

ANALYSIS ON THE NEW CHARACTERISTICS OF WIND WAVES AT SEA SURFACE

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ABSTRACT

In order to comprehend the characteristics of sea waves, starting from the features of fluid dynamics, the relationship between wave height and sea surface wind speed was found, the wind speeds over sea surface were calculated with SMB formula, so got the corresponding relationship between the peak frequency of wave height and the wind speed. Sea wave spectrum was introduced, the simulation results suggested when the wave number is smaller, changing of wave velocity has greater impact on sea wave spectrum, but when the wave number is larger, no significant effect do velocities have on wave spectrums. By using the resonance condition between radar working frequency and that of sea wave, sea surface wave number model was set up. Simulation results revealed that although the cutoff wave number changes with wind speeds and radar frequencies, but in the range of normal working frequencies of HF radar and general sea wind speeds, the cutoff wave number basically stabilizes as a constant quantity. Research conclusions can provide theoretical basis for quantitative research of the sea waves characteristics; it also is an important basis for the simulation of radar sea echoes.

Keywords: *Wind Waves, Sea Wave Spectrum, Wave Height, Resonance Condition, Cutoff Wave Number*

1. INTRODUCTION

Wind acting on the sea surface to produce wave, and the wind's continuing role would form waves of the sea. According to Fourier theory, the sea surface can be regarded as a superposition of waves of different amplitudes, phases, frequencies and directions of arrival. When the effect of wind on sea continues for some time, the energy which sea waves derives from the wind could compensate for the energy consumption by air resistance, the sea surface reaches a state of dynamic equilibrium - steady state, and now the waves heights only relevant with the wind speed; but not all of the surfaces are unconditionally to achieve stable state. There are certain limitations to quantitative study of sea wave characteristics [1], the research needs to be further strengthened.

The time-varying characteristics of sea surface waves are very complex; it can only be described by using statistical methods. According to Fourier theory, any surfaces including those completely random surfaces can be represented as a superposition of different amplitudes, phases, frequencies and wave arrival directions; these waves should meet the dynamics theory.

The deep-water gravity wave is the main object of study. Typically, if the depth of sea wave is greater than half the wind wavelength, the

condition of deep water wave is considered satisfied.

To a steady sea surface, the sea wave height h can be expressed as [2]:

$$\begin{aligned}
 h(x, y, t) &= \sum_{n=1}^{\infty} a_n (k_x x \cos \theta_n + k_y y \sin \theta_n - \omega_n t - \psi_n) \quad (1)
 \end{aligned}$$

Of which, a_n is the single-wave amplitude, ω_n , k_n , θ_n and ψ_n the angular frequency, wave number, propagation direction angle and initial phase respectively, meanwhile $0 \leq \theta_n \leq 2\pi$, probability distribution function ψ_n being uniform distribution, i.e.

$$p(\psi) = \frac{1}{2\pi}. \quad (2)$$

The wave height distributions to steady sea surface belong to Gaussian distribution:

$$p(h) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{h^2}{2\sigma_n^2}\right), \quad (3)$$

Where σ_n^2 is the variance of sea wave height h .

Although various wavelengths are included in the sea wave height of formula (1), but the most important are the gravity wave and capillary wave, the gravity one is our key research object.

2. CHARACTERISTICS OF SEA WAVES HYDRODYNAMICS

For deep-water gravity waves in steady state, there are the following relationships, which sea waves must comply with:

When sea surface getting to its steady state, the height of sea wave reaches its maximum [3]

$$H_{\max} = \frac{0.26}{g} v_{10}^2. \quad (4)$$

Where v_{10} is the wind speed 10m above the sea surface, g (m/s^2) is the gravitational acceleration. Characteristic velocity of sea wave:

$$v = \sqrt{\frac{g}{k}} = \sqrt{\frac{g\lambda_s}{2\pi}} = \frac{g}{\omega} \quad (5)$$

Gravity wave frequency:

$$f = \frac{v}{\lambda_s} = \sqrt{\frac{g}{2\pi\lambda_s}} \quad (6)$$

Sea wave and its wavelength:

$$\lambda_s = \frac{g}{2\pi f^2} \quad (7)$$

Sea wave and its wave number:

$$k_s = \frac{4\pi^2 f^2}{g}. \quad (8)$$

3. CALCULATION OF SEA WAVE PARAMETERS - SEA WAVE SPECTRA

Sea wave spectra is the wave energy spectra, it gives directly the relationship between sea wave energy, wave frequency and wave direction. Because it can be easily obtained by observation, so it is the most fundamental method to describe sea waves.

