

THE RESEARCHING OF PLOTTING LOGARITHMIC PHASE FREQUENCY RESPONSE CURVE

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ABSTRACT

In this paper, a simple method of plotting bode chart is introduced. Through composed of each typical links, the logarithmic phase frequency response curve for usually open-loop systems can be replaced by regular fold lines and the traditional method for the process is simplified, the velocity of the plotting logarithmic phase frequency response curve is fast. So the application value of plotting logarithmic phase frequency response curve for open-loop systems is greatly enhanced.

Keywords: *Frequency Response. Open-Loop Systems. Logarithmic Phase Frequency Response Curve.*

1. INTRODUCTION

The traditional method of plotting the Bode diagram of the phase frequency response curve is superposition, which specific practices is that plotting the true curve of every link of the system on a piece of logarithmic coordinate paper according to a known mathematical model of the system, and then superimpose each of the line[1-4]. It makes the plotting process trival, and brings adverse impact to application value of phase frequency response curve. This paper introduces a simple method of plotting the logarithmic phase frequency response curve for usually open-loop systems, which uses fold lines to replace the true curve in order to simplify the graph.

2. LOGARITHMIC PHASE FREQUENCY RESPONSE OF THE APPROXIMATE LINE

The Basic Requirements for Plotting the Phase Frequency Curve with Traditional Method. By taking seven kinds of typical aspects of the minimum phase open-loop system for example (including Proportional, integral, differential, inertia, first-order differential oscillation and second-order differential link), before discussing how to construct the phase-frequency response of the typical aspects of approximate line, it's necessary to examine the logarithmic amplitude frequency characteristic. Among seven kinds of typical part, three kinds of links (including proportional, integral and

differential) of the real line is straight lines, there's no need to construct approximate line. The approximate lines of another four kinds of links(including Inertia, first order differential and etc.) are two-segment broken lines: discount points is point $(1/T, 0)$, the left is a 0db straight line, the right line's slope is $\pm 20\text{db/dec}$ and $\pm 40\text{db/dec}$. These approximate lines have three important characteristics [5,6]:(1)Basically consistent with the true curve trend, it gradually approaches the true line at both infinity ends of the line, which ensures that the overall error is small; (2) Discount points are unified, and the slope is an integer multiple of the basic value, which is easy to calculate; (3) The approximate line not only simplifies the plotting, more importantly, the system curve can reflect the relationship between the links, which makes the research and analysis more easy. Based on the analysis above, logarithmic frequency response should refer to the amplitude-frequency response; these three features form the basic requirements of the constructed approximate line. Just as amplitude-frequency characteristic, the links that logarithmic frequency response need to construct approximate line is also the inertia and other four kinds of links. They are the first and second two sets of symmetric links. The real curve of logarithmic phase frequency response is very similar to approximate line, which is centro-symmetric, mirroring to each other. The following example focuses on inertia.

The Approximate Line of Inertia. The real curve of logarithmic frequency response of Inertia link $\angle G$ is a dashed line in Figure 1. After a specific analysis of three features of the

approximate line in Figure 1, we'll get the constitution and plotting method of the approximate line. Features (1) requires that the approximate line and the real curve basically have the same trend, that it gradually approaches the true line at both ends of infinity, and The left and right side respectively are horizontal line of 0° and -90°, in the middle of which there's a skew line with symmetry point O, and the same direction with real line. Approximate line is actually a cluster of lines due to the uncertainty slope of the middle section of three-stage polyline above. It can be seen from Figure 1, different slope of the middle section line leads to different error between approximate line and real line, the steeper the skew line is, the smaller error near the point of symmetry there is, and there is greater error near two-fold point and outside, while the skew line that is more flat is contrary. Only from the perspective of overall error, there's no clearly better or worse alternative for selecting a cluster of approximate lines. According to the requirements of characteristics (2), the abscissa denotes T is commonly used by indexing the logarithm, so the selection of the most ideal is to take the right and left two 10 times frequency of the symmetry point O - point A and B as the right and left fold point. The abscissa denotes of A, O, B are 0.1 / T, 1 / T, 10 / T, and the slope of middle section is just -45 / dec, that the calculation is most simple, and the four links can completely consistent to characteristics (2).

