Journal of Theoretical and Applied Information Technology
15th November 2012. Vol. 45 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

SYNCHRONIZATION OF DISCRETE CHAOTIC SYSTEMS BASED ON UNCERTAINTY COMPENSATED BY LS-SVR

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ABSTRACT

A chaotic system owns complex dynamics though it is with deterministic expression in mathematics, and it has attractived numerous study and application in many fields of the world. But for a concrete application, it is usually with parameters uncertainty while it is implemented. In this paper synchronization of discrete-time chaotic systems with parameters and/or structure uncertainty is researched, in which the uncertainty is modelled by using least square support vector regression(LS-SVR) to eliminate syncronization error. A novel synchronization control law that guarantees closed-loop robust stability is proposed. Synchronizing of the well-known Hénon chaotic system and Lozi chaotic system, Burgers' map and Holmes cubic map, are taken as illustrative examples. The chaotic systems in both examples are uncertain in parameters and structure. Experimental results demonstrate the effectiveness and feasiblility of the proposed synchronization method.

Keywords: *Chaotic Systems, Synchronization, Uncertainty, LS-SVR*

1 INTRODUCTION

Synchronization of chaotic dynamical systems has attracted an increasing attention [1-7] since the early work by Pecora and Carroll [8] on synchronizing of chaos. Many special issues on the control and synchronization of chaos have been published due to its possible application in various fields, such as application to control theory, secure communication, chemical reaction and encoding message [1, 2, 9, 10].

Most of the exiting synchronization methods for chaotic systems are based on very strict condition that the parameters of the concerned systems are exactly known [5-7, 11, 12]. Due to the tolerance and time-variant property of electronic components, we know that two identical physical circuits with exact same parameters can never be implemented. As a consequence, it is important to investigate the robustness of the considered synchronization technique. In that case the synchronization is defined as practical synchronization [13, 14]. For real systems, parameters can't be known exactly, so we can only estimate its approximate parameters. That is to say, only an approximate model can be used. Although the discrete-time chaotic mapping is an important family in chaos world, the current synchronization is mostly about continuous chaotic systems [5-7, 11, 15-17], and the chaotic systems do not contain the disturbance [15, 17].

To the best of the authors' knowledge, synchronization of discrete chaotic systems has been seldom studied [18]. So, in this article, synchronization of discrete chaotic systems with uncertainty and disturbance is studied. It does not need to know the exact model that is a prerequisite for most of the existing methods. We only need to estimate the uncertainty, which is much more easy to obtain with respect to exact model [19-25]. A novel synchronization control algorithm is proposed and designed by using LS-SVR to model the uncertainty part, which guarantees that the system synchronization error is globally uniformly ultimately bounded as long as the modeling error is bounded, and which will make synchronization error approach zero in the condition that the approximate error between model and real chaotic system is sufficiently small.

2 SYNCHRONIZATION OF UNCERTAIN DISCRETE CHAOTIC SYSTEMS USING LS-SVR FOR UNCERTAINTY COMPENSATION

2.1 Least Square Support Vector Regression (LS-SVR)

Least-squares support vector regression (LS-SVR) [26-29] proposed by Suykens is an

Journal of Theoretical and Applied Information Technology
15th November 2012. Vol. 45 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

alternate formulation of SVR. Consider a model in the primal weight space of the following form:

$$f(x) = \omega^T \varphi(x) + b\,,$$

where $x \in \mathbf{R}^m$, $\varphi(\cdot):\mathbf{R}^m \to \mathbf{R}^d$ is the mapping to the high dimensional and potentially infinite dimensional feature space, ω is weight vector, and b is bias term. Geven a training set of n points $\{x_k, y_k\}_{k=1}^n$ with input data $x_k \in \mathbf{R}^m$ and output data $y_k \in \mathbf{R}$, e_k is the deviate between the output data y_k and the model prediction $f(x_k)$, i.e. $e_k = y_k - f(x_k)$. Hence we can formulate the following optimization problem in the weight space

$$\min_{\omega,b,e} J_p(\omega,e) = \frac{1}{2}\omega^T\omega + \frac{C}{2}\sum_{k=1}^n e_k^2\,,$$

$$s.t.\ \ y_k = \omega^T\varphi(x_k) + b + e_k,\ k = 1,\dots,n$$

where C is penalty parameter and e_k is error variable.