Such as the adopted sea spectrum in actual use - the Pierson-Moskowitz spectrum [4, 5, 6] (PM), below is its undirected energy spectrum,

$$s(k) = \left(\frac{\alpha}{2}\right) k^{-4} \exp(-b(k_e/k)^2) \quad (9)$$

$$k_e = \frac{g}{v^2}, b = 0.74, v(h \geq 10m).$$

Where, α is Phillip constant quantity.

Figure 1 shows the undirected energy spectrum corresponding to velocity v of 4, 8, 12, 16 and 30m/s separately. Both horizontal and vertical coordinate axes of the figure choose the logarithm magnitudes.

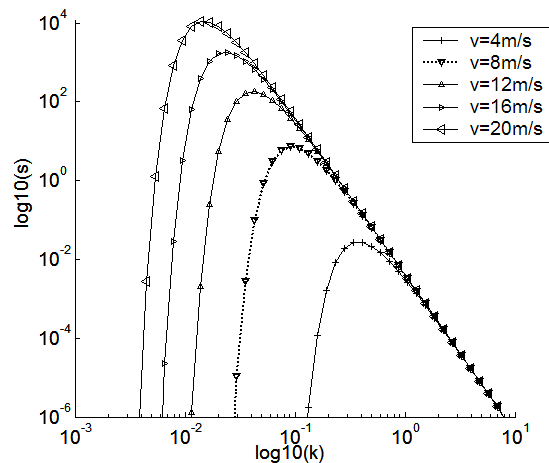


Figure 1: The Relationship Between Sea Wave Energy Spectrum Of PM Model $S(k)$ And The Sea Wave Number k

Simulation results show that for the PM wave: 1) the energy spectrum specific to different speed follows the same rule - It increases first and then decreases as wave number k keeping on increasing. 2) When with a higher speed, the wave energy spectrum has a larger maximum and mean value. 3) As the logarithm of wave number greater than 0.15, changing in wind speed has no differences to energy spectrum.

4. CALCULATION OF THE WIND SPEED ABOVE SEA SURFACE

For different heights above sea surface, the air density would vary with the heights, the wind pressure on the sea is also different from each height; the damping coefficient which atmosphere acting on wind is different too [7], so the wind speeds are not same even being over the same sea-surface. Figure 2 was the vertical distribution of longitudinal wind speeds at a vertical cross section BB' [8].

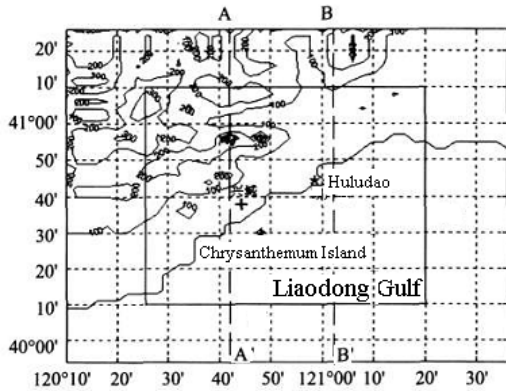


Figure 2: The Contour Map Of Terrain

The relation between wind speed and the height H above sea is as [9]:

$$V_H = V_{10} \{1 + [(C_{10})^{1/2} / k] \ln(H_E / 10)\} \quad (10)$$

Where V_H and V_{10} are the wind velocities at $H(m)$ and 10m above sea surface apart, k being Karman constant, C_{10} damping coefficient 10m above sea, and it having the relationship with V_{10} as the following [10]:

$$C_{10} = (0.80 + 0.11V_{10})10^{-3} \quad (11)$$

From the above two formulas, as long as V_{10} being established, the wind speed at any height above the sea can be determined.

The sea wave has two feature parameters; one is the valid wave height, the other the peak frequency of wave height spectrum. Both of these characteristic quantities are available through inversion of the second order radar cross section equation of sea echo Doppler spectrum. V_{10} can be computed by using SMB relational expression [11]:

$$\frac{gH_E}{V_{10}^2} = 0.26 \tanh[(f_M V_{10})^{-3/2} \frac{(3.5g)^{3/2}}{10^2}] \quad (12)$$

Where g is the gravitational acceleration, the numerical solution to V_{10} in transcendental equation (12), which is not easy to find, iteration method is used here; when solving it, first assuming the initial value of V_{10} , here setting 7, and substantially only iterating a few times, the result is stabilized. In addition, as sea waves needing to remain not broken, the effective height H_E and the

wavelength λ are demanded to meet $\frac{H_E}{\lambda} \leq 1/7$

[12].

