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A SOLUTION TO DECISION MAKING UNDER UNCERTAINTY

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ABSTRACT

Decision-making under uncertainty is the problem with the least known conditions, and also the most difficult problem to find the optimal solution. A. Wald and his contemporary V. Nouma first associate the decision theory with the game theory in the way of regarding the statistical decision as the two-person zero-sum game processed by the statistician and nature. A. Wald proves that we should choose the so called Max-min strategy in decision making. Based on the analyses of A. Wald matrix game and five classical rules of uncertain decision-making, this paper agrees that the max-min criterion is the relative scientific solution, and puts forward the general method of solving uncertain decision-making.

Keywords: *Uncertain Decision-making, Decision Method, Matrix Game*

1. INTRODUCTION

Decision-making is defined as three types including certain decision-making, risk decision-making and uncertain decision-making according to the amount of known information in the current literatures. In terms of uncertain decision-making, decision-makers having not sufficient information, lack profound understanding of various nature states involved and cannot predict the probability of nature state. On account of no optimal or satisfactory solution available based on the existing theoretical research outcomes, this kind of decision mostly depends on decision-makers' subjective judgment. Therefore, uncertain decision-making is the most common and the most difficult problem among all decision-making types.

Shackles of research means and research direction are partly the reason for the serious lag of theoretical research. In fact, as early as in 1950s, A. Wald made a research on relationship between decision theory and game theory and then put forward max mini criteria or pessimism decision criterion, one of the five criteria in uncertain decision-making [1]. But the further research in this regard long stopped at the achievements made at the time.

This paper discovers that uncertain decision-making problem can be solved based by matrix game through the study on decision criteria of uncertain decision-making and matrix game. At the same time because of popularization of computer techniques, linear programming solution process has become a piece of cake. Taking linear programming as basic method of solving matrix game makes it easier to get mathematical solutions. The solution is a more widely applied method than the current five decision criteria.

2. REVIEW OF RESEARCH ON UNCERTAIN DECISION-MAKING

Optimism decision criterion, compromised decision criterion, equality decision criterion, regret decision criterion and pessimism decision criterion have been regarded as a model in terms of uncertain decision-making research. These five criteria are considered as the best solutions in various related literatures. The subsequent studies are conducted mainly based on the five criteria.

2.1 Five Criteria

1. Optimism decision criterion or max-max criterion

Optimism decision criterion is always full of optimism for future development, taking the proceeds of the best programs into account. It has a lot of confidence to achieve the most ideal result in each decision program. All these expressions reflect optimism and spirit of adventure of the decision-makers [2].

Use S_i for strategy set, S_i^* for decision strategy， a_{ij} for return matrix elements (hereinafter the same)， its decision strategy can be expressed as follows:

$$S_i^* \to \max\max(a_{ij}) \qquad (1)$$

This decision is too risky and requires bearing the risk of the corresponding loss as the price obviously.

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2．Compromised decision criterion

It is also called optimism coefficient method or Hurwicz(a Nobel Prize winner) criterion. It is not as risky as optimists and not as conservative as pessimists, but first to identify an optimism coefficient a（0≤a≤1）according to experience, then to get a compromised benefit value in each program, last to compare compromised value H_i in all programs to choose the program with the biggest value as the best program. Its decision strategy can be expressed as follows:

$$S_i^* \to \max\left\{H_i\right\} \to \max\left\{\alpha * a_{i_{\max}} + (1-\alpha) * a_{i_{\min}}\right\} \quad (2)$$

Optimism coefficient mentioned in this method is with great subjectivity due to being determined by persons.

3．Equality decision criterion or Laplace method

It sees probability of nature state as equal. If there are n nature states, the probability of each nature state will be 1/n. Then calculate average value of return $E(S_i)$ in various nature state in each program, identify the biggest value. The program with the biggest average value is the best program. Its decision strategy can be expressed as follows:

$$S_i^* \to \max\left\{E(S_i)\right\} \to \max\left\{\frac{1}{n}\left(a_{i1} + a_{i2} + \cdots + a_{in}\right)\right\}$$
(3)

Apparently, it is biased to consider the probability of occurrence of uncertain various nature states as the same average value.