However, the primal problem is difficult to solve as ω is high dimensional. Therefore, let us proceed by constructing the Lagrangian and derive the dual problem. The Lagrangian is presented by

$$L(\omega,b,e;\alpha) = J_p(\omega,e) - \sum_{k=1}^n \alpha_k\left[\omega^T\varphi(x_k) + b + e_k - y_k\right]\,,$$

where α_k are Lagrange multipliers. The conditions for optimality

$$\begin{cases} \dfrac{\partial L}{\partial \omega} = 0 \to \omega = \sum_{k=1}^n \alpha_k \varphi(x_k) \\ \dfrac{\partial L}{\partial b} = 0 \to \sum_{k=1}^n \alpha_k = 0 \\ \dfrac{\partial L}{\partial e_k} = 0 \to \alpha_k = Ce_k,\ k = 1,\dots,n \\ \dfrac{\partial L}{\partial \alpha_k} = 0 \to \omega^T\varphi(x_k) + b + e_k - y_k = 0,\ k = 1,\dots,n \end{cases},$$

can be written as the solution to the following set of linear equations after eliminating the ω and e

$$\begin{bmatrix} 0 & \boldsymbol{g}^T \\ \boldsymbol{g} & \Omega + C^{-1}I \end{bmatrix}\begin{bmatrix} b \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{y} \end{bmatrix}, \qquad (1)$$

where $\boldsymbol{y} = [y_1,\dots,y_n]^T$, $\boldsymbol{g} = [1,\dots,1]^T$, $\boldsymbol{\alpha} = [\alpha_1,\dots,\alpha_n]^T$, and I is an identity matrix. The kernel trick is applied here as following

$$\Omega_{kl} = \varphi(x_k)^T\varphi(x_l) = K(x_k, x_l).\quad k,l = 1,\dots,n$$

Where $K(x_k, x_l)$ is the kernel function, the RBF kernel function is used in the paper because of the fact that RBF kernel has a strong approximation capacity, i.e. $K(x_k, x_l) = \exp[-(\|x_k - x_l\|^2)/(2\sigma^2)]$, σ is the width for RBF kernel.

Then we can get α and b from (1). Therefore, the result of LS-SVR model is

$$f(X) = \sum_{k=1}^n \alpha_k K(x, x_k) + b\,.$$

2.2 Approximating The Uncertainty Of Chaotic Systems Using LS-SVR

Considering the following two dynamic systems:

Driving system

$$x(k+1) = f\left[x(k),\alpha\right] + d_1(k)\,. \qquad (2)$$

Response system

$$y(k+1) = h\left[y(k),\beta\right] + d_2(k) + u(k)\,, \qquad (3)$$

where $x,y \in R^n$ are the state vectors of the driving system and response system respectively; f and h are both $n\times1$ bounded continuous functional matrices; d_1 and d_2 are both $n\times1$ unknown bounded structural uncertainty; $\alpha \in \mathbf{R}^p$ and $\beta \in \mathbf{R}^q$ are the unknown parameter vectors of the systems; $u \in \mathbf{R}^n$ is control input.

Assume that $\tilde{\alpha}$ and $\tilde{\beta}$ are the rational pre-estimated value of α and β respectively. So the model error between the pre-estimated models and the real systems are as follows respectively:

$$\Delta f\left[x(k),\alpha,\tilde{\alpha}\right] = f\left[x(k),\alpha\right] + d_1(k) - f\left[x(k),\tilde{\alpha}\right], \qquad (4)$$

$$\Delta h\left[y(k),\beta,\tilde{\beta}\right] = h\left[y(k),\beta\right] + d_2(k) - h\left[y(k),\tilde{\beta}\right]. \qquad (5)$$

