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# SINE MOVABLE TEETH PROFILE EQUATION ESTABLISHMENT AND SIMULATION

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**ABSTRACT**

The driving shaft as well as the tooth profile equation of the shell spatial sine raceway has been established according to the spatial meshing theory. And the tooth profile simulation has been also conducted. It is required to carry on the force analysis on the single tooth and establish the spatial mechanical model of the cylindrical sine movable teeth drive based on the deformation compatibility conditions. At the same time, the principal curvatures of the contact point between the sine raceway and the movable tooth have been calculated and the distribution rules have been also analyzed. What's more, the spatial motion of movable teeth has been discussed and the formula of sliding rate between the conjugate profiles has been given. And the influence of the changes of structure parameters of movable teeth drive on the sliding rate has been also qualitatively analyzed in this paper.

**Keywords:** *Tooth Profile Equation, Movable Teeth Drive, Conjugate Profile, Spatial Meshing*

## 1. INTRODUCTION

With the advancement in networking and the cylindrical sine movable teeth drive refers to a movable drive which has the self-balancing structure. The inertial force can be balanced by only employing the structure of the single-row shock wave device, which can shorten the drive kinematic chains, reduce the power loss and improve the efficiency and the movement stability of drive system. The cylindrical sine movable teeth drive belongs to the spatial meshing transmission [1]. Currently, the study on the spatial meshing theory of movable teeth drive is not perfect.

In this paper, the tooth profile equation of cylindrical sine movable teeth drive has been established after analyzing the calculation of DOF, the movement possibility, the continuous drive conditions and some others. Meanwhile, the complete design theory and the spatial meshing theory of cylindrical sine movable teeth drive have been also established. Through adopting this theory, the force analysis as well as the study on the variation of sliding rate between the conjugate profiles of movable teeth drive can be conducted, which can provide the theoretical basis for the strength design and the optimization design of this drive.

## 2. ESTABLISHMENT OF COORDINATE SYSTEM

The sine raceway in the cylindrical sine movable teeth drive is composed by the movement of the center of movable teeth along the spatial sine trajectory curve. In order to facilitate the processing and the manufacturing of this drive mechanism as well as the further study of theoretical analysis, it is required to establish the tooth surface equation of sine raceway.

Assuming that $\sigma_1$ ( $o$ ; $\vec{i}_1$ , $\vec{j}_1$ , $\vec{k}$ ), $\sigma_2$ ( $o$ ; $\vec{i}_2$ , $\vec{j}_2$ , $\vec{k}$ ), $\sigma_3$ ( $o$ ; $\vec{i}_3$ , $\vec{j}_3$ , $\vec{k}$ ) and $\sigma_4$ ( $o$ ; $\vec{i}_4$ , $\vec{j}_4$ , $\vec{k}_4$ ) are respectively the coordinate systems fixed with the driving shaft, the leading frame, the shell and the movable tooth [2]. The coordinate relationship can be shown in Figure 1.

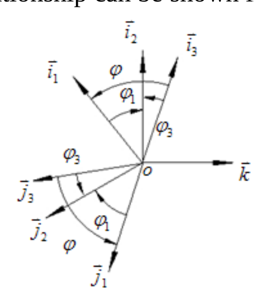


*Figure 1: The Transformation Of Coordinate Systems*

The common coordinate origin of $\sigma_1$, $\sigma_2$ and $\sigma_3$ is $o$ and the all movable tooth are distributed on

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the cylindrical surface whose radius is $R$. The tooth surface $\Sigma_1$ of driving shaft, the tooth surface $\Sigma_2$ of leading frame, the tooth surface $\Sigma_3$ of shell are composed by the tooth surface $\Sigma_4$ of movable tooth. The shell is fixed, $\sigma_3$ is the fixed coordinate system and $\varphi$ is the rotation angle from the coordinate system of driving shaft to the fixed coordinate system [3]. Assuming that the coordinate system of leading frame is the reference coordinate system, $\varphi_1$ and $\varphi_3$ are respectively the rotation angles from the coordinate systems of driving shaft and shell to the reference coordinate system.

$$\varphi_1 = -\frac{Z_3}{Z_1+Z_3}\varphi + \frac{\pi}{Z_1+Z_3} \quad \varphi_3 = \frac{Z_1}{Z_1+Z_3}\varphi + \frac{\pi}{Z_1+Z_3}$$