The second radar cross section equation of Doppler spectrum of sea surface echo is used to invert the wave height parameter [6, 13, and 14], formula (12) is employed to calculate wind speed V_{10} . Computed results as shown in table 1, from it can be got the conclusions: the peak frequency of wave height spectrum f_m changes only depending on wind speed quantity, and no relation to the wind direction. The greater, the wind speed, the smaller f_m is.

TABLE 1: PEAK FREQUENCY f_M , EFFECTIVE WAVE HEIGHT, H_E AND THE CALCULATED WIND SPEED V_H

f_M / Hz	0.08	0.146	0.275
	0.08	0.146	0.275
H_E / m	5.50	2.32	0.28
	2.67	1.54	0.34
$V_H / \text{m} \cdot \text{s}^{-1}$	15.1185	10.5892	3.2786
	10.0707	7.9275	3.6459

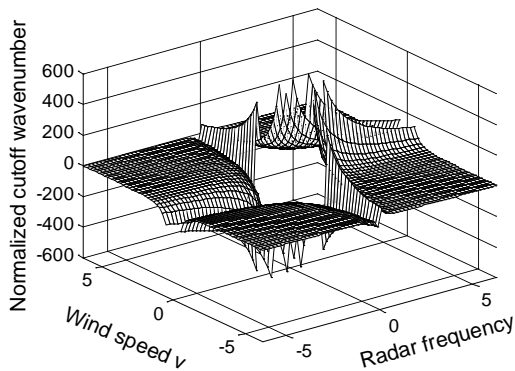
5. SEA WAVES CUTOFF WAVE NUMBER OF RADAR

The cutoff wave number of sea wave is specific to narrow-beam radar. Radar Frequency and that of sea waves must meet the resonance condition. From formulas (5) and (6), a corresponding wave number k_c can be found, the wave number k_c is

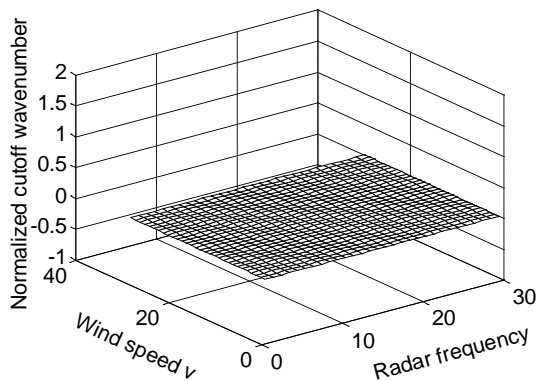
normalized by using $K_C = \frac{k_c}{2k_0}$, so got:

$$K_C = \frac{g}{v^2} \cdot \frac{1}{2k_0} = \frac{g}{v^2} \cdot \frac{c}{4\pi f_0} \quad (13)$$

The normalized cutoff wave number alters with the velocity v ; it is also associated with the radar frequency f_0 . This relationship is described in figure 3, figure 3_a only reflects the pure mathematical relationship, figure 3_b reflects a specific relationship which being in usual sea wind, normal working HF radar frequency.



A: This Figure Only Reflects The Mathematical Relationship



B: In The Range Of Normal Working Frequencies Of HF Radar And General Sea Wind Speeds
Figure 3: The Relationship Between Normalized Cutoff Wave Number And Wind Speed, Radar Frequency

By figure 3_b, when in HF radar operating frequencies and general sea wind speeds, K_c fluctuates very little, so is applied as a constant. Of course, in specific case, sea surface wind speed is also relevant to the sea state; figure 3_b does not reflect this meaning.

6. CONCLUSION

Using SMB formula to calculate wind speed, the peak frequency of wave height spectrum changes only depending on wind speed quantity, the greater, the wind speed, the smaller the peak frequency. Simulation results showed: when the logarithm of wave number k is much smaller than 0.15, the sea wave energy spectrum increases significantly with the increasing of sea wind speed, but when the logarithm of greater than 0.15, wind speeds cause no diversities..

The quantitative research on the sea wave characteristics is mainly organized on the basis of measurement. With the measured HF radar sea echo data and the gradual accumulation of data, the establishment of a more practical model will be important research tendency in the future.

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