According to the discussion above, the approximate line of logarithm phase frequency

response of inertial link < G is determined for a three stages of the line.

$$\angle G'(j\omega) = \begin{cases} 0^\circ, T\omega \leq 0.1 \\ -45^\circ(\lg T\omega + 1), 0.1 < T\omega < 10 \\ -90^\circ, T\omega \geq 10 \end{cases}$$

The full line in Figure1 stands for the approximate line of logarithm phase frequency response of inertial link, and its system characteristics can completely consistent to characteristics (3).

The Approximate Line of Typical Link. The approximate line of first order differential, oscillation and second order differential can be got from similar links. Table 1 lists the calculation formula of 4 typical links' approximate lines (including inertia, first order differential, oscillation and second order differential), and related factors.

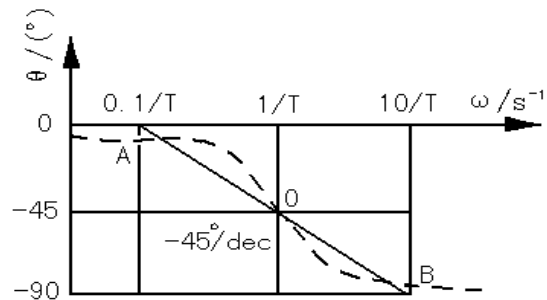


Figure1 The Real Curve And Approximate Line Of Logarithm Phase Frequency Response Of The Inertial Link

Table 1 The Approximate Line Of Logarithm Phase Frequency Response Of Typical Link

link	Inertia, first order differential	Oscillations, second order differential
Calculation formula	$\angle G'(j\omega) = \begin{cases} 0^\circ, T\omega \leq 0.1 \\ \pm 45^\circ(\lg T\omega + 1), 0.1 < T\omega < 10 \\ \pm 90^\circ, T\omega \geq 10 \end{cases}$	
Slash slope	$\pm 45^\circ / \text{dec}$	$\pm 90^\circ / \text{dec}$
Symmetric point	$(1/T, \pm 45^\circ)$	$(1/T, \pm 90^\circ)$
Fold point (left/right)	$(0.1T, 0^\circ), (10/T, \pm 90^\circ)$	$(0.1T, 0^\circ), (10/T, \pm 180^\circ)$



3. THE CORRECTION OF APPROXIMATE LINE OF TYPICAL LINK

The approximate line has been constructed, and phase frequency responses are the same with amplitude frequency response. Plotting logarithm phase frequency response approximate line can show system graphics, because the approximate line

is more convenient and more practical to study and analysis than the real curve. But in a few instances, it requires to do some correction to change approximate line to the real curve. This paper gives revised errors of four approximate lines and real curves of four links, and when necessary, the approximate lines can be modified for real curves through the look-up table.

Table 2 The Correction Table Of Logarithm Phase Frequency Characteristic Curve Of Inertial Link

ωT	0.01	0.02	0.04	0.1	0.2	0.4	0.6	0.8	1
G'	0	0	0	0	-13.55	-27.09	-35.02	-40.64	-45
G	-0.57	-1.13	-2.18	-5.71	-11.31	-21.80	-30.96	-38.66	-45
Δ	-0.57	-1.13	-2.18	-5.71	2.24	5.29	4.06	1.98	0

Note: G' stands for approximation, G stands for the real value, Δ stands for error.

All we need is to build two correction tables for two groups of the four links are symmetrical. Table 2 lists the correction table of inertia link, it can also be used for first order differential link after symmetric conversion; Table 3 lists the correction table of oscillation link, it can also be used for second order differential link after symmetric

conversion. Two tables only list some points' value which lays the left symmetric point, and we can get the value of the right points through symmetry checking. In Table 2, Table 3 ωT and ξ take only part of the value; other values can be obtained by interpolation.

Table 3 The Correction Table Of Logarithmic Phase Frequency Response Curve Of Oscillation Links