## 3. THE MESHING EQUATION

It is necessary to establish the meshing equation and the tooth surface equation of cylindrical sine movable teeth drive through taking the meshing of driving shaft and movable tooth as the example. According to the spatial meshing theory [4, 5], the meshing equation and the meshing function of the conjugate surface $\Sigma_1$ and $\Sigma$ can be shown as

$$\vec{n}\cdot\vec{v}_{41} = 0 \qquad \vec{n}\cdot\vec{v}_{41} = \Phi$$

(1)

In order to establish the meshing equation of this drive and calculate the vectors in different coordinate systems, it is required to convert the all vectors into a unified coordinate system. To facilitate the solution, it is also necessary to convert the all vectors into the coordinate system $\sigma_2$ of leading frame.

The movable tooth in the cylindrical sine movable teeth drive is the regular sphere and the tooth surface equation is the spherical equation [6]. The coordinate system of movable tooth can be shown in Figure 2.

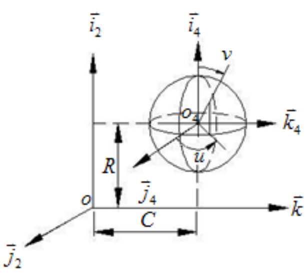


*Figure 2: The movable teeth coordinate system*

The tooth surface equation of movable tooth can be expressed by the spherical coordinates

$$\vec{r}_q = r\cos(v)\vec{i}_4 + r\cos(u)\sin(v)\vec{j}_4 + r\sin(u)\sin(v)\vec{k}_4$$

(2)

$$\begin{cases} \dfrac{\partial\vec{r}_q}{\partial u} = -r\sin(u)\sin(v)\vec{j}_4 + r\cos(u)\cdot\sin(v)\vec{k}_4 \\ \dfrac{\partial\vec{r}_q}{\partial v} = -r\sin(v)\vec{i}_4 + r\cos(u)\cos(v)\vec{j}_4 + r\sin(u)\cos(v)\vec{k}_4 \end{cases}$$

(3)

According to the differential geometry, the unitary normal vector of each point on the spherical surface is

$$\vec{n} = \frac{\partial\vec{r}_q}{\partial u}\times\frac{\partial\vec{r}_q}{\partial v} \Big/ \left|\frac{\partial\vec{r}_q}{\partial u}\times\frac{\partial\vec{r}_q}{\partial v}\right|$$

(4)

After the coordinate transformation we can get

$$\vec{n} = \cos(v)\vec{i}_2 + \cos(u)\sin(v)\vec{j}_2 + \sin(u)\sin(v)\vec{k}$$

(5)

The relative speed of the meshing point between the driving shaft and the movable tooth is

$$\vec{v}_{41} = \frac{d\vec{\xi}}{dt} + \vec{\omega}_{41}\times\vec{r}_{q2} - \vec{\omega}_1\times\vec{\xi}$$

(6)

The relative speed $\vec{v}_{41}$ of the meshing point can be got after taking $\vec{\xi}$, $\frac{d\vec{\xi}}{dt}$, $\vec{\omega}_{41}$, $\vec{\omega}_4$ and $\vec{r}_{q2}$ into the formula

$$\vec{v}_{41} = \frac{Z_3}{Z_1+Z_3}\omega r\cos(u)\sin(v)\vec{i}_2 - \frac{Z_3}{Z_1+Z_3}\omega\cdot(r\cos(v)-R)\vec{j}_2 - \frac{Z_1Z_3}{Z_1+Z_3}A\omega\cos(Z_1\varphi_1)\vec{k}$$

(7)

Then the meshing function can be got after classification

$$\Phi(u,v;\varphi) = -\frac{Z_3}{Z_1+Z_3}\omega\sin v(R\cos uAZ_1 + \sin u\cos(Z_1\varphi_1))$$

(8)

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## 4. THE ESTABLISHMENT OF THE TOOTH SURFACE EQUATION OF SINE RACEWAY