Now, we use LS-SVR to approximate the uncertainty as shown above, and they are denoted as $f_{\text{SVR}}\left[x(k),\alpha,\tilde{\alpha}\right]$ and $h_{\text{SVR}}\left[y(k),\beta,\tilde{\beta}\right]$ respectively. The approximate error are denoted as follows:

$$f_e\left[x(k),\alpha,\tilde{\alpha}\right] = \Delta f\left[x(k),\alpha,\tilde{\alpha}\right] - f_{\text{SVR}}\left[x(k),\alpha,\tilde{\alpha}\right], \qquad (6)$$

$$h_e\left[y(k),\beta,\tilde{\beta}\right] = \Delta h\left[y(k),\beta,\tilde{\beta}\right] - h_{\text{SVR}}\left[y(k),\beta,\tilde{\beta}\right]. \qquad (7)$$

We denote

Journal of Theoretical and Applied Information Technology
15th November 2012. Vol. 45 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\eta(k) = h_e\left[y(k),\beta,\tilde{\beta}\right] - f_e\left[x(k),\alpha,\tilde{\alpha}\right]. \quad (8)$$

Because LS-SVR is used to learn the model error systems, good approximate ability can guarantee the fllowing result.

$$\left\|\eta(k+1)-\eta(k)\right\| \le \gamma, \qquad k=1,2,3,\cdots. \quad (9)$$

in which, □is a suffiecient small number that is usually the approximation bound of $\left\|\eta(k+1)-\eta(k)\right\|$, which is theoretically zero according to the functional approximation theory, see Ref. [30].

2.3 Design Of Synchronization Law

Syncronization law can be induced as a theorem as follows:

Theorem For the system (2) and (3), if the following synchronization control action u is used,

$$u(k) = f\left[x(k),\tilde{\alpha}\right] + f_{\rm SVR}\left[x(k),\alpha,\tilde{\alpha}\right] - h\left[y(k),\tilde{\beta}\right] - h_{\rm SVR}\left[y(k),\beta,\tilde{\beta}\right] + Bu_1(k), \quad (10)$$

$$u_1(k) = u_1(k-1) + (CB)^{-1}(A-I)Ce(k)\,, \quad (11)$$

where, $u_1(k)$ is an auxiliary control action; $A\in\mathbf{R}^{n\times n}$ is a matrix chosen by designer that satisfies $\left\|A\right\|<1$; $B\in\mathbf{R}^{n\times m}$ is a direct input matrix; $C\in\mathbf{R}^{m\times n}$ is chosen such that CB is nonsingular; I is identity matrix with proper dimension; and vector $s(k)=\boldsymbol{C}e(k)$ is linear switching surface of sliding mode.

Then, the error of the synchronization system (2) and (3) is globally uniformly ultimately bounded.

Proof The error of the synchronization system (2) and (3) is

$$\begin{aligned} e(k+1) &= y(k+1)-x(k+1) \\ &= h\left[y(k),\beta\right]+d_2(k)-f\left[x(k),\alpha\right]-d_1(k)+u(k) \\ &= h_e\left[x(k),\alpha,\tilde{\alpha}\right]-f_e\left[y(k),\beta,\tilde{\beta}\right]+Bu_1(k) \\ &= \eta(k)+Bu_1(k), \end{aligned} \quad (12)$$

so

$$e(k) = \eta(k-1)+Bu_1(k-1)\,,$$

and

$$Ce(k) = C\eta(k-1)+CBu_1(k-1)\,. \quad (13)$$

For the convienience of analysis, we denote $s(k)=\boldsymbol{C}e(k)$.