4．Regret decision criterion or Savage method

It selects the best value in each state as the ideal goal and defines the difference N_{ij} between the other benefit value and the best value as the regret value when failing to reach the ideal value. Then find out the biggest regret value from each program, identify the program with the smallest regret value as the best program.

Its decision strategy can be expressed as follows:

$$S_i^* \to \min\max\left\{N_{ij}\right\} \to \min\max\left\{\max_{1\le k\le m} a_{kj} - a_{ij}\right\} \quad (4)$$

It is easy to validate that the best strategy of this criterion lacks “independence” while strategy set change. So it is impossible to determine the best strategy [3].

5．Max min criterion, also pessimism decision criterion or conservative criterion

It identifies the program with the biggest benefit value selected among the smallest values generated in each nature state, as the best program. Its decision strategy can be expressed as follows:

$$S_i^* \to \max\min(a_{ij}) \quad (5)$$

It can be understood that this criterion takes the worst condition into account and makes efforts for the best. Many literatures address that this criterion is a more reliable method. The research on uncertain decision-making by A. Wald in early years also deals with this problem.

An obvious problem can be found from the statements of five criteria. For an uncertain decision-making to be solved, the conclusions resulting from calculation based on the above five criteria are different, which usually makes people disoriented. In practice, the action strategy is selected on the basis of personal preferences of different decision-makers, which causes confusion of scientific decision. That is because diversified decision methods and different conclusion are equal to no solution.

2.2 Other Research Conclusions

A. Wald, who is the famous statistician in the V. Nouma contemporary, first relates the decision theory to the game theory [4]. He regards the statistical decision as the two people zero-sum game processed by the statistician and nature. By mathematical means, he proves that we should choose the so called “Maximin” strategy in decision making.

Under the foundation of illustrating the general decision making problem, A. Wald strictly proves that the decision making problem can be interpreted as the two people zero-sum game according to the game theory of V. Nouma. He extends this theory, and deduces the function theory of statistical consequently. He considers that the experimenter hopes to decrease the risk (F, &) to the minimum in a decision making problem, but the risk is the function of two variables, which are F and &, the experimenter can only choose the decision marking function & and cannot choose the F. F is selected by nature, and the selection of nature cannot be known by the experimenter. The condition is very similar to the two-person game.

A. Wald explains the decision marking problem as the two people zero-sum game through a series of corresponding relations, However, he points out the distinctions between them meanwhile: the decider hopes that the risk can become minimal, but it is difficult for him to say that nature requires risk (F, &) to the minimum. The selection of nature cannot be known by the decider. Consequently, the decider could believe that nature hopes the maximum risk. Except for this point, he thinks that the decision marking problem and two people zero-sum game problem are totally similar [5].

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J. O. Berger, the American statistician, considers that the substance of the maxmin criterion is the guard and protection of the worst situation. The condition which is fit for this situation is that situation is decided by the smart opponent, and the opponent will maximize your personal loss. And then, the worst situation you expected to yourself will happen, and consequently, you should find a way out to cope with your opponent. The decision marking research which aims at this situation can be called the game theory. Thus, it is evident that the decision marking and the dice game are one problem of two aspects.

Under the condition of lacking prior information, the decider cannot carry out the nature decision marking principle. The scientific method is the maxmin criterion. Meanwhile, the maxmin criterion can be proved reasonable. We should pay attention to two points: the first point is that the maxmin criterion may lose an optimal decision marking; the second point is that it may have a lot of difficulties in carrying out the maxmin criterion. However, it may be much easier to adopt Bayes method which lacks the prior information. And generally, this method will obtain better or equally good results. In fact, in terms of the decision marking problem of carrying out the maxmin criterion, the results of the two methods tend to be identical.

