

# BACKSTEPPING CONTROL FOR NONLINEAR SYSTEMS OF OFFSHORE PLATFORMS

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## ABSTRACT

The vibration control problem of offshore jacket platforms is studied. According to Morison equation, the total wave force acting on the offshore structure is obtained and is generated by an exosystem. Considering the nonlinear interaction of pile-soil, which is described by a fifth order anti-symmetric polynomial, the nonlinear dynamical model of offshore platforms affected by the wave load disturbances is established and its state-space described model is constructed by introducing the state vector. Based on Lyapunov stability theory, two Lyapunov candidate functions are chosen, then a back stepping controller is developed for the nonlinear systems of offshore structures under wave loading, and the nonlinear vibration control systems of the offshore platforms is exponentially stable. Numerical simulations illustrate the effectiveness of the proposed controller.

**Keywords:** *Nonlinear Systems, Backstepping Control, Offshore Platforms*

## 1 INTRODUCTION

Offshore structures such as jacket platform, mobile offshore base have received much interest due to their diverse roles. Some passive control systems [1] have been proposed and several active control techniques such as active mass dampers [2] and friction dampers [3] have been proposed.

To keep them safe and comfortable enough to guarantee the production and life of workers, the vibration control problem of offshore jacket platforms is needed to be considered. However, there are not many articles have dealt with the vibration control of offshore platforms. In [4], a damping isolation system was developed to mitigate earthquake and ice-induced vibrations of jacket offshore platforms. The experiments on steel rubber vibration isolator were carried out to investigate the compressive properties and fatigue properties in different low temperature conditions [5]. In [6], a tuned mass damper was proposed to reduce ice-induced vibration of an offshore platform. The optimal control is investigated for linear systems affected by external harmonic disturbance and applied to vibration control systems of offshore steel jacket platforms [7].

In most of the previous studies on offshore structural control, linear control theory has been used though there is nonlinear fluid structure interaction [2-7]. However, the nonlinear dynamics of offshore platforms has attracted much attention over the last several years. In [8], dynamic response analysis of a tension leg platform to deterministic

first order wave forces is studied and the analysis considers nonlinearities produced due to changes in cable tension and due to nonlinear hydrodynamic drag forces. An intelligent control technique using a neural network is proposed for seismic protection of offshore structures and a non-linear equation of motion considering fluid-structure interaction is derived and used to verify controller performance in numerical simulations [9]. In [10], the comparison of the two alternative nonlinear normal mode analysis techniques is completed and the effect of nonlinearity to a floating offshore platform is investigated.

In this paper, the offshore platform model is considered as non-linear systems with the nonlinear interactions of the pile-soil and the effect of the wave load disturbances. A back stepping controller is developed for the offshore structures under wave loading for the first time. Then, the performance of back stepping controller was evaluated in numerical examples.

In the following section, wave force model acting on the offshore structure is established. A dynamic equation of offshore structure under wave loading is derived. Then, a back stepping controller is presented. Finally, numerical verification was shown.

## 2 WAVE FORCES

According to Morison equation, the horizontal wave force  $p_u$  acting on the circle cross section of per unit height shown in Figure 1 is as follows:

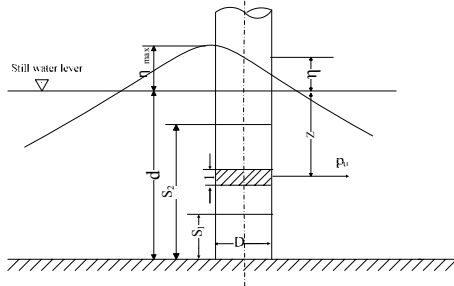


Figure 1: The Wave Forces Acting On The Pile

$$p_u = \frac{\rho C_D D |v| v}{2g} + \frac{\pi \rho C_M D^2}{4g} \frac{\partial v}{\partial t}, \quad (1)$$

where  $C_D$  is the coefficient of the resisting force,  $C_M$  is the coefficient of inertia,  $\rho$  is the density of sea water,  $g$  is gravitation acceleration,  $D$  is the equivalent characteristic diameter of the offshore structure legs,  $v$  and  $\frac{\partial v}{\partial t}$  are the horizontal water particle velocity and acceleration respectively, which are described by

$$v = \frac{\pi H}{T} \frac{\cosh kz}{\sinh kd} \cos \theta, \quad (2)$$

$$\frac{\partial v}{\partial t} = \frac{2\pi^2 H}{T^2} \frac{\cosh kz}{\sinh kd} \sin \theta,$$

in which  $T$  is the wave period,  $H$  is the wave height,  $k$  is the wave number,  $\theta = kx - \omega t$ ,  $x$  is the abscissa,  $z$  is the ordinate.