Then $u_1(k\text{-}1)$ can be written as

$$\begin{aligned} u_1(k-1) &= (CB)^{-1}\left[Ce(k)-C\eta(k-1)\right] \\ &= (CB)^{-1}\left[s(k)-C\eta(k-1)\right]. \end{aligned} \quad (14)$$

By substituting formula (14) into formula (11), we will obtain

$$u_1(k) = (CB)^{-1}\left[As(k)-C\eta(k-1)\right]. \quad (15)$$

According to formula (12), (13) and (15), we can get

$$\begin{aligned} s(k+1) &= Ce(k+1) \\ &= C\eta(k)+CBu_1(k) \\ &= As(k)+C\left[\eta(k)-\eta(k-1)\right]. \end{aligned} \quad (16)$$

It's obviously that

$$\begin{aligned} \left\|s(k+1)\right\| &\le \left\|A\right\|\cdot\left\|s(k)\right\|+\left\|C\right\|\cdot\left\|\eta(k)-\eta(k-1)\right\| \\ &\le \left\|A\right\|\cdot\left\|s(k)\right\|+\left\|C\right\|\cdot\gamma. \end{aligned}$$

According to mathematical induction principle, we can obtain

$$\left\|s(k+1)\right\| \le \left\|A\right\|^k\cdot\left\|s(1)\right\|+\frac{1-\left\|A\right\|^k}{1-\left\|A\right\|}\cdot\left\|C\right\|\cdot\gamma\,.$$

Due to the assumption $\left\|A\right\|<1$ in the **Theorem**, we get

$$\lim_{k\to\infty}\left\|s(k)\right\| \le \frac{1}{1-\left\|A\right\|}\cdot\left\|C\right\|\cdot\gamma\,.$$

So, according to formula (10), (11) and $s(k)=\boldsymbol{C}e(k)$, synchronization control action u is convergent. According to (9), the synchronization error of the system (2) and (3) is globally uniformly ultimately bounded.

The synchronization error of system (2) and (3) can infinietely approach zero as long as $f_e\left[x(k),\alpha,\tilde{\alpha}\right]$ and $h_e\left[y(k),\beta,\tilde{\beta}\right]$ infinietely approach zero, which can be guaranteed by model error approximator [30].

3 ILLUSTRATIVE EXAMPLES

Example 1 Synchronizing of Hénon chaotic system and Lozi chaotic system

Driving system: Hénon chaotic system [31, 32] is described as

$$x(k+1) = f\left[x(k),\alpha\right]+d_1(k)\,,$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

in which,

$$x(k+1)=\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix},$$

$$f\left[x(k),\alpha\right]=\begin{bmatrix} x_2(k) \\ \alpha_1+\alpha_2 x_1(k)-x_2^2(k) \end{bmatrix},$$

and

$$d_1(k)=\begin{bmatrix} d_{11}(k) \\ d_{12}(k) \end{bmatrix}.$$

Response system: Lozi chaotic system [32] is expressed as

$$y(k+1)=h\left[y(k),\beta\right]+d_2(k)+u(k).$$

in which,

$$y(k+1)=\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \end{bmatrix},$$

$$h\left[y(k),\beta\right]=\begin{bmatrix} y_2(k) \\ \beta_1+\beta_2 y_1(k)+\beta_3 \mid y_2(k) \mid \end{bmatrix},$$

and

$$d_2(k)=\begin{bmatrix} d_{21}(k) \\ d_{22}(k) \end{bmatrix}.$$

In the experimental study, the real system parameters are α_1=1.29 and α_2=0.3 for Hénon chaotic system, β_1=3, β_2=−1.8 and β_3=0.4 for Lozi chaotic system. Figure 1 shows the chaotic characteristic curves of x_i and y_i (i=1,2) with initial conditions x=[0,0]^T and y=[0.4,0.4]^T. These initial conditions will also be used in the other studies in the fllowing experiments for the sake of comparison. And in the following four figures, solid lines stand for x_i(i=1,2) and dot lines stand for y_i(i=1,2) in all sub-figures (a) and (b) of each figure. The synchronization errors $e=y-x$ are shown in sub-figures (c) and (d) of Figure 3.

Suppose the pre-estimated model parameters for synchronization controller design are $\tilde{\alpha}_i$ =1.1α_i and $\tilde{\beta}_i$ =0.9β_i (i=1,2). And for simplicity but without loss generality, we suppose the external uncertainty to the driving system and to the response system are d_1=[0,0]^T and d_2=0.01[2+sin(3k),2+cos(k/4)]^T respectively, i.e. there isn't any structural uncertainty act on the driving system, and the external disturbances on the response system are time-varing. Figure 2 shows the chaotic curves of x_i and y_i (i=1,2) in this case.

