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RESEARCH ON CENVERGENCE OF ANT ALGORITHM

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ABSTRACT

A basic ant algorithm based on elite strategy to resolve the shortest path problem is given, In terms of stochastic processes and functional analysis methods; the abstract description of the basic ant algorithm is established. By using the basic ant algorithm, the iteration feature of problem-solving is studied, the conclusion can be drawn that the solution sequence is asymptotic convergence.

Keywords: *Ant Algorithm; Markov Process; Convergence; Fixed Point*

1. INTRODUCTION

The ant algorithm is learning the behavior characteristics of ants and bionic algorithm by simulating insect kingdom ant foraging behavior. Ant algorithm has been widely used in various combinations, optimization problems, but still lacks theoretical foundation.

Based on the similarity of the step-by-step iteration and transfer mechanisms of evolutionary class of algorithms and discrete Markov process, simulated annealing, genetic algorithms have used the principle of the Markov chain in analyzing the convergence characteristics of the algorithm. Due to the transition probability of the ant algorithms are constantly changing, there are certain difficulties for the use of discrete Markov chain converges to the steady state theory.

Ant algorithm applied in a variety of problems has a different deformation, start from a basic ant algorithm, with the help of a major theorem in functional - the contraction mapping principle as a research tool to analyze the convergence of the ant algorithm.

2. THE BASIC PROCESS OF THE ANT ALGORITHM

2.1 The Basic Assumption

For convenience of analysis, the basic ant algorithm of solving the shortest path algorithm on acyclic graph G = (V, E) is given, which has the following assumptions:

(1) G is a non-self-acyclic graph (V-set of points-set of edges);

(2) Elitist strategy is included;

(3) The starting point and the end point is sole;

(4) The target of ants is finding a shortest path from the starting point to the end point;

(5) There is at least a feasible path;

(6) The objective function of each feasible option is sum of each edge length on the path;

(7) The best path is only one.

This setting is only to facilitate the following algorithm convergence analysis; the application of the algorithm may not have fully accorded to the operation which is a basic form. In fact, different ant algorithms can be formed by the different tracks to update the equation.

2.2 Main Algorithm Parameters Set

The transition probability p_{ij}

$$p_{ij} = \frac{\left[\tau_{ij}\right]^{\alpha}\left[\eta_{ij}\right]^{\beta}}{\sum_{k}\left[\tau_{ik}\right]^{\alpha}\left[\eta_{ik}\right]^{\beta}} \tag{1}$$

Parameter α represents the relative importance of track pheromone, β represents the relative importance of the path visibility.

(2)The path visibility η_{ij}

η_{ij} Can be set to the reciprocal of the distance between v_i and v_i .Other function is allowed which

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compliances with the principle that path is the shorter visibility of the higher.

(3)The section tracks pheromone of single ant searching $\Delta\tau_{ij}{}^{k}$

$$\varDelta\tau_{ij}{}^{k} = \begin{cases} \varPhi\left(Z^{k}\right), & (i,j)\in w^{k} \\ 0, & \text{otherwise} \end{cases}$$

$$\left(\Phi\left(Z^{k}\right) = \begin{cases} C\ (>0), & Z^{k} \le Zn \\ 0, & Z^{k} > Zn \end{cases}\right)$$

(2)

Tracks pheromone will be released only when the ant's path is better than Wn.

(4) Trajectory pheromone update value τ after the each ant finishes its pathfinding

$$\tau = \sum_{(i,j)\in E} \sum_{k=1}^{AS} \Delta\tau_{ij}^{k} \tag{3}$$

(5) the track strength update equation

$$\tau_{ij}{}^{n} = \begin{cases} \tau_{ij}{}^{n-1}, & if\ \tau = 0 \\ (1-\rho)\tau_{ij}{}^{old} + \dfrac{\rho}{\tau}\sum_{k}\tau_{ij}{}^{k}, & \text{i f } \tau > 0 \end{cases} \tag{4}$$

ρ Represents The pheromone decay rate.

2.3 The Detailed Algorithm Process

Begin

Iteration times are given its initial value: $n \leftarrow 0$;

The ant population is settled at AS on the start point;

Initialize the track strength on every edge (v_i, v_j): $\tau_{ij} \leftarrow 1/L$ (L: total number of E);

Initialize the pheromone strength on edge (v_i, v_j) left by ants : $\Delta\tau_{ij}^{k} \leftarrow 0$;

Initialize $Z_1 \leftarrow L$, $W_1 \leftarrow \varphi$;

While $n <$ the default number of iterations and without degradation behavior begin

Moving to next point with a probability of p_{ij}, K-th ant finishes one random walk w^{k}, $\Delta\tau_{ij}^{k}$ is calculated;

Get functional value Z^{k} of every ant;

Putting $W(n)$ to be equal to the best path of the n-th searching, $Z(n) = Z(W(n))$;

By comparing $W(n)$ and $Wn-1$, putting Wn to be equal to the best path of the first n searching, Zn is corresponding functional value;

Figure up τ, modification track strengths with track update equations;

$n \leftarrow n+1$;

End

Output the best solution and the optimal path

End

Used $\left(\tau(n), W(n), Z_n\right)$ to record the iterative process of the ant algorithm, in which $\tau(n)$ represents all edges of the arc of the track of the pheromone vector τ_{ij} in the n-th round of the iterative algorithm. The track pheromone can be updated at the end of the iteration round.

3. CONVERGENCE ANALYSIS OF THE ANT ALGORITHM

If the iterative process of the ant algorithm is described as a finite Markov process, it can be proved, in case under certain conditions, the entire iteration reaches a steady state after a period of time and a certain means that ant algorithms can converge to asolution.

