

TUBULAR SURFACES EXTRACTION WITH MINIMAL ACTION SURFACES

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ABSTRACT

This paper presents a new fast approach for surfaces extraction of tubular structures, such as blood vessels, tracheas, and colons. Since they are usually long and thin, when being extracted with the traditional Minimal Action Surface method, edges leakage problem could not be avoided. So a front freezing criterion based on the front's average energy is developed. To freeze the tail of the front whose energy is less than the average energy, and to continue propagating the head of the front whose energy is larger than the average energy. While the head front gets the end of the structure, the whole tubular structure surfaces are extracted. Experimental results show the freezing criterion based on the front's average energy avoids the edges leakage problem effectively. The criterion is based on the global information of the front, so can enhance the robustness to image noise and the local minimum.

Keywords: *Minimal Action Surface, Fast Marching Method, Tubular Structure Surfaces, Average Energy Criterion*

1 INTRODUCTION

For decades, the extraction of tubular surfaces such as blood vessels, trachea, colon, or other tube-like structures has attracted the attention of more and more researchers[1]. Much work has been done on surface segmentation since the introduction of active contour models[2,3]. The global minimum for active contour models introduced by Cohen and Kimmel[4] presents a set of Minimal Action Surfaces of a given single point inside tubular structure, which can be seen as the front propagation process and can be used to extract the tubular surfaces. The Fast Marching Method is introduced to implement the Minimal Action Surfaces of a given point, inflating a long balloon from the given point in the tubular object.

However, since the tubular structures are usually rather long and thin, the front propagation may blow up through the structure edge closed to the given point, while the front has not reached the end of the structure. In order to avoid the edges leakage problem, Deschamps and Cohen[5] proposed a freezing criterion based on geodesic distance to stop those points that should not have any influence on the front propagation any more, and to keep the front propagating inside the desired object. But the freezing criterion is based on the geodesic distance to the given point, which is the local information of the front, so it is sensitive to image noise and the

local minimum. The front's average energy includes the statistic information of all points on the front, which is the global information[6,7]. Lu and Zhuang make use of the characteristic of it to remove all the over-segmentation points when segmenting the low contrast medical images[8]. But the over-segmentation method is not suitable for extracting the tubular surfaces, which are always circuitous and strip. When the method is used to segment them, the front propagation should go through the boundary of the structure, so resulting in error segmentation, especially when two parts of anatomical object are much closed.

In this paper, we present a front freezing criterion based on the front's average energy to stop those points of lower energy, so as to extract tubular surfaces with the Minimal Action Surfaces effectively. Firstly, we analyze the problem of edges leakage when using the Minimal Action Surfaces method to extract tubular surfaces. Then the front's average energy is defined and its characteristic is analyzed. Lastly, a front freezing criterion based on the average energy is developed and its Fast Marching Method is given. Experimental results show that our freezing criterion based on the global energy information of the front is robust to image noise and the local minimum.

2 MINIMAL ACTION SURFACES

2.1 The Definition

An active contour model can map into an energy curve $C(s): \Omega \rightarrow R^2$, where s is the arc-length parameter, $\Omega = [0, L]$ is its domain of definition, and L is the length of the curve. Cohen and Kimmel [4] define the energy of the model as below:

$$E(C) = wL + \int_{\Omega} P(C) d_s = \int_{\Omega} \tilde{P} d_s \quad (1)$$

where $\tilde{P}(p) = w + P(p)$ is potential function, regularization term w maintains the smoothness of the front propagation.

The Minimal Action Surface U is defined as the minimal energy integrated along a path between a starting point p_0 and any point p :

$$U_0(P) = \inf_{C(L)=p} \left\{ \int_{\Omega} \tilde{P} d_s \right\} = \inf_{A_{p_0,p}} E(C) \quad (2)$$

where $A_{p_0,p}$ is the set of all paths between p_0 and p .