The other application of the maxmin criterion is that it provides a scale to the Bayes stability research. The maxmin criterion is the most stable principle to the prior regulations. The obtained stability of Bayes principle is partly indicated by the comparison with the mini max theorem.

Certainly, the reason of the vastly popularization of the maxmin criterion is that it is more abundant in math color than the Bayes theory rather than the above reasons. The mathematic foundation of game theory is the max min criterion, among which, the two people zero-sum game theory can be regarded as the result of maxmin criterion directly applied under the strictly statistic background. People can image that the loss of decider is the acquisition of nature, whereas nature is the smart opponent, namely, it can optimize the loss of the decider (O. Berger, 1985). Several basic theories on matrix game supports these analyses very well [6].

Consequently, making use of the method to solve uncertain decision-making problem has stable mathematical theory foundation.

In the last few years, other mathematical methods are gradually introduced in this research field to work out the uncertain decision-making problems. For example, a scholar Areeg Abdalla brings Monte Carlo methods into fuzzy game theory, which also gives some clues to solve decision making under uncertainty [7].

3. GENERAL SOLUTION TO UNCERTAIN DECISION-MAKING PROBLEMS

As has been stated above, A. Wald explains the decision making problem in 1950 as the two people zero-sum finite game (it can also be called the matrix game) problem. Meanwhile, he exerts the minimax theorem of the two people zero-sum finite game to the research of uncertain decision-making problem and forms the maxmin criterion. The research finds that the maxmin criterion of A. Wald aims at the uncertain decision-making of matrix game with saddle point. How to solve the uncertain decision-making problem which corresponds to the matrix game without saddle point do not induce the importance from the later generations and the later generations even confuse these two problems. Under this circumstance, this paper continues the ideas of A. Wald and puts forwards the general solution of solving the uncertain decision-making problem. This measure is the improvement to the maxmin criterion of A. Wald. Some scholars in China also raise different solutions to uncertain-decision [8, 9].

3.1 An Application Example

Suppose the weather condition is K, we can gain 30 thousand profits from planting the crop G, and we can gain 50 thousand profits from planting the crop H. Suppose the weather condition is N, we can gain 60 thousand profits from planting the crop G, and we can gain 40 thousand profits from planting the crop H. The detail information is shown in Table I.

TABLE I: Profits Chart of Crop Planting Unit: Ten Thousand

nature / farmer	Weather K	Weather N
Crop G	3	6
Crop H	5	4

In this decision making problem under uncertainty, we can regard the cropper as the player 1 and regard nature as the player 2(regard it as the rational player for processing it).

Owing to

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$$\max\min a_{ij} = 4 < 5 = \min\max a_{ij}, \quad (6)$$

there is no solution to this decision making problem under uncertainty on the pure strategy sense, namely, the matrix which corresponds to this decision making problem under uncertainty do not have saddle. Therefore, suppose X=(x_1, x_2), which is the mixing strategy of player 1, and suppose Y=(y_1, y_2), which is the mixing strategy of player 2.

Then:

$$S_m^* = \{(x_1, x_2) | x_1, x_2 \geq 0, x_1 + x_2 = 1\} \quad (7)$$

$$S_n^* = \{(y_1, y_2) | y_1, y_2 \geq 0, y_1 + y_2 = 1\} \quad (8)$$

The expected value gained by player 1 is:

$$\begin{aligned} &E(x,y) \\ &= 3x_1y_1 + 6x_1y_2 + 5x_2y_1 + 4x_2y_2 \\ &= -4\left(x - \frac{1}{4}\right)\left(y - \frac{1}{2}\right) + \frac{9}{2} \end{aligned}$$

Suppose

$$X^* = \left(\frac{1}{4}, \frac{3}{4}\right), Y^* = \left(\frac{1}{2}, \frac{1}{2}\right),$$

Then

$$v = E(x,y) = \frac{9}{2}$$

Therefore, X^*= (1/4, 3/4) and Y^*= (1/2, 1/2) is the optimal strategy of player 1 and player 2 respectively. The decision making expected value (the expected value gained by player 1) is V=E(x, y) = 9/2=4.5 ten thousand.

