cubic polynomial of α is obtained. Then use α to compute F.

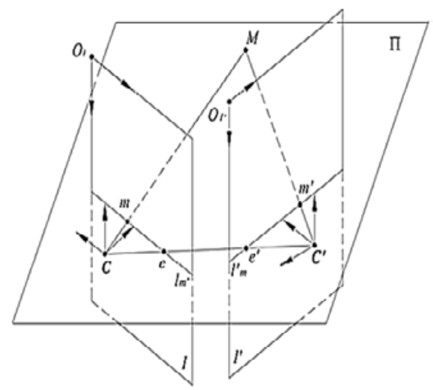

Figure 1: The Epipolar Geometry

3. AN IMPROVED METHOD FOR ESTIMATING THE FUNDAMENTAL MATRIX

In order to address the problems of the 7-point algorithm mentioned in the introduction, I propose an improved method based on the 7-point algorithm and can be used when $n \geq 7$. Given that there are N point correspondences. Divide the point correspondences randomly into n subsets. Each subset has 7 point correspondences. If N is not a multiple of 7, then some point matches must be deleted to make this happen. The 7-point algorithm is used to estimate the fundamental matrix of each subset, denoted by $F_i (i = 1...n)$. An optimal F_t is selected according to the mean, maximum and standard deviation of the discrepancy of the distance between points and their corresponding epipolar lines. F_t can make these three parameters relatively low. In order to describe the method clearly, a flow chart is shown in Fig. 2. The corresponding steps are as follows:

Step1. Obtain the entire set of point correspondences: M_{ini}.

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Step2. Divide M_{ini} randomly into n subsets. Each subset has 7 point matches. Initialize $i=1$.

Step3. Get subset i from M_{ini}, use the 7-point algorithm to compute Fi and put it into an array [F].

Step4. For each Fi, compute the mean, maximum and standard deviation of the discrepancy between all points in M_{ini} and epipolar lines and put them separately into three arrays: [Mean], [Stddev], [Max].

Step5. Set $i=i+1$ and repeat steps 3 and 4 until i reaches n.

Step6. Sort the three arrays in ascending order and put their indexes into another three arrays in the same order. These three new arrays are named as [Nomean], [Nostd], [Nomax].

Step7. Find the median and the minimum of [Mean]. The median minus minimum is r. Find all the elements of [Mean] smaller than r and record the total number n_r.

Step8. Initialize $j=1$. Give the value of element j in [Nomean] to a parameter t.

Step9. Find if there is an element in [Nostd] equal to t. If not, set $j = j+1$ and jump back to step6. If so, move to the next step.

Step10. Find if there is an element in [Nomax] equal to t. If not, set $j = j+1$ and jump back to step6. If so, find the element t in [F], which is the final result F_t.

