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THREE-DIMENSIONAL NUMERICAL CALCULATION ANALYSIS OF STRESS INDUCED BY PORE PRESSURE VARIATION

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ABSTRACT

Refracturing is an important technique for developing low-permeability reservoir economically and effectively. It is the stress field around well bore and fracture that plays a crucial role in determining new fracture creation and propagation. The calculation analysis of the stress induced by pore pressure variation is critical to evaluate the stress field distribution. By employing finite element analysis software ABAQUS, three-dimensional numerical calculation model for stress induced by pore pressure variation is established. It is proved that on the path orthogonal to the initial fracture, the stress induced by pore pressure variation will reduce the value of difference between two horizontal principal stresses. For a point near the initial fracture, the induced stress is mainly generated at the initial stage after the first fracturing treatment.

Keywords: *Numerical Calculation, Refracturing, Pore Pressure, Induced Stress, Stress Reversal*

1. INTRODUCTION

Influenced by some factors, for fractured oil and gas wells, the original fracture would gradually loose its effectiveness. At this time, refracturing is necessary. The stress field around the well bore and fracture is crucial in deciding the fracture initiation and propagation. During the period between initial fracturing and refracturing, the variation of reservoir pore pressure may help forming a stress reversal region (as Fig.1 shows) around the well bore and fracture. That is to say, the directions of two horizontal principle stresses in this region reverse [1]. The calculation analysis of the stress induced by pore pressure variation is critical to evaluate the stress field distribution.

Based on Fluid-solid Coupling Theory, Elbel and Mack conducted research on the distribution features of pore pressure around well bore and fracture after initial fracturing. In addition, they also studied the effect of formation parameter on refracturing [2]. Siebrits suggested that pore pressure variation will result in stress field changes, causing reorientation of two horizontal principle stresses. Thus the new fracture generated by refracturing would propagate along the direction perpendicular to the original fracture [3]. Roussel carried out research on stress reorientation variation with production time due to stress induced by propped hydraulic fracture and by pore pressure variation [4]. Rahman analyzed influence of stress induced by pore pressure variation on stress field nearby the fracture tip, using FLAC3D (fast lagrangian analysis of continua software) [5].

Two-dimensional model was commonly used in previous studies and the model size differs too much from the size of real one. In this paper, the research employs the finite element method analysis software ABAQUS and establishes finite element model more similar to research object. The calculation of stress induced by pore pressure variation before refracturing is modeled. Based on Fluid-solid Coupling Theory, a numerical analysis is conducted on stress induced by pore pressure before refracturing. The results further prove the existence of stress reversal region. Meanwhile, the research also reveals some new variation features of stress induced by pore pressure, which helps to advance further research on refracturing mechanism and numerical simulation.

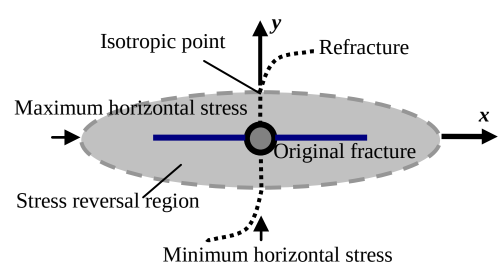

Figure 1: Schematic Plot Of Stress Reversal Region

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2. FLUID-SOLID COUPLING MATHEMATICS DISCRETE MODEL

The oil and gas reservoir rock is a kind of porous media with complicated internal structure [6, 7]. It consists of solid phase matrix, complicated connected pore space and fluid (oil, gas and water) stored in the pore space. During oil and gas production, there exists interaction and mutual influence between fluid seepage in reservoir rock and stress field variation of the reservoir rock. In order to analyze the stress induced by pore pressure before refracturing, there is the need to consider coupling mechanism of seepage and stress.

2.1 Effective Stress Principle Of Porous Media

In terms of research on mechanical behavior of oil and gas reservoir, the concept of effective stress is commonly used [8, 9]. In saturated reservoir rock, the fluid pressure is defined as stress undertaken and passed by fluid. The effective stress refers to stress passed through solid phase matrix.

If using σ , p and σ' to respectively represent total stress, fluid pressure and effective stress of one point in oil and gas reservoir rock, their relation can be presented below:

$$\sigma' = \sigma - \alpha p \tag{1}$$

where α represents the effective stress coefficient.

