

DECISION MAKING IN INVESTING: APPLICATION OF INTERVAL-PROMETHEE BASED ON THE COMPOSITE WEIGHT

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ABSTRACT

This paper introduces the interval-PROMETHEE method based on the composite weight to the decision making process of value investing. We use five performance criteria to measure the growth potential of the firms. They are represented by interval numbers from the real financial data. And the weights of the five criteria are compounded through the information entropy and the subjective method. The interval-PROMETHEE method is applied to rank all the stocks in Shanghai Stocks Exchange. The portfolio with the top 10 stocks was proven to have higher return than the top 20 stocks, and they all have higher return than the Shanghai index which represents the average performance of the Chinese stock market within the 22 month investment period. The empirical research shows the effectiveness of this method in the investment decision making.

Keywords: *Value Investing, Multi-Criteria Decision-Making, Interval-PROMETHEE; Composite Weight*

1. INTRODUCTION

The MCDM (Multiple Criteria Decision Making) often deals with ranking of many concrete alternatives from the best to the worst ones based on multiple conflicting criteria. As a MCDM problem, Value investing has been proven to be a successful investment strategy and paid more and more attention in the last three decades. It was initially proposed by Graham and well developed later by Buffet. As suggested, the main process consists of two phases. The first is to find the equities with extraordinary overall performance based on several financial ratios. The second is to evaluate the intrinsic value of each selected equity. We will lay the emphasis on the first phase in this paper. That is, we will focus on the overall performance evaluation of equities based on the five criteria suggested by Buffet.

PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) is one of the most recent MCDM methods that were developed by Brans for the first time in 1982 at a conference in Canada and further extended by Vincke and Brans [1]. The PROMETHEE is an outranking and simple ranking method for a finite set of alternative actions to be ranked and selected among criteria. The PROMETHEE family initially included the PROMETHEE I for partial ranking of the alternatives and the PROMETHEE II for

complete ranking of the alternatives. Then other versions of the PROMETHEE methods were presented [2]. But the PROMETHEE II is fundamental to implement the other PROMETHEE methods.

The PROMETHEE method has successively been applied in many fields, especially in the investment analysis and performance evaluation. Mareschal and Brans[3], Vranegl et al.[4], Babic and Plazibat[5], Bouri et al.[6] and Albadvi et al.[7] all applied PROMETHEE as a decision making tool to solve the different problems in the field of finance.

In the above mentioned application in finance, the performance parameters are all represented by single value numbers. However, finance investment decision making is a complex process due to some uncertain nature of financial markets, or because the markets are not well understood which is called non-random uncertainty. It is hard to convey the profitability information just by a single-valued number or a simple average of the past. So in this paper, we will introduce interval number to model the uncertainty in value investing decision making problem, and apply the PROMETHEE II method to select firms with outstanding performance in the decision making process of value investing.



Moreover, determination of the weights is an important step in multi-criteria methods. But PROMETHEE II assumes that the decision-maker can weigh the criteria appropriately, at least when the number of criteria is not too large [8]. So the weights are usually provided by expert. This method of determining the weight seems too subjective. We will introduce the composite weight in this paper.

The rest of this paper is organized as follows: in section 2 the original PROMETHEE II method is recalled and then generalized to interval-PROMETHEE in section 3, at the same time, the composite weight is introduced simply; in section 4, the multi-criteria ranking problem of ranking outstanding firms is presented, and the application of interval-PROMETHEE method based on the compound weight is performed for stocks based on five criteria in Shanghai Stock Exchange; finally the conclusions are drawn in section 5.

2. PROMETHEE II

Suppose $A = \{a_1, a_2, \dots, a_n\}$ is a set of alternatives to rank, $F = \{f_1, f_2, \dots, f_m\}$ is a set of criteria, which have to be optimized according to their potential contributions to the final results. If the higher of the performance evaluation for a criterion, the higher of the ranking, this criterion will be maximized; if the higher of the performance evaluation, the lower of the ranking, this criterion should be minimized. A pair-wise comparison between any two alternatives a_i and a_j is implemented, and