By comparison of Figure 1 and Figure 2, the characteristic curves of response system (Lozi chaotic system) are different because the uncertainty exists. The synchronization results are illustrated in Figure 3 with the matrix A=0.5, B=[0,1]^T and C=[c_1,c_2]=[0.5,1].

Remark 1 The synchronization tracking performance can be obtained with a wide range of the matrix A, B and C, i.e. the three matrixes are easy to set, which will be demonstrated in both Example 1 and Example 2 by using the same value for the three matrices.

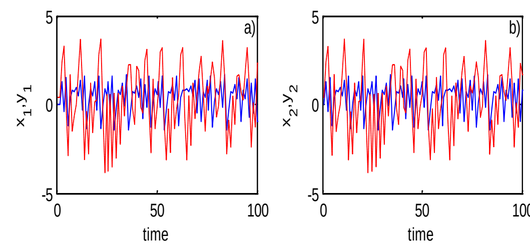

Figure 1 Chaotic curves of Example 1: a) x_1, y_1; b) x_2, y_2

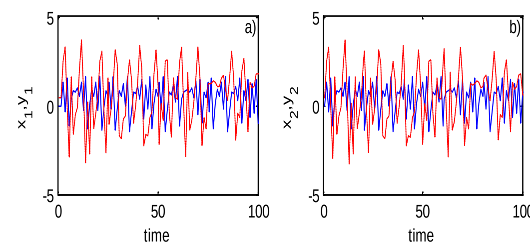

Figure 2 Chaotic curves of Example 1 with uncertainty: a) x_1, y_1; b) x_2, y_2

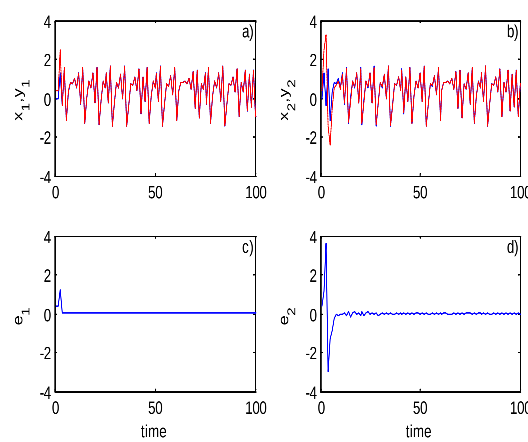

Figure 3 Synchronization results of Example 1 with uncertainty: a) x_1, y_1; b) x_2, y_2; c) e_1; d) e_2

Example 2 Synchronizing of Burgers' map and Holmes cubic map

Driving system: Burgers' map [33] is written as

Journal of Theoretical and Applied Information Technology
15th November 2012. Vol. 45 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$x(k+1)= f\left[x(k),\alpha\right]+d_1(k).$$

in which,

$$x(k+1)=\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix},$$

$$f\left[x(k),\alpha\right]=\begin{bmatrix} \alpha_1 x_1(k)-x_2^2(k) \\ \alpha_2 x_2(k)+x_1(k)x_2(k) \end{bmatrix},$$

and

$$d_1(k)=\begin{bmatrix} d_{11}(k) \\ d_{12}(k) \end{bmatrix}.$$

Response system: Holmes cubic map [33] is described as

$$y(k+1)=h\left[y(k),\beta\right]+d_2(k)+u(k),$$

in which,

$$y(k+1)=\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \end{bmatrix},$$

$$h\left[y(k),\beta\right]=\begin{bmatrix} y_2(k) \\ -\beta_1 y_1(k)+\beta_2 y_2(k)-y_2^3(k) \end{bmatrix},$$

and

$$d_2(k)=\begin{bmatrix} d_{21}(k) \\ d_{22}(k) \end{bmatrix}.$$

In the experimental study, the real system parameters are α_1=0.75 and α_2=1.75 for Burgers' map, β_1=0.2 and β_2=2.77 for Holmes cubic map. Figure 4 shows the chaotic curves of x_i and y_i (i=1,2) with initial conditions $x=[-0.1,0.1]^T$ and $y=[1.6,0]^T$. These initial conditions will also be used in the other studies in the fllowing experiments for the sake of comparison. And in the following four figures, solid lines stand for x_i(i=1,2) and dot lines stand for y_i(i=1,2) in all sub-figures (a) and (b) of each figure. The synchronization errors $e=y-x$ are shown in sub-figures (c) and (d) of Figure 6.