Then, the total wave force acting on the vertical cylinder is as follows:

$$\bar{p}(t) = \int_{S_1}^{S_2} p_u dz, \quad (3)$$

in which  $S_1$  and  $S_2$  are the heights of the two points over the seabed. Substituting (1) and (2) into (3) and letting  $x=0, S_1=0, S_2=d$  yield the total force acting on the offshore structure

$$\bar{p}(t) = \frac{\rho g D H^2}{8} \left[ \frac{C_D (2kd + \sinh 2kd)}{2 \sinh 2kd} |\cos \omega t| \cos \omega t - \pi D C_M \tanh(kd) \sin \omega t \right] \quad (4)$$

For the vibration controller designing convenience, we rewrite wave force (4) in a dynamic exosystem. Denote

$$\bar{p}_j = A_j \sin(\omega_j t + \varphi_j), \quad j = 1, 2, \dots, r, \quad (5)$$

$$\tilde{p}(t) = [\bar{p}_1, \bar{p}_2, \dots, \bar{p}_r]^T,$$

in which  $\omega_j$  is frequency of the  $j$  th wave component,  $\varphi_j$  is random phase angles uniformly distributed in  $0 \leq \varphi_j < 2\pi$ . Then we have

$$\ddot{\bar{p}}_j = -\omega_j^2 \bar{p}_j, \quad j = 1, 2, \dots, r, \quad (6)$$

$$\ddot{\tilde{p}}(t) = -\bar{\Omega}^2 \tilde{p}(t).$$

where  $\bar{\Omega} = \text{diag}\{\omega_1, \omega_2, \dots, \omega_r\}$ .

Letting  $w(t) = [\bar{p}(t) \quad \dot{\tilde{p}}(t)]^T$ , then

$$\dot{w}(t) = \bar{G}w(t), \quad \tilde{p}(t) = [I_r \quad 0]w(t), \quad (7)$$

$$p(t) = [1, \dots, 1]\tilde{p}(t) = \tilde{F}w(t),$$

where

$$\bar{G} = \begin{bmatrix} 0 & I_r \\ -\bar{\Omega}^2 & 0 \end{bmatrix} w(t), \quad \tilde{F} = [1, \dots, 1, 0, \dots, 0] w(t),$$

$I_r$  is the unit matrix,  $0 \in R^{r \times r}$  is the zero matrix.

So the total wave force acting on the offshore structure can be generated by the exosystem

$$\dot{w}(t) = \bar{G}w(t), \quad (8)$$

$$p(t) = \tilde{F}w(t).$$

### 3 EQUATION OF CONTROLLED SYSTEM

It is very complex of the actual offshore platform system, and it has different simplified model for the concrete study. Because the first mode response contributes most to the dynamical model, the offshore structure is simplified into single degree of freedom. Usually the approximation is adequate for the purpose of the vibration control. The sketch of the system is shown in Figure 2.

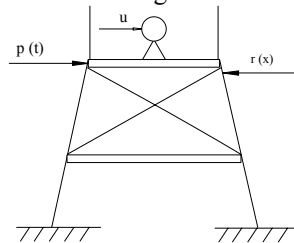


Figure 2: Sketch Of Offshore Platform

Then, we give nonlinear motion equations of the jacket-type offshore platform. The nonlinear interaction of pile-soil is expressed by  $r(\cdot)$  which is a fifth order anti-symmetric polynomial. The motion equations of the structure can be expressed as

$$m\ddot{x} + c\dot{x} + kx + r(x) = p(t) + u, \quad (9)$$

$$x(0) = x_0,$$

where  $x$ ,  $\dot{x}$ , and  $\ddot{x}$  are structural displacements, velocities and accelerations, respectively.  $m$ ,  $c$  and  $k$  are mass, damping and stiffness matrices of the system, respectively;  $u$  is control forces;  $r(x) = a_1x + a_2x^3 + a_3x^5$  is the interaction of pile-soil.