$$d_k = f_k(a_i) - f_k(a_j) \tag{1}$$

is determined firstly, where $f_k(a_i)$ is the evaluation of alternative a_i corresponding to the k -th criterion f_k . Then the intensity of the preference of an alternative a_i over another alternative a_j is denoted as $P_k(d_k)$ which is called the preference function. Six different types of the preference function for the k -th criterion f_k are recommended by Brans et al. The decision makers can also define their own preference function. A linear preference function [1] is selected in this paper:

$$P_k(d_k) = \begin{cases} 0, & \text{if } d_k < 0 \\ \frac{d_k}{q_k}, & \text{if } 0 \leq d_k \leq q_k \\ 1, & \text{if } d_k > q_k \end{cases} \tag{2}$$

This shows that the intensity of the decision maker's preference between the alternatives a_i and a_j which increases linearly with the growth of d_k up to q_k . After the threshold q_k , the preference will be equal to 1. For ranking purposes, q_k can be set according to the real situation. The value of the preference scales varies from 0 (no preference) to 1 (strong preference).

The preference of alternative a_i and a_j is evaluated for each criterion and the preference index is determined by

$$\begin{aligned} \pi(a_i, a_j) &= \sum_{k=1}^m w_k P_k(d_k) \\ &= \sum_{k=1}^m w_k P_k(f_k(a_i) - f_k(a_j)), \end{aligned} \tag{3}$$

where $\forall a_i, a_j \in A$, $w_k \in W = \{w_1, w_2, \dots, w_m\}$ is a weight for the k -th criterion f_k which is a measure for the relative importance of each criterion and $\sum_{k=1}^m w_k = 1$. The leaving flow of a_i is a measure of the alternative a_i over all the other alternatives and it is given by

$$\phi^+(a_i) = \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n \pi(a_i, a_j) \tag{4}$$

And the entering flow of a_i is a measure of all the other alternatives over a_i and it is given by

$$\phi^-(a_i) = \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n \pi(a_j, a_i) \tag{5}$$

The leaving flow and the entering flow represent the preference of all the other alternatives over the alternative being examined. The basic premise is that the higher the leaving flow and the lower the entering flow, the better the alternative. PROMETHEE II method is a total ranking method based on the evaluation of the net flow obtained by subtracting the entering flow from the leaving flow,

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i), \forall a_i \in A \tag{6}$$

The higher the net flow the better the alternative.

3. INTERVAL-PROMETHEE

As we can see from above, the original PROMETHEE method is designed for a single-



valued number. When some uncertainties inherent are represented as interval numbers, the interval-PROMETHEE method is then required. This comes from the fact that in most cases the input data cannot be defined within a reasonable degree of accuracy. This imprecision is sometimes treated as an interval number. So the regular PROMETHEE algorithm will be generalized to the interval-PROMETHEE.

3.1 Interval Number

An interval number x has such a form: $x = [a, b]$, $a < b$, where a and b are all real number. The interval number set is recorded as $I(R)$. Obviously for $x = [a, b] \in I(R)$, if $a = b$, then $x = a = b$ is an ordinary real, so $R \subset I(R)$. The basic operations with interval numbers are summarized in TABLE 1.

TABLE 1
THE BASIC OPERATION WITH INTERVAL NUMBERS

Addition	$[a, b] + [c, d] = [a+c, b+d]$
Subtraction	$[a, b] - [c, d] = [a-d, b-c]$
Multiplication	$[a, b] \cdot [c, d] = [\min\{ac, bd, ad, bc\}, \max\{ac, bd, ad, bc\}]$
Division	$[a, b] / [c, d] = [\min\{a/c, b/d, a/d, b/c\}, \max\{a/c, b/d, a/d, b/c\}], 0 \notin [c, d]$

Especially, if $a > 0, b > 0, k \geq 0$, then

$$[a, b] \cdot [c, d] = [ac, bd],$$

$$[a, b] / [c, d] = [a/d, b/c],$$

$$k \cdot [a, b] = [ka, kb].$$

3.2 Interval-PROMETHEE

When the performances of a_i and a_j corresponding to the criterion f_k are represented by interval numbers, the $d_k = f_k(a_i) - f_k(a_j)$ is the interval number which is recorded as (u, v) temporarily, and $P_k(d_k)$ in (2) between a_i and a_j based on f_k is expressed as:

$$P_k(d_k) = \begin{cases} 0, & \text{if } u < 0 \\ \frac{d_k}{q_k}, & \text{if } 0 \leq u, v \leq q_k \\ 1, & \text{if } v > q_k \end{cases} \quad (7)$$

Where q_k can be expressed as interval, but for simplicity we regard it single value number here.