In order to compute this map U , the evolution curve is treated as a function of time t , that is $C(s,t)$, then the Minimal Action Surface $U_0(P)$ becomes its level set function, and represents the height of the level set of U , so it can be seen as the front propagation process and satisfies the Eikonal equation:

$$\|\nabla U\| = \tilde{P} \quad (3)$$

The Fast Marching Method (FMM) which relies on a one-sided derivative that looks in the up-wind direction of the moving front can get stable results. Deschamps and Cohen [9] extended the FMM to 3D to compute the minimal action U starting from an initial infinitesimal front around start point p_0 in 3D.

The finite differences is:

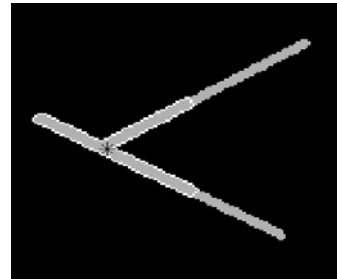
$$\begin{aligned} & (\max\{u - U_{i-1,j,k}, u - U_{i+1,j,k}, 0\})^2 \\ & + (\max\{u - U_{i,j-1,k}, u - U_{i,j+1,k}, 0\})^2 \\ & + (\max\{u - U_{i,j,k-1}, u - U_{i,j,k+1}, 0\})^2 = \tilde{P}_{i,j,k}^2 \end{aligned} \quad (4)$$

giving the correct viscosity-solution u for $U_{i,j,k}$.

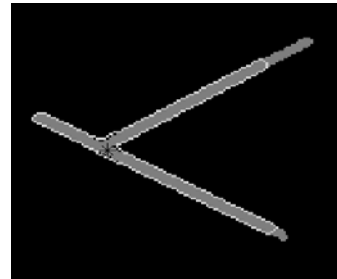
2.2 The Problem

When applying the Minimal Active Surfaces to extract tubular surfaces, we may assume that the potential is smaller inside the tubular object than on its boundary. Therefore, the front inside the tubular object march faster than that is located close to its boundary. We could call the head of the front with the faster speed, and call the tail of the front with the

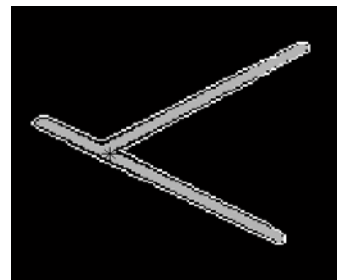
slower speed. Because tubular structures are thin and long, when most part of the front reaches the boundary and its speed is slow, there is still a small part of front in the central cavity is marching to the depths of tubular object by faster speed. After the front propagating process, an elongated shape along the tubular surface would be formed. However, the non-negative potential energy makes the tail of the front, which is already located on the edge of tubular structures, continue propagating, and leading to edges leakage. A synthetic example of a thin and long structure is shown in Fig. 1, where “*” is the start point given by user. Fig.1 (a) shows the initial phase of the front propagation. There is no front passed through the boundary, and the head of the front is rapidly marching forward. Fig.1 (b) shows the intermediate phase, although the head has not reach the end of the structure, the edges leakage has occurred partly in the tail of the front. Fig.1 (c) shows the result of the final front, when the entire front has almost stopped marching, and the leak phenomenon near the start point is very obvious.



(A) Iteration 600



(B) Iteration 900



(C) Iteration 1200

Figure 1: Synthetic Test Problem

3 THE AVERAGE ENERGY FREEZING CRITERION

Obviously, it is the non-negative potential \tilde{P} resulting in the edges leakage. However, the definition of potential function comes from the gray intensity or gradient information of image. It is generally difficult to design a potential function which makes the energy of the front be zero on the boundary of the desired object.