By introducing the state vector  $x_1 = x$ ,  $x_2 = \dot{x}_1$ , we can obtain the state-space model

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{m}x_1 - \frac{c}{m}x_2 - \frac{1}{m}r(x) \\ &\quad + \frac{1}{m}p(t) + \frac{1}{m}u \end{aligned} \quad (8)$$

#### 4 CONTROL LAW DESIGN

In this paper, we employ back stepping procedure to design the controller  $u$ .

Then, we have the main result.

**Theorem 1.** If we design the controller  $u$  as

$$u = -2m(x_1 + x_2) + kx_1 + cx_2 + r(x) - p(t) \quad (11)$$

then the controlled system (10) exponentially stable.

**Proof.** The back stepping procedure includes two steps in this paper.

Step 1. Let  $z_1 = x_1$ , we define a Lyapunov candidate

$$V_1 = \frac{1}{2}z_1^2 \quad (12)$$

Its derivative along the solutions of system (10) is

$$\begin{aligned} \dot{V}_1 &= z_1\dot{z}_1 = x_1x_2 \\ &= -x_1^2 + x_1(x_1 + x_2) \\ &= -z_1^2 + z_1(x_1 + x_2) \end{aligned} \quad (13)$$

Step 2. Let  $z_2 = x_1 + x_2$ , we form the Lyapunov function as

$$V = V_1 + \frac{1}{2}z_2^2 = \frac{1}{2}(z_1^2 + z_2^2). \quad (14)$$

The time derivative along the solutions of (10) is

$$\begin{aligned} \dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 \\ &= -z_1^2 + z_1(x_1 + x_2) + z_2(\dot{x}_1 + \dot{x}_2) \\ &= -z_1^2 + z_1(x_1 + x_2) + (x_1 + x_2)(\dot{x}_1 + \dot{x}_2) \\ &= -z_1^2 + (x_1 + x_2)(z_1 + \dot{x}_2) \\ &= -z_1^2 + (x_1 + x_2)(x_1 + x_2 + \dot{x}_2) \\ &= -z_1^2 + (x_1 + x_2)(x_1 + x_2 - \frac{k}{m}x_1 - \frac{c}{m}x_2 \\ &\quad - \frac{1}{m}r(x) + \frac{1}{m}p(t) + \frac{1}{m}u) \\ &= -(z_1^2 + z_2^2) \\ &= -2V \end{aligned} \quad (15)$$

Therefore, by the control input (11), we can have

$$\dot{V} = -(z_1^2 + z_2^2) = -2V < 0 \quad (16)$$

From (8) we can yield  $V \leq V(0)e^{-2t}$ , i.e.,  $\|x_i\| \leq 2V(0)e^{-2t}$  ( $i = 1, 2$ ). Hence, the controlled system (10) system is exponentially stable.

#### 5 SIMULATION

In this section, the approximate optimal control law is applied to a jacket platform for illustration of its effectiveness. The jacket platform located in Bohai Sea is made of jacket structure, piles and double-deck decks. The model parameters have the values listed in Table I [2, 7].

Table I: Main Structural Parameters And Design Sea State

Platform	Total mass	2 708 900 kg
	First model mass	2 371 100 kg
	Damping ratio	4%
	First model period	2.086 s
	Total height	41.1 m
	Equivalent diameter	1.7 m
Wave load	Equivalent drag coefficient	2.0
	Equivalent inertia coefficient	1.2
	Average water depth	13.2 m
	Wave period	4.5 s
	Significant wave height	2.5 m

Then we present wave load, displacement and velocity curves of the jacket platform in Figures 3–5, respectively.