Suppose the pre-estimated model parameters for synchronization controller design are $\tilde{\alpha}_i$ =0.97α_i and $\tilde{\beta}_i$ =0.97β_i (i=1,2). And for simplicity as well as for the sake of comparison but without loss generality, we suppose the external uncertainty to the driving system and the response system are $d_1=[0,0]^T$ and $d_2=0.015[1+\sin(3k),1+\cos(k/4)]^T$ respectively, i.e. there isn't any structural uncertainty act on the driving system and the external disturbances on the response system are time-varing. Figure 5 shows the chaotic curves of x_i and y_i (i=1,2) in this case.

By comparison of Figure 4 and Figure 5, the curves of response system (Holmes cubic map) are different because the uncertainty exists. The synchronization results are illustrated in Figure 6 with the matrix A, B and C are the same as those in Example 1.

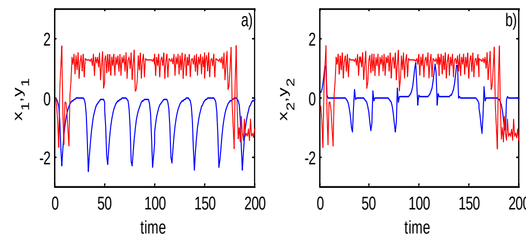

Figure 4 Chaotic curves of Example 2: a) x_1, y_1; b) x_2, y_2

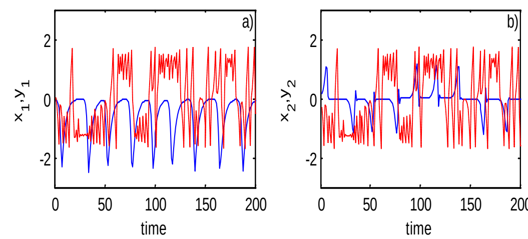

Figure 5 Chaotic curves of Example 2 with uncertainty: a) x_1, y_1; b) x_2, y_2

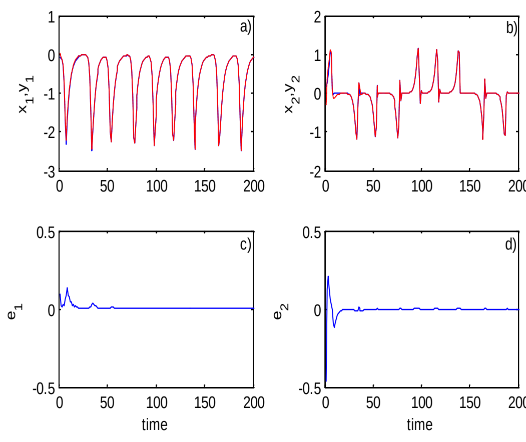

Figure 6 Synchronization results of Example 2 with uncertainty : a) x_1, y_1; b) x_2, y_2; c) e_1; d) e_2

4 CONCLUSIONS

The synchronizations of chaotic systems with models uncertainty are studied for discrete-time chaotic systems. The approximate models were used

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

firstly for the design of synchronization controllers. The modeling uncertainty of chaotic dynamics is adaptively learned on line by LS-SVR. One of the many merits of the proposed synchronization method is that robust control performance can be obtained. The synchronization of Hénon chaotic system and Lozi chaotic system, and the synchronization of Burgers' map and Holmes cubic map were illustrated as the examples, which demonstrated the synchronization performance of proposed method.

ACKNOWLEDGEMENTS

This work was project supported by the National Natural Science Foundation of China (Grant No. 61203041) and by the Fundamental Research Funds for the Central Universities (Grant No.11MG49).

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ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

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