	ωT	0.01	0.02	0.04	0.1	0.2	0.4	0.6	0.8	1
ξ	G'	0	0	0	0	-	-	-	-	-90
0.2	G	-0.11	-0.23	-0.46	-1.16	-2.39	-5.44	-	-	-90
	Δ	-0.11	-0.23	-0.46	-1.16	24.70	48.75	59.41	57.32	0
0.4	G	-0.23	-0.46	-0.92	-2.31	-5.41	-	-	-	-90
	Δ	-0.23	-0.46	-0.92	-2.31	21.68	43.41	54.11	51.54	0
0.6	G	-0.34	-0.68	-1.38	-3.47	-7.12	-	-	-	-90
	Δ	-0.34	-0.68	-1.38	-3.47	19.97	38.24	40.67	28.15	0
0.8	G	-0.46	-0.92	-1.84	-4.62	-9.46	-	-	-	-90
	Δ	-0.46	-0.92	-1.84	-4.62	17.63	33.34	33.16	20.64	0
1.0	G	-0.57	-1.15	-2.39	-5.77	-	-	-	-	-90
	Δ	-0.57	-1.15	-2.39	-5.77	15.32	28.73	26.88	15.51	0

Note: G' stands for approximation, G stands for the real value, Δ stands for error.

4. THE PLOTTING METHOD OF LOGARITHMIC FREQUENCY CHARACTERISTIC CURVE OF SYSTEM

The Characteristics of the Approximate Line of Logarithmic Frequency Response of System.

The entire system log phase frequency response can be plotted continuously at once after creating the approximate lines of links. Just as the amplitude frequency response, the approximate line of logistic system phase frequency response has the following features: (1) The left side is a y-coordinate $v \ 90^\circ$ for-the horizontal line (v for integral link number or negative differential link number); (2) The lines on the right side followed by the line in (1) change in slope regularly according to the following pattern (unit for 1 / dec). Inertia: "left"-45, "right" + 45; First order differential: "left" + 45, "right"-45; Oscillation: "left"-90, "right" + 90; second order differential: "left" + 90, "right"-90. ("Left" means the fold point which is on the left side of the symmetric point; "right" means the fold point which is on the right side of the symmetric point).

The Steps of Plotting the Approximate Line of the System Log Phase Frequency Response.

(1) Calculate the abscissa value of the symmetry points and left, right fold points of inertia, the first derivative, the oscillation, the second-order differential of the system. Then mark every slope that needs to be changed respectively from small to large, meanwhile mark every fold point in the coordinate.

(2) First plot a- $v \ 90^\circ$ of horizontal line at the most left end.

(3) Plot lines from left to right, and change the slope at the abscissa of left and right fold points according to the note, until the last link ends.

5. EXAMPLES

The transfer function of open-loop system is as follows, plotting its phase frequency characteristic curve (approximate line).

$$G(s) = \frac{20(s+5)(s+40)}{s(s+1)(s^2+4s+400)}$$

Solution:

(1) There are two first order differential links, a inertia link and a oscillation link, and the abscissa values of symmetric points, left and right fold points are as the following:

First order differential: symmetric point $\omega=5$, left fold point $\omega=0.5$, right fold point $\omega=50$;

Second order differential: symmetric point $\omega=40$, left fold point $\omega=4$, right fold point $\omega=400$;

Inertia link: symmetric point $\omega=1$, left fold point $\omega=0.1$, right fold point $\omega=10$;

Oscillation link: symmetric point $\omega=20$, left fold point $\omega=2$, right fold point $\omega=200$.

Sort the value of 8 left and right fold points from small to large, and mark every slope that needs to be changed respectively, meanwhile mark every fold point in the coordinate: 0.1 (inertia left:-45°), and 0.5 (first order differential left: + 45°), 2 (oscillation left:-90°), 4 (first order differential left: + 45°), 10 (inertia right: + 45°), 50 (first order differential right:-45°), 200 (oscillation right: + 90°), 400 (first order differential right:-45°).

(2) The number of integral link $v=1$ in this system, plot a $-1 \times 90 = -90^\circ$ horizontal line in the most left.

(3) Change the slope and plot new lines from left to right at the 8 fold points according to the requirements above, and the approximate line of log phase frequency response of open-loop system is shown in Figure 2.

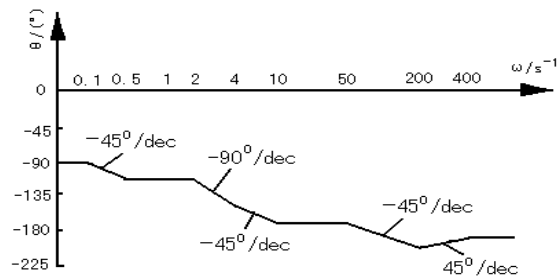


Figure 2 The Approximate Line Of Log Phase Frequency Response Of Open-Loop System That The Instance Seek

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