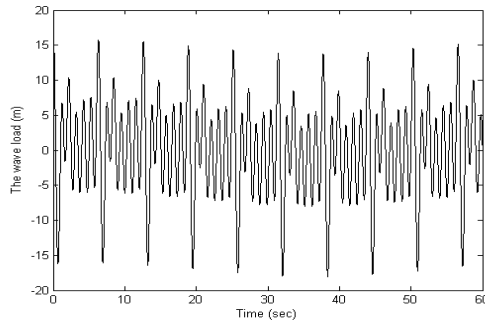


Figure 3: Wave Load

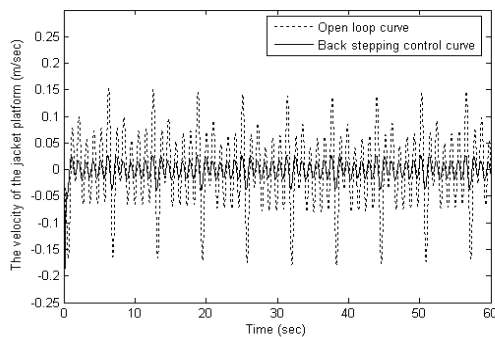


Figure 4: Velocity Curves Of The Jacket Platform

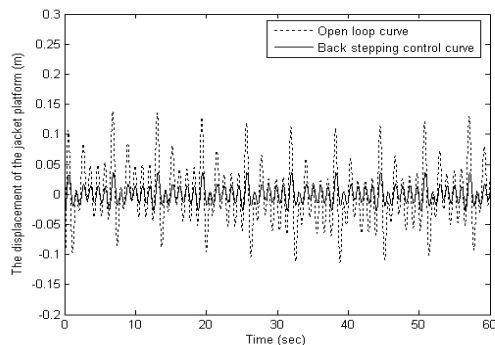


Figure 5: Displacement Curves Of The Jacket Platform

In Figures 4-5, it is demonstrated clearly that the approximate optimal control law is efficient in reducing the displacement and velocity of the jacket platform subjected to irregular wave forces.

## 6 CONCLUSION

In this paper, to study the vibration control problem of offshore jacket platforms, a nonlinear

dynamical platform model has been established. Then, a back stepping controller is developed for the nonlinear systems of offshore structures under wave loading. Numerical simulations illustrate the effectiveness of the proposed controller.

## REFERENCES:

- [1] K. C. Patil, R. S. Jangid, "Passive control of offshore jacket platforms", *Ocean Engineering*, Vol. 32, No. 16, 2005, pp. 1933-1949.
- [2] H. Ma, G.-Y. Tang, Y.-D. Zhao, "Feedforward and feedback optimal control for offshore structures subjected to irregular wave forces", *Ocean Engineering*, Vol.33, No.5, 2006, pp. 1105-1117.
- [3] A. A. Golafshani, A. Gholizad, "Friction damper for vibration control in offshore steel jacket platforms", *Journal of Constructional Steel Research*, Vol. 65, No. 1, 2009, pp. 180-187.
- [4] J.- P. Ou, X. Long, Q. S. Li, Y. Q. Xiao, "Vibration control of steel jacket offshore platform structures with damping isolation systems", *Engineering Structures*, vol.29, No.3, 2007, pp. 1525-1538.
- [5] Y. J. Xu, Y. B. Liu, C. Z. Kan, Z. H. Shen, Z. M. Shi, "Experimental research on fatigue property of steel rubber vibration isolator for offshore jacket platform in cold environment", *Ocean Engineering*", Vol. 36, No. 8, 2009, pp. 588-594.
- [6] Q. J. Yue, L. Zhang, W. S. Zhang, T. Kärnä, "Mitigating ice-induced jacket platform vibrations utilizing a TMD system", *Cold Regions Science and Technology*, Vol. 56, NO. 2, 2009, pp. 84-89.
- [7] W. Wang, G.-Y. Tang, "Feedback and Feedforward Optimal Control for Offshore Jacket Platforms", *China Ocean Engineering* Vol. 18, No. 5, 2004, pp. 515-526.
- [8] A. K. Jain, "Nonlinear coupled response of offshore tension leg platforms to regular wave forces", *Ocean Engineering*", Vol. 24, No. 7, 1997, pp. 577-592.
- [9] H. K. Dong, "Neuro-control of fixed offshore structures under earthquake", *Engineering Structures*", Vol. 31, No. 2, 2009, pp. 517-522.
- [10] J. M. Falzarano, R. E. Clague, R.S. Kota, "Application of nonlinear normal mode analysis to the nonlinear and coupled dynamics of a floating offshore platform with damping", *Nonlinear Dynamics*, Vol. 25, No. 2, 2001, pp. 255-274