Theorem 1 Variables ($\tau(n)$, $W(n)$, Z_n) $(n = 1,2,\cdots)$ form a finite Markov chain.

Theorem-proving:

Firstly, in accordance with the track update guidelines $\tau(n)$ is determined by the state of the first $n-1$ rounds. Secondly, the probability distribution of $W(n)$ is depends on $\tau(n)$, so indirectly depends on the $n-1$-th round. at last,

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$Z_n = \min\{Z(W(n)), Z(n-1)\}$.

Thus the iterative process of search results is a Markov chain.

Satisfactory solution in optimizing care only about the change process, therefore, define the basic ant algorithm is mapped as $w_n = T(w_{n-1})$,to record the best path of the first $(n-1)$ round of changes.

Definition 1 Mapping $d: W \times W \to R$, $d(w_i, w_j) = |Z_i - Z_j|$, $w_i, w_j \in W$.

Theorem 2 (W,d) forms a metric space. ($d: W \times W \to R$, $d(w_i, w_j) = |Z_i - Z_j|$, $w_i, w_j \in W$)

Theorem-proving:

W is a non-empty, and the follows are satisfied :

① $d(w_i, w_j) \ge 0$, if and only if $w_i = w_j$, $d(w_i, w_j) = 0$

②
$$d(w_i, w_j) = |Z_i - Z_j| = |Z_j - Z_i| = d(w_j, w_i)$$

③
$$\begin{aligned} d(w_i, w_j) &= |Z_i - Z_j| \\ &= |(Z_i - Z_p) + (Z_p - Z_j)| \\ &\le |Z_i - Z_p| + |Z_p - Z_j| \\ &= d(w_i, w_p) + d(w_p, w_j) \end{aligned}$$

Theorem 3 The metric space (W,d) is complete。

Theorem-proving:

For any Cauchy sequence in W, the following condition is met: $\forall \ \varepsilon > 0$, $\exists \ N$,make any $i, j > N(\varepsilon)$, $d(w_i, w_j) < \varepsilon$.

As W is a finite state space, Cauchy sequence W must be a constant sequence from the N item. In other words, the Cauchy sequence $\{w_i\}$ converges, so (W,d) is complete.

Theorem 4, the operator generated by basic ant algorithm ($T: W \to W$) is contractive, furthermore, there exists a unique fixed point on the complete metric space W, of which the operator asymptotic convergences.

Because of the elitist strategy, $0 \le Z_{n+1} \le Z_n$, $\{Z_n\}$ forms a monotonically decreasing sequence. So $\{Z_n\}$ must have the lower bound, while decreasing sequence with a lower bound is inevitable convergence.

therefoe, $\{Z_i - Z_j\}$ is An infinitesimal series, $\forall \varepsilon > 0$, $\exists N$,while $i, j > N(\varepsilon)$, $|Z_{i+N} - Z_{j+N}| < \varepsilon$。

specially, when $\varepsilon = \theta|Z_i - Z_j|$, $(0 \le \theta < 1)$,

$$\begin{aligned} & d(T^N w_i, T^N w_j) = |Z_{i+N} - Z_{j+N}| \\ & < \varepsilon = \theta \cdot |Z_i - Z_j| = \theta \cdot d(w_i, w_j) \end{aligned}$$

So , There must be the only fixed point w^* which is the only optimal path in complete metric space T , and T asymptotic convergences w^*.

Based on the above analysis, the basic ant algorithm to solve the iterative process is a random contraction mapping, the algorithm must converge to the only fixed point of the objective function to reach the global minimum Z^*, and the iterative process is convergent.

4. CONCLUSION

Due to the constantly changing feature of the transition probability of the ant algorithms , there are certain difficulties in the application of discrete Markov chain theory.

Some research to be translated into the calculation of the ant algorithm to converge to the optimal solution of minimum probability.

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In this paper, the theory of random functional analysis is used to proof the convergence of the basic ant algorithm. The result demonstrates the ant algorithm is gradually converge to the fixed point, which provide a theoretical basis for the application and promotion of the ant algorithm

REFERENCES

[1] Dorigo M, Maniezzo V, Colorni A. Ant system: optimization by a colony of cooperating agents[J]. IEEE Trans. on SMC, 1996, 26 (1): 29-41.

[2] Ma Liang, Wang Longde. A new algorithm for solving multicriteria shortest path problem[J]. Journal of Systems Science and Systems Engineering, 1999, 8 (3): 335-339.

[3] Ma Liang, Xiang Pei-Jun.Ant algorithm in combinatorial optimization [J]. Management Science, 2001, 4 (2): 32-37.

[4] Cuixue Li, Ma Liang. Shortage restrictions VRP ant algorithm [J]. Shanghai University of Technology, 2003, 25 (1): 39-44.

[5] Zheng Jin-hua, Lu Hui, Wu Jun. Space-based mating convergence analysis of the genetic algorithm [J]. Pattern Recognition and Artificial Intelligence, 2010 (7) :639-645.

[6] Feng Yurong, ZHU Jun Yan. The simulated annealing algorithm Convergence [J]. Fujian computer, 2006 (12) :46-47.

[7] Gutjahr W J. A Graph-based Ant System and its Convergence[J]. Future Generation Computer Systems, 2000, 16 (8): 873-888.

[8] Apostol T M. Mathematical analysis[M]. Menlo Park, California: Addison-Wesley, 1974.

[9] Xu Tian-Zhou. Application of functional analysis [M] Beijing: Science Press, 2002