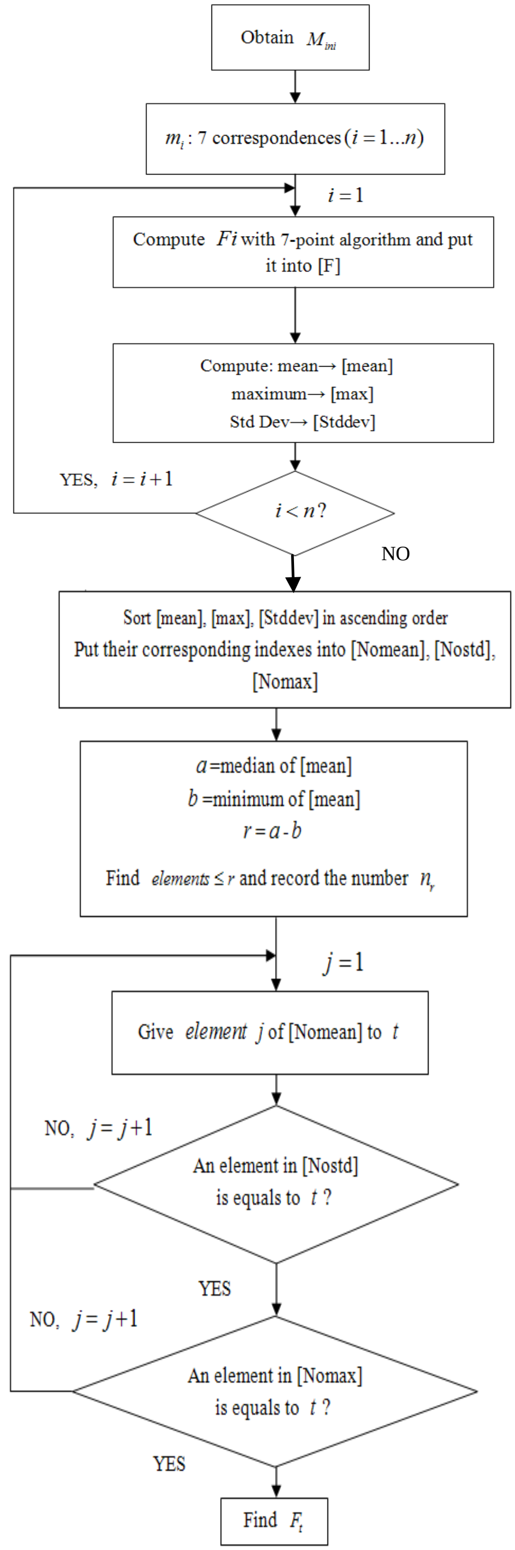

Figure 2: Flow Chart Of The Proposed Method

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4. EXPERIMENTAL RESULTS

In this section, I show some experimental results of the proposed method and compare them with the results of the 7-point algorithm. Both synthetic data and real images are used to complete the experiments. A standard must be made in order to analyze the two methods. In this paper, the accuracy of each method is shown as the mean and standard deviation of the discrepancy between points and epipolar lines.

4.1 Experiments with synthetic data

In the experiments with synthetic data, the point matches are randomly generated by space points in 3D space which are visible to two different positions of a synthetic camera [2]. The number of point matches is 80, and only 7 random correspondences are used in the experiments of the 7-point algorithm. Fig. 3 shows the correspondences of synthetic data in the form of arrows. The point of an arrow means a point in the first image and the end of it means the matching point in the second image. F_{syn} is generated by synthetic data using projection matrix. $F_{7\,pts}$ is computed by the 7-point algorithm. F_{pro} is computed by the proposed method. The three fundamental matrices are as follows:

$$F_{syn}=\begin{pmatrix} 6.1042e{+}000 & -5.8434e{+}000 & -3.2308e{+}000 \\ 1.6287e{+}001 & -1.7059e{+}001 & 2.5486e{-}001 \\ 1.3582e{+}001 & -1.4719e{+}001 & 3.1951e{+}000 \end{pmatrix}$$

$$F_{7-pts}=\begin{pmatrix} 1.9399e{-}004 & -6.7609e{-}005 & -4.2140e{-}002 \\ -2.7880e{-}004 & 1.3411e{-}004 & 5.5683e{-}002 \\ -6.6568e{-}004 & -6.8057e{-}003 & 1.0745e{+}000 \end{pmatrix}$$

$$F_{pro}=\begin{pmatrix} 4.6524e{-}005 & 1.4706e{-}004 & 8.6461e{-}002 \\ -4.7720e{-}005 & -1.6505e{-}004 & -9.7597e{-}002 \\ -2.9317e{-}002 & -4.4666e{-}003 & 8.7074e{-}001 \end{pmatrix}$$

The sensitivity to noise of different methods is studied in this experiment. Four different ranges of Gaussian noise are added to the point correspondences, whose means are 0 and variances vary from 0 to 1. The noise-containing data and the three fundamental matrices are used to compute the means and standard deviations of the discrepancy between points and epipolar lines, as shown in Table 2. In order to compare the time consumed by different methods, the experiment is repeated for 100 times and put the average value into Table 2. Through computation I find that the rank of the fundamental matrices using both methods is 2. Table 1 and Table 2 shows that the proposed method provides a better accuracy at the cost of time and it is less sensitive to noise compared with the 7-point algorithm.

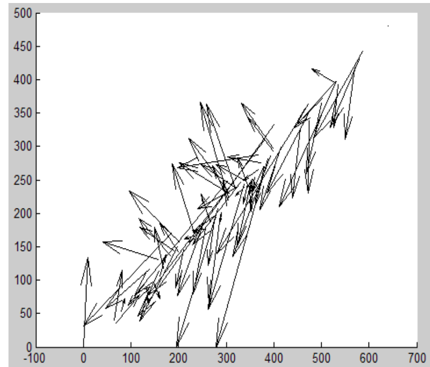

Figure 3: Correspondences Used In Synthetic Data

Table 1
Means (1st Row) And Standard Deviations Of The Discrepancy (2nd Row) Between Points And Epipolar Lines Using Synthetic Data