If the change of K and N is uncertain, the decider can ensure the optimal mixed strategy by calculating in order to insure the incomes of the minimum limitation (no matter the weather is K or N). Through calculating, we can ensure that the frequency of G is 1/4, and the frequency of H is 3/4. That is to say, we can utilize 1/4 land to plant the crop G and utilize 3/4 land to plant the crop H for expecting to obtain the minimum profits 45 thousand Yuan.

3.2 Meaning Of General Solution

X^*=(x_1,…, x_m) represents the decision making project that should be made by the decider. It does not purely select a certain project, and it may be the mixing strategy or correction strategy of several projects. Consequently, in terms of the decider, the general solution provides more selection methods. No matter what project the decider selects, the result of the general solution is the most break-even selection method.

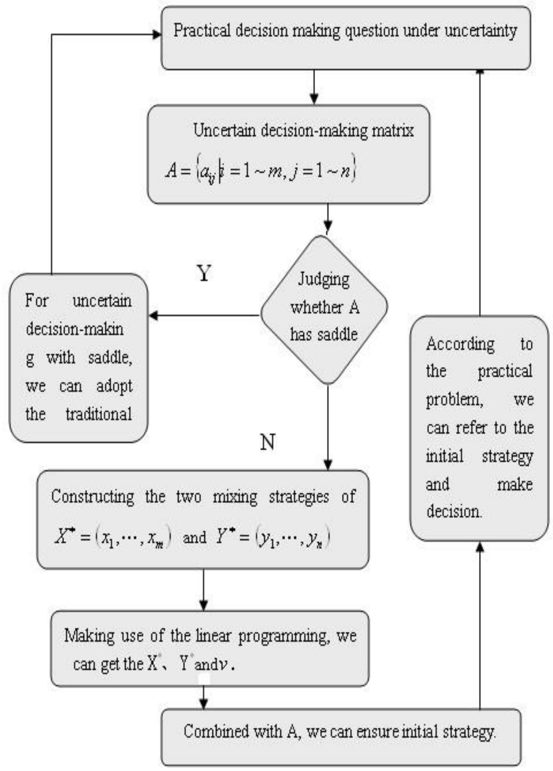

Figure 1: Solution Course of the General Uncertain Decision-making

Y^*=(y_1,…, y_n) represents the decision making reaction of the other side in the decision making chart. The decider can process the nature condition to the rational player. In the practical problem, the meaning of this result is the transformation of the uncertainty to the risk decision making. In some uncertain decision-making problems of more nature conditions, the probabilities of these nature conditions may be zero. They provide more information for the decider to judge the good and bad of nature condition and provide the struggling orientation to the decider.

V represents the decision value of the decider under the condition of X^*=(x_1,…,x_m) and Y^*=(y_1,…,y_n). This value may not be found in the decision chart, but it stands for the minimum expected value which the decider can obtain. The worst result of the decision making cannot lower than this value [10].

Figure 1 shows the illustration of the general solution to the decision making problem under uncertainty.

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4. CONCLUSION

Uncertain decision-making is a common problem in economic activities and management activities. Some discussions have been done between lots of scholars and experts [11]. Unfortunately, existing literatures still have not given a final solution. This research discovers that it is scientifically reasonable to transform decision-making problem into zero-sum two-person game in the general decision problem explaining by A. Wald. But A. Wald and his followers didn't continue this idea to discuss the problem thoroughly. This paper puts forward the solution to uncertain decision-making problems. It is just a general science project, which doesn't pick project by decision makers' preference like "Five Criteria". Due to space constraints, a further application study on this problem will be elaborated in another paper.

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