2.2 Stress Balance Equation Of Reservoir Rock

According to effective stress principle and virtual work principle, when setting the value of α as 1 and carries out time derivation calculus of the virtual work equation, the stress balance equation can be gained as the following:

$$\begin{aligned}&\int_V \delta\varepsilon^T D_{ep}\frac{d\varepsilon}{dt}dV+\int_V \delta\varepsilon^T D_{ep}\left(m\frac{(s_o+p\xi)}{3K_s}\frac{dp}{dt}\right)dV-\\&\int_V \delta\varepsilon^T m(s_o+p\xi)\frac{dp}{dt}dV=\int_V \delta u^T\frac{df}{dt}dV+\int_S \delta u^T\frac{d\tilde{t}}{dt}dS\end{aligned}\tag{2}$$

where $\tilde{t}$ is surface force, f is body force, $\delta\varepsilon$ is virtual strain, δu is virtual displacement, D_{ep} is elastic-plastic matrix, S is area, V is volume, s_o is fluid saturation, $m=[1,1,1,0,0,0]^T$.

2.3 Fluid Flow Continuity Equation

By making stress balance equation discrete, finite element mesh of solid phase material is derived. Meanwhile, in order to make fluid flow through the mesh, the fluid flow should satisfy the continuity equation. That is to say, the quantity of flow in the mesh unit in a certain time increment is equal to increasing rate of fluid volume. In accordance with Darcy's law, fluid flow continuity equation can be derived [10]:

$$\begin{aligned}&s_o\left(m^T-\frac{m^T D_{ep}}{3K_S}\right)\frac{d\varepsilon}{dt}-\nabla^T\left[k_o k_r\left(\frac{\nabla p}{\rho_o}-g\right)\right]+\\&\left\{\xi\phi+\phi\frac{s_o}{K_o}+s_o\left[\frac{1-\phi}{3K_S}-\frac{m^T D_{ep} m}{(3K_S)^2}\right](s_o+p\xi)\right\}\frac{dp}{dt}=0\end{aligned}\tag{3}$$

where k_o is the product of initial permeability tensor timing density, k_r is the proportion permeability, g is the acceleration of gravity, K_o is the fluid bulk modulus, K_S is the modulus of compressibility of solid phase matrix, ξ is the variation rate of saturation, ϕ is the porosity, ρ_o is the fluid density.

2.4 Finite Element Discretization Of The Equation

The shape function is presented below:

$$\begin{cases}u=N_u\bar{u}\\ \varepsilon=B\bar{u}\\ p=N_p\bar{p}\end{cases}\tag{4}$$

where $\bar{u}$ is the nodal displacement of element, $\bar{p}$ is the nodal pore pressure of element.

The FEM formulation of reservoir rack solid phase can be derived as following by substituting the formula (4) into the formula (2)：

$$K\frac{d\bar{u}}{dt}+C\frac{d\bar{p}}{dt}=\frac{df}{dt}\tag{5}$$

In the formula:

$$K=\int_V B^T D_{ep} B dV\tag{6}$$

$$\begin{aligned}C=&\int_V B^T D_{ep} m\frac{(s_o+\xi p)}{3K_S}N_p dV\\&-\int_V B^T(s_o+\xi p)mN_p dV\end{aligned}\tag{7}$$

$$df=\int_V N_u^T df dV+\int_S N_u^T dt dS\tag{8}$$

As for analysis on flow field, boundary condition should be introduced:

$$-n^T k k_r\left(\frac{\nabla p}{\rho_o}-g\right)=q_o\tag{9}$$

where n is the unit normal of flow boundary, k is the permeability, q_o is the flow rate in unit time at the boundary.

Using the Galerkin method：

$$\int_V a^T\bar{A}dV+\int_S b^T\bar{B}dS=0\tag{10}$$

where a , b are arbitrary functions； $\bar{A}$ is the governing equation； $\bar{B}$ is the continuity equation through the boundary. Through taking formula (3)

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as $\overline{A}$, formula (9) as $\overline{B}$, substituting shape function into formula (10) and making a is equal to $-b$, the formula can be simplified as the following:

$$E\frac{d\overline{u}}{dt}+F\overline{p}+G\frac{d\overline{p}}{dt}=\tilde{f} \quad (11)$$

In this formula:

$$E=\int_V N_p^T\left[s_o\left(m^T-\frac{m^T D_{ep}}{3K_S}\right)B\right]dV \quad (12)$$

$$F=\int_V\left(\nabla N_p\right)^T kk_r\nabla N_p dV \quad (13)$$

$$G=\int_V N_p^T\left\{s_o\left[\left(\frac{1-\phi}{K_S}-\frac{m^T D_{ep}m}{(3K_S)^2}\right)\right]\cdot\left(s_o+p\xi\right)+\xi\phi+\phi\frac{s_o}{K_o}\right\}N_p dV \quad (14)$$

$$\tilde{f}=\int_S N_p^T q_o dS-\int_V\left(\nabla N_p\right)^T kk_r g dV \quad (15)$$