The weights of (3) are compounded through the entropy weights (denoted as “E-weight” or “E”) and

the subjective weights (“Sub-weight” or “S”). The method is as follows.

$$C = \beta \times E + (1 - \beta) \times S \quad (8)$$

Where “C” represents the composite weight and $\beta \in [0, 1]$.

Then the procedures of the PROMETHEE method described in (3)-(6) are followed step by step according to the interval number calculations. And other parameters are all considered as regular data with precise numerical values.

Finally, when interval data are included, the net flows are all interval numbers, so the final ranking problem is boiled down to the ranking of interval numbers. Thus the comparison of two intervals plays a crucial role. Shakhnov [9] presented three models of ranking interval-defined objects based on pairwise relations of domination with respect to probability, mathematical expectation, and utility. Xu and Da [10] presented a methodology for comparison of two interval numbers based on possibility degree formula. Zhang and Su [11] reviewed the existing methods for ranking two intervals. An enhanced ranking approach for interval numbers presented by Li [12] was used in this paper which is based on the possibility degree which represents the degree of one interval is greater than another interval. This method is briefly described below.

Let $I_a = [a, \bar{a}]$ and $I_b = [\underline{b}, \bar{b}]$ are two intervals, if $\underline{a} = \underline{b}$ and $\bar{a} = \bar{b}$, then $I_a = I_b$; if the possibility degree of $I_a > I_b$ denoted as $P_{I_a > I_b}$ is greater than 0, then $I_a > I_b$; if $P_{I_a > I_b} < 0$, then $I_a < I_b$. The possibility degree of $I_a > I_b$ is defined as

$$P_{I_a > I_b} = \begin{cases} 1, & \text{if } \underline{a} \geq \bar{b} \\ \frac{(\bar{a} - \bar{b}) - (\underline{b} - \underline{a})}{\bar{a} - \underline{a}}, & \text{if } \underline{a} \leq \underline{b} < \bar{b} \leq \bar{a} \\ \frac{\bar{a} - \bar{b}}{\bar{a} - \underline{a}} + \left(\frac{\bar{b} - \underline{a}}{\bar{a} - \underline{a}} \right) \cdot \left(\frac{\underline{a} - \underline{b}}{\bar{b} - \underline{b}} \right), & \text{if } \underline{b} \leq \underline{a} < \bar{b} \leq \bar{a} \end{cases} \quad (9)$$

If $I_b = [\underline{b}, \bar{b}]$ degrades to a single value, i.e., $I_b = [\underline{b}, \bar{b}] = b$, the possibility degree of $I_a > I_b$ is given by



$$P_{I_a > I_b} = \begin{cases} 1, & \text{if } b < \underline{a} \\ \frac{(\bar{a} - b) - (b - \underline{a})}{\bar{a} - \underline{a}}, & \text{if } \underline{a} < b \leq \bar{a} \\ -1, & \text{if } b > \bar{a} \end{cases} \quad (10)$$

If $I_a = [\underline{a}, \bar{a}] = a$ and $I_b = [\underline{b}, \bar{b}] = b$, the comparison of two intervals becomes ranking two real numerical values. In this scenario, the possibility degree is then defined as

$$P_{I_a > I_b} = \begin{cases} 1, & \text{if } a > b \\ 0, & \text{if } a = b \\ -1, & \text{if } a < b \end{cases} \quad (11)$$

The comparison matrix of possibility degree for m interval numbers is determined by

$$P = \begin{pmatrix} 0 & P_{12} & \dots & P_{1m} \\ P_{21} & 0 & \dots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \dots & 0 \end{pmatrix} \quad (12)$$

Where P_{ij} is the possibility degree of $I_{a_i} > I_{a_j}$ and meets $-1 \leq P_{ij} \leq 1$ and $P_{ij} + P_{ji} = 0$. Let

$$r_i = \sum_{k=1}^m P_{ik}, i = 1, 2, \dots, m \quad (13)$$

Once we get $R = (r_1, r_2, \dots, r_m)^T$, the comparison of interval numbers then becomes ranking r_i , i.e., the higher the r_i , the higher the interval number.