Combining with the performance of the Minimal Active Surface, we can freeze the tail of the front which is located close to the boundary, that is, set its propagation speed to zero, and only to propagate the head until reaching the end of the structure or a specific place. Deschamps and Cohen [5] proposed a freezing criterion based on geodesic distance. It accords to the current maximum geodesic path length d_{max} in the front propagation process to remove the voxel v from the front by these two criterions:

$$D(v) < d_{max} / \alpha, \text{ with } \alpha \geq 1 \quad (5)$$

$$D(v) < \max(d_{max} - \tilde{d}, 0) \text{ with } \tilde{d} > 0 \quad (6)$$

where α or \tilde{d} is the user-defined value.

Obviously, the criterion is based on the local information of the front, so it is sensitive to image noise and the local minimum.

We shall analyze the characteristic of the front energy in the front propagation process, and then design a freezing criterion based on the front's average energy.

3.1 Analysis Of The Front Energy

According to the Eikonal equation of U, Eq.(3), the front energy is inversely proportional to the potential, then we define the energy of a voxel v on the front as:

$$E_{front}(v) = \frac{1}{\tilde{P}} \quad (7)$$

The average energy of the front is defined as:

$$E_{mean}(t) = \frac{1}{N_{front}} \sum_{v \in front} E_{front}(v) \quad (8)$$

where $front$ is the front on time t , and N_{front} is the number of voxels of the front.

Fig.2 shows the change process of the propagating front's average energy with the evolution time during extracting the colon surface with the Minimal Active Surface method. The

potential is defined as: $\tilde{P} = |v(x) - 200|^2 + 10$.

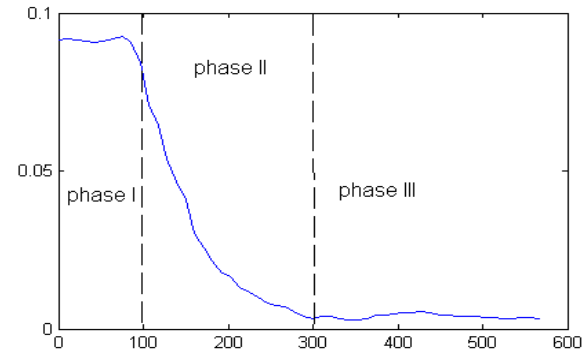


Figure 2. Propagating Front's Average Energy Vs Time

According to Fig.2, we can divide the front's average energy into three phases during the front propagation process:

Phase I: The front starts from the given point, and move to the boundary of tubular object. The energy of the front is enough large to move the front with almost constant speed.

Phase II: Part of the front is closed to the boundary of the object, and the front's average energy decline rapidly.

Phase III: Most of front is closed to the boundary of the object, and the average energy is almost zero. Only the head of the front maintains a larger energy. With the evolution time increasing, the front's average energy fluctuates slightly, which indicates a part of the front is likely to go through the edge of the object.

Based on the above analysis, it is necessary to freeze the tail front focus on the third stage. In fact, the section of a tube-shaped object must be small with respect to its length, so most of the front should be the tail. The potential is large around the target boundary, and accordingly, the corresponding energy is lower than the front's average energy. But the energy of the head front is greater than the average energy, so we can use the front's average energy to design a freezing criterion, which is described as:

$$E_{front}(v) < E_{mean}(t) \text{ while } high \geq E_{mean}(t) \geq low \quad (9)$$

where $high$ and low are thresholds controlling the $tail$ front in the third stage to be frozen.

3.2 3D FMM Based On The Average Energy Criterion

Definition:

– Accepted is the set of all grid points at which the u -value have been reached and will not be changed;

- Trial is the set of grid points to be examined. Their u -values have been computed using Eq.4;
- Far is the set of all the other grid points. Their u -values have never been computed.
- Initialization:
 - Setting the starting point p_0 as trial set with $u(p_0) = 0$, compute its energy by Eq.7, and storing it in both min-heap structures TR and ER ;
 - Setting all the other grid points with $u = \infty$.
- Loop:
 - Let p_{min} be the Trial point with the smallest u -value, the roof of TR ;
 - Move it from the Trail to the Accepted set, and delete it in both TR and ER ;
 - For each six-neighbor p of p_{min} :
 - If p is the Far set, set it with the Trial set, add it into both TR and ER ;
 - If p is the Trial set, update both TR and ER with the new action values for u -value and energy computed;
 - If TR is not empty, compute the average energy E_{mean} of all the points in it, or stop loop;
 - If $E_{mean} > high$, return Loop; $E_{mean} < low$, stop loop; otherwise continue;
 - Let q_{min} be the roof point of ER , if $E(q_{min}) < E_{mean}$, delete it in both TR and ER , and move it to the Accepted set;
 - If TR is empty, stop loop, otherwise continue.