By joining formula (5) and (11), coupled stress-flow formulation is derived below:

$$\begin{bmatrix}K & C\\ E & G\end{bmatrix}\frac{d}{dt}\begin{Bmatrix}\overline{u}\\ \overline{p}\end{Bmatrix}+\begin{bmatrix}0 & 0\\ 0 & F\end{bmatrix}\begin{Bmatrix}\overline{u}\\ \overline{p}\end{Bmatrix}=\begin{bmatrix}\frac{df}{dt}\\ \tilde{f}\end{bmatrix} \quad (16)$$

3. FINITE ELEMENT METHOD MODEL

3.1 Physical Model

It is assumed that the fracture height is equal to the reservoir thickness and the width of fracture keeps unchanged in the direction of fracture height. Then the reservoir is constrained by upper and lower layers in normal direction and tangential displacement remains unconstrained. The whole model is established in size of 450 m×450 m. With consideration of symmetry, A quarter of the overall model area is taken as analysis object (Fig.2). Normal displacement is constrained on boundary face BCIH and face EFGA; displacement is fixed on boundary face CDJI and DEFJ; the circular arc boundary face ABHG is free-displacement boundary. BLKH is hydraulic fracture intermediate cross-section. The well bore radius is 0.15 m with length of fracture is 70 m, height 5 m and width of 0.005 m.

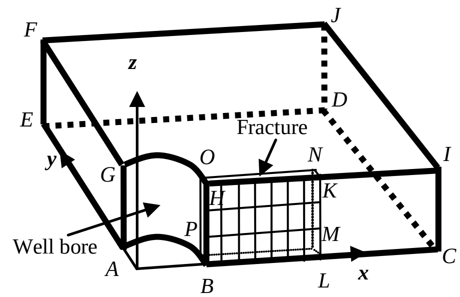

Figure 2: Schematic Plot Of Physical Model

3.2 Meshing And Solving

Based on physical model and in view of fluid-solid coupling mathematics discrete model, finite element analysis model is set up through finite element discretization. The finite element mesh and subdivision near the well bore is shown in Fig.3.

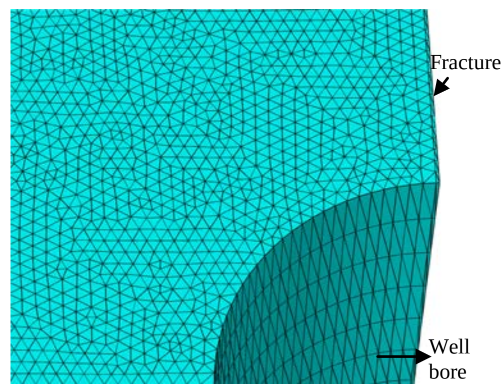

Figure 3: The Finite Element Mesh Near The Well Bore

The model adopts hexahedral ten nodes pore pressure element, with the top and bottom surface impermeable for flow, the wall permeable for flow and constant bottom-hole pressure. The pore pressure of fracture and its intermediate interface element nodes can change with production. All element nodes are given initial pore pressure. The permeability of reservoir rock and fracture are set to be different, with the former is lower than the latter. The fluid seepage in model unit mesh meets Darcy's law. According to the fluid-solid coupling mathematics discrete model mentioned earlier in the second part, ABAQUS can conduct continuous solution in consequence of time. The flow field and stress field is coupled directly. That is to say, by conducting spatial discretization on nodal displacement and nodal pore pressure of element, the stress balance equation of solid phase in reservoir rock and fluid phase continuity equation are written in matrix form. The time integration is introduced and coupled governing equation is

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derived. With combination of displacement and flow boundary conditions, the equation is solved at each time step.

4. CALCULATION RESULTS

Table 1 : Calculation Parameters

Parameters	Value
Overburden stress	42.5 MPa
Maximum horizontal stress	35.5 MPa
Minimum horizontal stress	33.7 MPa
Elastic modulus	11.3 GPa
Poisson ratio	0.23
Fluid viscosity	1.2 mpa•s
porosity	21%
Reservoir permeability	7 mD
Fracture permeability	175 mD
Reservoir pore pressure	17 MPa
Well bore pressure	10 MPa

Using the established finite element analysis and calculation model, the features of stress distribution near well bore and fracture before refracturing after two-month production under constant pressure can be calculated (Fig. 4~Fig. 6).

(1) In line with Fig.4, along the y axis path, as distance from the well bore increases, the absolute value of stress induced by pore pressure variation in direction of two horizontal principal stresses gradually reduces. The stress in the direction of the maximum horizontal stress (x-axis direction) is greater than that in the direction of the minimum horizontal stress (y-axis direction). In the oval shaped drainage region controlled by fracture, the degree of pressure depletion in the direction of the maximum horizontal principal stress is greater than that in the direction of minimum horizontal stress(as Fig.5 shows, the pore pressure equivalence contour around fracture takes on the oval shape).