Methods	7-point algorithm	Proposed method
σ=0.0	1.0644e-010	4.4958e-012
	1.0165e-010	3.5029e-012
σ=0.1	34.9476	0.1816
	34.7465	0.2022
σ=0.5	72.7667	1.0191
	98.8069	1.1512
σ=1.0	42.8833	4.0878
	56.7205	6.8437

Table 2
Time Consumed By Different Methods

Methods	7-point algorithm	Proposed method
time	3.9086e-004	0.0303

4.2 Experiments with real images

In this section two images are used to estimate the fundamental matrix. The correspondences of the images have already been detected and matched [3] [8] [9]. 347 pairs of correspondences are marked in Figure 4 in the form of arrows. Use all 347 point matches to compute the fundamental matrix with the proposed method and choose random 7 correspondences from the whole set for the 7-point algorithm. The results are as follows:

$$F_{7-pts}=\begin{pmatrix} -1.0644e{-}006 & 6.7839e{-}005 & -1.7468e{-}003 \\ -6.8751e{-}005 & -9.5049e{-}007 & 2.2910e{-}002 \\ -9.4397e{-}004 & -2.2793e{-}002 & 1.0201e{+}000 \end{pmatrix}$$

$$F_{pro}=\begin{pmatrix} -8.4538e{-}007 & 5.8004e{-}005 & 6.6191e{-}003 \\ -6.0932e{-}005 & -1.9853e{-}006 & 2.4793e{-}002 \\ -8.7405e{-}003 & -2.3957e{-}002 & 9.9678e{-}001 \end{pmatrix}$$

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Table 3 shows the means and standard deviations of the discrepancy between points and epipolar lines under different methods. In order to analyze the relationship between accuracy and amount, the number of point correspondences involved increases from 40 to 200. As the 7-point algorithm can only use 7 correspondences, the experiment deals with the proposed method only. The time of estimating the fundamental matrix with ranging number of points is computed for 100 times to get the average value, as shown in Table 4. These tables show that the accuracy of the proposed method is much better than that of the 7-point algorithm. With the number of point correspondences increasing, more accurate results are received and more time are consumed.

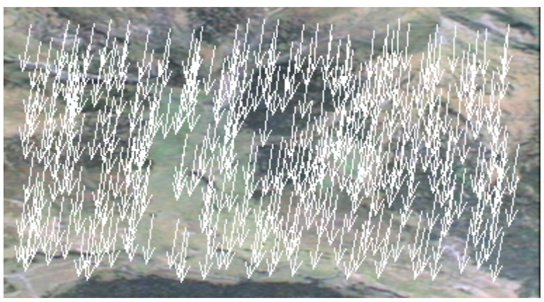

Figure 4: Correspondences Used In Real Images

Table 3 : Means And Standard Deviations Of The Discrepancy Between Points And Epipolar Lines Using Real Images

Methods	Mean	Standard deviations of the discrepancy
7-point algorithm	1.9395	2.0149
Proposed method	0.2488	0.2303

Table 4: Means (1st Row) And Standard Deviations Of The Discrepancy (2nd Row) Between Points And Epipolar Lines Using Different Number Of Correspondences

No.	N=40	N=80	N=120	N=140	N=200
proposed method	6.8748 7.0918	0.4421 0.4242	0.4194 0.3846	0.3160 0.3214	0.2722 0.2742

Table 5 : Time Consumed With Different Numbers

No.	N=40	N=80	N=120	N=140	N=200
time	4.1255 e-003	1.4677 e-002	3.2348 e-002	4.3939 e-002	8.3031 e-002

5. CONCLUSIONS

In this article I proposed an improved method for estimating the fundamental matrix based on the 7-point algorithm. Unlike the 7-point algorithm, this method can be applied when the number of point correspondences is more than 7. It improves the accuracy of the 7-point algorithm at the cost of time and better results can be obtained if more point matches are involved. The proposed method has a better anti-noise performance, while the 7-point algorithm performs badly. However, the drawbacks of this method cannot be ignored. Because the division of the initial set of correspondences is random, so I can only choose subsets passively. The fundamental matrix is computed by the best subset among all the subsets rather than the best seven points among all the point correspondences. And even the best subset may have bad locations and false matches. A further research on how to select good points is needed to improve the accuracy of the fundamental matrix.

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