4 EXPERIMENTAL RESULTS

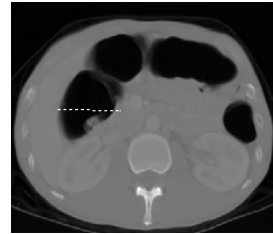
4.1 Building A Potential

When extracting tubular surfaces with the Minimal Action Surfaces method, the potential function must satisfy the following conditions [9]: Firstly, the potential must be lower inside the anatomical object in order to propagate the front faster, and to avoid crossing the edges of the anatomical object. Secondly, around of the walls of the anatomical object, the value of potential is as high as possible, so as to prevent the front going through the walls. Thus the potential is designed as:

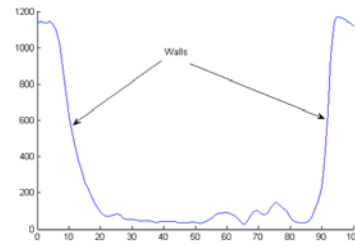
$$\tilde{P}(x) = |I(x) - I_{mean}|^\alpha + w \quad (10)$$

where the scalar $\alpha > 1$, so as to increase the potential value around the walls of object. In Fig.3 (a)

is shown a slice of a colon volumetric data set. Fig.3(b) shows the grey level profile along the line drawn in Fig.3 (a). As we can see that the grey intensity inside the colon is around 200, while edges are around 1200. Thus we define: $\tilde{P}(x) = |I(x) - 200|^2 + 10$, where the constant 10 can maintain the smoothness of the front.



(A) Slice Of A Colon CT Scan



(B) Colon Gray Information

Figure 3. Profile Of The Colon Volume

4.2 Colon Surfaces Extraction

In the experiment to evaluate our average energy freezing criterion, a series of 2D 512*512 abdomen CT images are used, which are taken with a GE Genesis Signa HiSpeed CT/I system at the Guangzhou Clifford Hospital. The experiment is performed on a PC with Pentium D 2.8 GHz, 2 G RAM.

We use the improved Minimal Action Surface method based on our average energy freezing criterion to extract the colon surfaces. The starting point is selected from three orthogonal views shown in Fig.4. According to the analysis of the propagating front's average energy (Fig.2), we set the freezing thresholds as: $high = 0.5$, $low = 0.01$. Fig.5 shows the extraction process of the colon surfaces with our method. As we can see in Fig.5(a) there is a part of the front has been reached the boundary of the tubular structure when E_{mean} is less than high. It indicates that the front propagation comes into the third phase, when the freezing criterion should be implemented. In Fig.5(b), our freezing criterion has frozen those voxels whose front energy are less than E_{mean} , while the head of the front is marching still. When the head reaches a specific location or E_{mean} is less than low, the energy

of the head is enough small, which indicate the head has reached the end of the structure, then stop propagating the front, like Fig.5(c). The experimental results show that the traditional Minimal Action Surfaces method has obvious edge

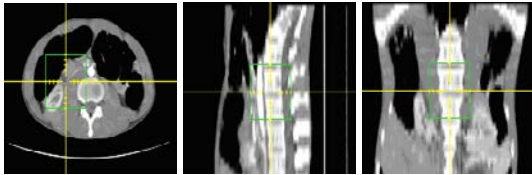
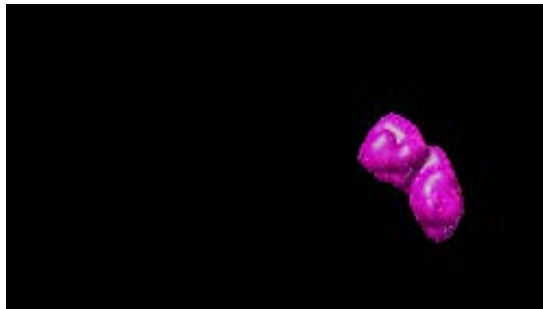