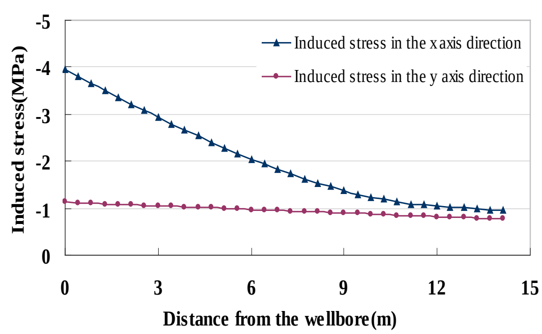

Figure 4: The Characteristics Of Induced Stress On The Path Of Y-Axis

Figure 5: Pore Pressure Equivalence Contour Around The Hydraulic Fracture

(2) From Fig.6, it can be seen that along the y-axis path, variation of the maximum horizontal stress is greater than that of the minimum principal stress. At the distance of about 3.25 m from well bore, two equal horizontal principal stresses are equal. This is a stress isotropic point, where the difference of two induced stress in the direction of two horizontal principal stresses is equal to the difference of the two original horizontal principal stresses.

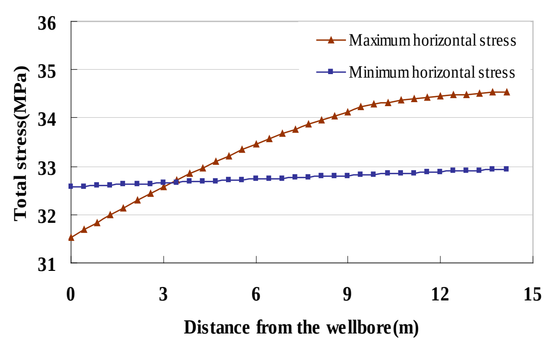

Figure 6: The Characteristics Of Two Principal Stresses On The Path Of Y-Axis

(3) The space point reflected in Fig.7 is located in stress reversal region. As the initial stage of the pressure drop, the pressure difference of formation at this point and well bottom is relatively large. Therefore, the induced stress at this point changes rapidly at the initial stage. Then with the further lowering of pore pressure, it tends to be stable. In addition, the extent of the maximum horizontal stress reducing is larger than that of minimum horizontal stress reducing. This is also due to the uneven changes of formation pressure in the oval shaped region around fracture.

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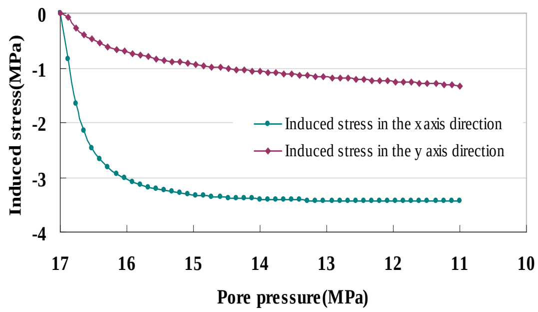

Figure 7: The Characteristics Of Induced Stresses Near The Well Bore (1.5 M From The Well Bore On The Y-Axis)

(4) As Fig.8 shows, at a point near well bore, during production, the reducing degree of maximum horizontal principal stress is greater than that of the minimum one, and the horizontal stresses reverse at this point.

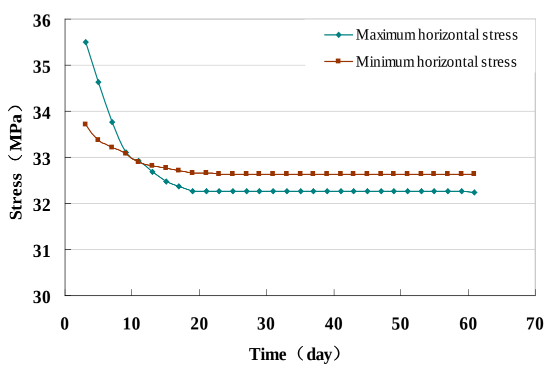

Figure 8: The Characteristics Of Horizontal Stresses Near The Well Bore (2 M From The Well Bore On The Y-Axis)

5. CONCLUSION

By employing the finite element calculation method, the finite element calculation model of stress induced by pore pressure is established. This model makes it possible to conduct numerical analysis on stress induced by pore pressure after initial fracturing. In the direction perpendicular to the initial fracture, absolute value of induced stress in the direction of maximum horizontal stress is greater than that in direction of the minimum horizontal stress. Therefore, stress induced by pore pressure will reduce the difference between the two horizontal principal stresses. For a space point near fracture, pore pressure induced stress tends to an extreme value with production. At this moment, if the induced stress difference is larger than the difference of two original horizontal principal stresses, then stress reversal will occur at this space point, otherwise it will not occur.

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