4. APPLICATION TO VALUE INVESTING

The philosophy behind value investing is that the intrinsic value determines the stock price of a firm and the stock price fluctuates around the value; the outstanding performance suggests that the firm has great potential and ability to grow and profit more, which lead to a higher intrinsic value, and thus the current below-intrinsic-value price is expected to rise, therefore if an investor buys and holds it now and he or she will surely make money in a long time horizon.

The five criteria for measuring the growth potential of a firm suggested by Warren Buffet are return on assets(denoted as f_1), increasing ratio of sales(f_2), increasing ratio of equity(f_3), increasing

ratio of earnings per share(f_4), increasing ratio of free cash flow(f_5). The five criteria provide a relatively overall evaluation of the ability of a firm to grow and profit. They are listed in TABLE 2.

Table 2: Five Criteria For Selecting Stocks In Value Investing
(Unit : Percentage)

Criterion	Definition	Min /Max
f_1 Return on assets	$\frac{\text{Earnings before interest and tax}}{\text{total assets}}$	Max
f_2 Increasing ratio of sales	$\frac{\text{Sales}_t - \text{Sales}_{t-1}}{\text{Sales}_{t-1}}$	Max
f_3 Increasing ratio of equity	$\frac{\text{Equity}_t - \text{Equity}_{t-1}}{\text{Equity}_{t-1}}$	Max
f_4 Increasing ratio of earnings per share	$\frac{[\text{earnings per share}]_t - [\text{earnings per share}]_{t-1}}{[\text{earnings per share}]_{t-1}}$	Max
f_5 Increasing ratio of free cash flow	$\frac{[\text{free cash flow}]_t - [\text{free cash flow}]_{t-1}}{[\text{free cash flow}]_{t-1}}$	Max

The performance can be calculated based on the firms' financial statements. In this paper, the original finance data are obtained through CSMAR4.0.

In order to obtain the composite weight of the five criteria, we adopt the entropy weights in [13]. The subjective weights are obtained from the experts' advice. Finally, we can compute the composite weights according to (8), where $\beta = 0.6$. The three weights are listed in TABLE 3.

Table 3: The Weight

Criterion	f_1	f_2	f_3	f_4	f_5
E-weight	0.196	0.188	0.226	0.155	0.235
Sub-weight	0.30	0.15	0.25	0.20	0.10
Com-weight	0.2376	0.1728	0.2356	0.173	0.181

Table 4 The Ranking Result

Rank	Code	Net flow
1	600395	32.2476
2	600216	32.2411
3	600892	31.5398
4	600551	30.8765
5	600712	30.5263
6	600252	30.3905
7	600242	27.5853
8	601607	27.5163
9	600166	26.9302
10	600546	26.4537

With the interval-PROMETHEE methods based on the composite weight presented above, we rank all the stocks in Shanghai Stocks Exchange and get the order. Peculiarly, considering some policy factors and so on, we eliminate some abnormal firms here. So we only list the code of the top10 among them according to the composite weights in TABLE 4.

The higher the rank of the firm, the higher the potential of the stock price to grow. The price with rank 1 is supposed to have higher potential to grow than that with rank 2. For the complex nature of finance market, it is not realistic to expect the stock with rank n to grow faster than that with rank $n+1$. But it is natural to expect that the portfolio value with top m stocks grows faster than that with top n stocks, where m is less than n . Thus, the effectiveness of this interval ranking method can be verified by the investment effect of portfolios with top m stocks and top n stocks.

Now we construct portfolios with top10, top20 according to the composite weights, and the two portfolios are denoted as top10 and top20 respectively, and each stock with 1000 shares. Suppose we invested the two portfolios right after at the end of 2009, and on the data Jan 04, 2010. We kept them until October 31, 2011 without any changing of the portfolios.