The tooth surface equation of driving shaft can be obtained after converting the tooth surface equation (8) into $\sigma_1$ and combining with the meshing equation, namely

$$\begin{cases} \vec{r}_1 = \mathbf{M}_{21} \cdot \mathbf{M}_{42} \cdot \vec{r}_q \\ \tan(u) = -R/\left(AZ_1 \cos(Z_1\varphi_1)\right) \end{cases} \quad (9)$$

In this formula, $\mathbf{M}_{21}$ —— the coordinate transformation matrix from $\sigma_2$ to $\sigma_1$.

$$\mathbf{M}_{21} = \begin{bmatrix} \cos(\varphi_1) & -\sin(\varphi_1) & 0 & 0 \\ \sin(\varphi_1) & \cos(\varphi_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The tooth surface $\Sigma_1$ equation of driving shaft can be written in component forms

$$\begin{cases} x = r\cos(\varphi_1)\cos(v) - r\sin(\varphi_1) \\ \quad \cdot\cos(u)\sin(v) + R\cos(\varphi_1) \\ y = r\sin(\varphi_1)\cos(v) + r\cos(\varphi_1) \\ \quad \cdot\cos(u)\sin(v) + R\sin(\varphi_1) \\ z = r\sin(u)\sin(v) + C \\ \tan(u) = -R/(AZ_1\cos(Z_1\varphi_1)) \end{cases} \quad (10)$$

Similarly, the equation of the tooth surface $\Sigma_3$ of shell is

$$\begin{cases} x = r\cos(\varphi_3)\cos(v) - r\sin(\varphi_3) \\ \quad \cdot\cos(u)\sin(v) + R\cos(\varphi_3) \\ y = r\sin(\varphi_3)\cos(v) + r\cos(\varphi_3) \\ \quad \cdot\cos(u)\sin(v) + R\sin(\varphi_3) \\ z = r\sin(u)\sin(v) + C \\ \tan(u) = -R/(AZ_3\cos(Z_3\varphi_3)) \end{cases} \quad (11)$$

When $\varphi_1$ and $\varphi_3$ are the set values, the above two tooth surface equations will be the contact line equations and the contact line of this drive will be a part of the movable teeth circle[7]. In actual processing, considering the influence of processing precision and the contact conditions, the radius $r'$ of raceway should be slightly larger than the radius $r$ of movable tooth. Generally speaking, $r' = (1.04 \sim 1.11)r$, which indicates that the cylindrical sine movable teeth drive is the spatial contact meshing transmission in actual processing [8,9].

At present, there are two common raceway surfaces at home and abroad: the single-arc raceway and the double-arc raceway [10, 11]. If the single-arc is employed, the actual tooth profile equations of the sine raceway of driving shaft and shell are respectively:

$$\begin{cases} x = r'\cos(\varphi_1)\cos(v) - r'\sin(\varphi_1) \\ \quad \cdot\cos(u)\sin(v) + R_1\cos(\varphi_1) \\ y = r'\sin(\varphi_1)\cos(v) + r'\cos(\varphi_1) \\ \quad \cdot\cos(u)\sin(v) + R_1\sin(\varphi_1) \\ z = r'\sin(u)\sin(v) + C \\ \tan(u) = -R_1/(AZ_1\cos(Z_1\varphi_1)) \end{cases} \quad (12)$$

$$\begin{cases} x = r'\cos(\varphi_3)\cos(v) - r'\sin(\varphi_3) \\ \quad \cdot\cos(u)\sin(v) + R_3\cos(\varphi_3) \\ y = r'\sin(\varphi_3)\cos(v) + r'\cos(\varphi_3) \\ \quad \cdot\cos(u)\sin(v) + R_3\sin(\varphi_3) \\ z = r'\sin(u)\sin(v) + C \\ \tan(u) = -R_3/(AZ_3\cos(Z_3\varphi_3)) \end{cases} \quad (13)$$