Figure 4. Three Orthogonal Views Of A Volumetric CT Data Set Of The Colon



(A) Iteration 2000, Time-Consuming 6.932s



(B) Iteration 5000, Time-Consuming 11.489s



(C) Iteration 10152, Time-Consuming 18.221s

Figure 5. The Colon Extraction Process With Minimal Action Surfaces Based On The Average Energy Freezing Criterion

leakage (see Fig.1). Conversely, our average energy freezing criterion could effectively control the front propagating and obtain the desired results.

5 CONCLUSION

By analyzing the character of the front's average energy information, we developed a freezing criterion based on the front's average energy, which prevent the edges leakage when extracting tubular surfaces with the Minimal Action Surfaces method. The average energy freezing criterion can freeze the tail voxels, whose energy is lower than the front's average energy and which are always closed to the edges of tubular structures. Since the freezing criterion is based on the global energy information of the front, it is robust to image noise and the local minimum. Table 1 gives the performance compare-son between the geodesic distance criterion in [5] and our average energy criterion. By defining an appropriate potential function, the improved Minimal Action Surfaces method based on average freezing criterion can be applied directly to all the tubular surfaces extraction.

TABLE 1 Performance Comparison Between The Two Criterion

Freezing criterion	Advantage	Shortcoming
Geodesic distance criterion	Small amount of calculation	Sensitive to noise and local minimum
Average energy criterion	Global energy information, robust to noise	Need to compute the front's average energy

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REFERENCES:

- [1] D. Lesage, Elsa D. Angelini, I. Bloch, et al, "A review of 3D vessel lumen segmentation techniques: Models, features and extraction schemes", Medical Image Analysis, Vol. 13, No. 6, 2009, pp.819-845.
- [2] Gao X., "Fast extraction of tubular structures surfaces", Computer Engineering and Applications, Vol. 46, No. 11, 2010, pp.158-160, (in Chinese).



- [3] Wang Y., N. A., B. Roysam, "Novel 4-D Open-Curve Active Contour and curve completion approach for automated tree structure extraction", IEEE international conference on Computer Vision and Pattern Recognition (CVPR), IEEE Conference Publishing Services, 2011, pp. 1105-1112.
- [4] Cohen L.D., Ron Kimmel, "Global Minimum for Active Contour Models: A Minimal Path Approach", International Journal of Computer Vision, Vol. 24, No. 1, 1997, pp. 57-78
- [5] Deschamps T., Cohen L.D., "Fast extraction of tubular and tree 3D surfaces with front propagation methods", In: Proceedings of the 16th ICPR, 2002, pp. 731-734.
- [6] Yan J., Zhuang T., Zhao B., et al, "Lymph node segmentation from CT images using fast marching method", Computerized Medical Imaging and Graphics, Vol.28, No.122, 2004 , pp. 33 - 38.
- [7] Adrian B., Michael S., Xu X., et al, "Automatic Detection and Segmentation of Lymph Nodes from CT Data", IEEE Transactions on Medical Imaging, Vol.31, NO.2, 2012, pp. 240-251.
- [8] Lu D., Zhuang T., "Fast Marching Method Based Segmentation in Low Contrast Medical Images", Journal of Shanghai Jiaotong University, Vol.40, No.8, 2006, pp.1444-1447, (in chinese).
- [9] Deschamps T., Cohen L.D., "Fast extraction of minimal paths in 3D images and applications to virtual endoscopy", Medical Image Analysis, Vol.5, No.4, 2001, pp. 281-299.