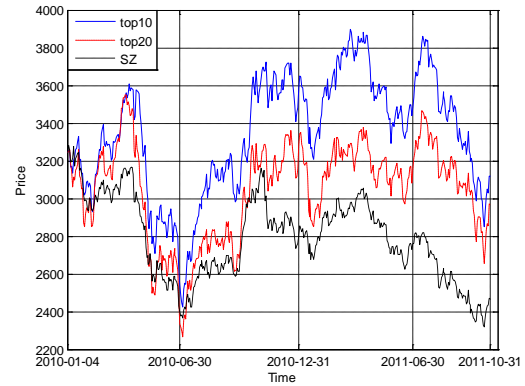


Figure 1: The Portfolio Values

For comparing the investment effect, the portfolios are adjusted to be equal at the right beginning and adjusted with the same ratio in the following. For example, suppose we take the Shanghai Index value 3000, on Jan 04 2010 as the base point, the portfolio value is 30000, then we divide the portfolio value by 10 ($30000/3000=10$), and thus the adjusted portfolio value is 3000 equal to the Shanghai index at the very beginning, then for each day after, the adjusted portfolio is set to the real value divided by 10, so that they are comparable. We adjusted the two portfolios as and the evolution of the portfolios are plotted in Figure 1.

As we can see from Figure 1, starting from the same point, the portfolio value with the top 10 stocks (top10) stays above that with the top 20 stocks (top20) within the 22-month investment period. By the way, the two portfolios are almost stay above the Shanghai index (denoted as SZ in Figure 1) which represents the average performance of the Chinese stock market throughout the investment period. This fact suggests that the interval-PROMETHEE based on the composite weight we used is effective.

Furthermore, we compute the distribution of the annual rate of the top10, the top20, from the beginning up to 2010-06-30, 2010-12-31, 2011-06-30 and 2011-10-31 respectively. The corresponding investment periods are 6 months, 12 months, 18 months and 22 months respectively. The mean values for the annual return distribution are listed in TABLE 5 and the standard deviations for the annual return distribution are listed in TABLE 6.

Table 5: MEAN

Investment period	Mean		
	top10	top20	SZ
6 months	-0.4223	-0.5436	-0.6035
12 months	0.1179	0.0239	-0.1446
18 months	0.0790	0.0109	-0.0998
22 months	0.0144	-0.0044	-0.1339

Table 6: STANDARD DEVIATION

Investment period	std		
	top10 (Com-w)	top20 (Com-w)	SZ
6 months	0.0195	0.0213	0.2270
12 months	0.0185	0.0203	0.2238
18 months	0.0174	0.0193	0.2080
22 months	0.0171	0.0190	0.2064

When the investment period is 6 months (up to 2010-06-30) the average value of the return of the top 10 portfolio is -42%, the top 20 portfolio has the average return rate -54%, although they are both loss, but the top 10 loss is less than the top 20 loss. Up to 2011-01-04 (12 months), the top 10 portfolio has the average return rate 11.79 % which is higher than that of the top 20 portfolio which is 2.39%. Until 2011-10-31 (22 months), the top 10 portfolio has higher average return rate than the top 20 portfolio within the whole investment period. Similarly we can see the better performance of the top 10 portfolio and the top 20 portfolio up to 2011-10-31 than the Shanghai index (SZ).

On the other hand, the risk of a portfolio is measured by the standard deviation (std) of the return rate. The smaller the std, the less risky the portfolio. For the above mentioned 4 investment period, the stds of the top 10 portfolio are all less than those of the top20 portfolio, therefore, the top 10 portfolio is less risky than the top 20 portfolio. All in all, the top 10 portfolio brings higher return but less risk than the top 20 portfolio.

5. CONCLUSIONS

This paper applied the interval-PROMETHEE method based on the composite weight to the investment decision-making process of value investing. Five criteria were used to evaluate the growth-potential performance suggested by Buffet and were represented by interval numbers based on the real financial data gathered from the data base CSMAR4.0. The interval-PROMETHEE method was applied to rank all the stocks in Shanghai Stock Exchange. The portfolio with the top 10 stocks was

proven to have higher return than the top 20 stocks within the whole 22-month period.

The empirical study showed the effectiveness of the interval-PROMETHEE method based on the composite weight in the decision making process of value investing.

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