In this formula, $R_1$ ——the radial radius of the raceway spatial sine curve of driving shaft (mm). $R_3$ ——the radial radius of the raceway spatial sine curve of shell(mm).If the double-arc raceway is employed, the tooth profile equation of this double-arc sine raceway (taking the driving shaft as the example) can be shown as[12,13]

$$\begin{cases} x = (r'\cos(v) - \Delta r\cos(\beta))\cos(\varphi_1) - (r'\cos(u)\cdot \\ \quad \sin(v) - \Delta r\cos(u)\sin(\beta))\sin(\varphi_1) + R_1\cos(\varphi_1) \\ y = \left(r'\cos(v) - \Delta r\cos(\beta)\right)\sin(\varphi_1) + (r'\cos(u)\cdot \\ \quad \sin(v) - \Delta r\cos(u)\sin(\beta))\cos(\varphi_1) + R_1\sin(\varphi_1) \\ z = r'\sin(u)\sin(v) - \Delta r\sin(u)\sin(\beta) + C \\ \tan(u) = -R_1/(AZ_1\cos(Z_1\varphi_1)) \end{cases} \quad (14)$$

In this formula, $r'$ ——the radius of raceway (mm); $\Delta r$ ——the difference of the radius of raceway and the radius of movable tooth (mm); $\beta$ —— the contact angle (rad).

The tooth profile equation of the right tooth surface is equal to the one of the left tooth surface, in which the contact angle $\beta$ takes the negative value.

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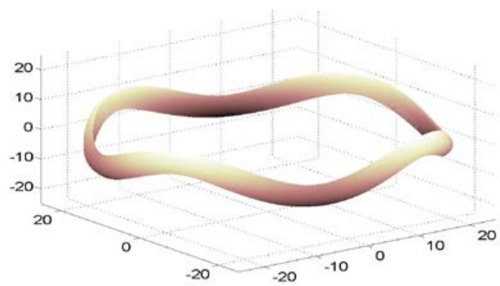
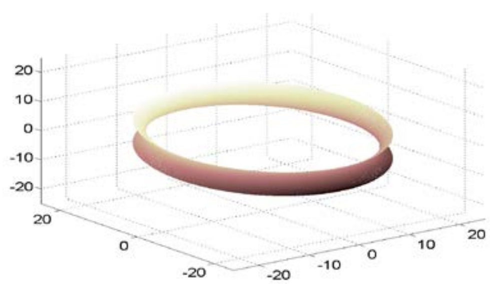
## 5. THE TOOTH SURFACE SIMULATION

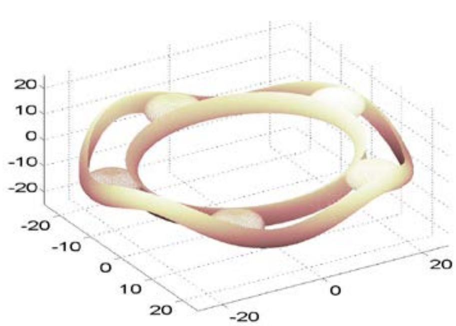
It is required to adopt the Matlab engineering software to carry on the tooth surface simulation for the cylindrical sine movable teeth drive. In order to simulate the three dimensional surface, it is required to divide 100 units on contact lines and divide 1001 units in the circumferential direction of tooth surface. Then the tooth profile surface can be divided into $100 \times 1001$ grids. It is necessary to calculate the corresponding values of each grid node based on the mathematical model and then the grid graph of tooth profile surface will be obtained. Afterwards, it is needed to employ the two-dimensional linear interpolation method to get the surface simulation and the meshing simulation of the whole tooth profile, which can be shown in Figure 3 (a), (b) and (c) [14].




*(A)Outer Sine Raceway Tooth Profile Simulation*




*(B) Inner Sine Raceway Tooth Profile Simulation Of Shell*




*(C)The Teeth Profile Meshing Simulation*

*Figure 3: The Surface Simulation And The Meshing Simulation Of Tooth Profile*

## 6. CONCLUSION

The structure composition and the drive principle of cylindrical sine movable teeth drive has been discussed and the computational formula of drive ratio as well as the correct continuous drive conditions of movable teeth drive has been also derived. At the same time, the tooth profile equation of the tooth surface of spatial sine raceway has been also established and the tooth profile surface has been also simulated in this paper.

Based on the force analysis of the single tooth and the deformation compatibility conditions, the spatial mechanical model of the cylindrical sine movable teeth drive has been established. Accordingly, the numerical solution of the contact force of each component on the movable tooth can be calculated and the force computational formula of other components of this drive can be established, which will provide the basis for the optimization design and the strength analysis of lindrical sine movable teeth drive. Then it will also prove that the cylindrical sine movable teeth drive is a structural system with the self-balancing capability.

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