ON-LINE PORTFOLIO SELECTION VIA MEAN REVERSION STRATEGY

1, 2 LI GAO, 1 WEIGUO ZHANG

1 School of Business Administration, south China University of Technology, Guangzhou 510640, China
2 School of Science, South China University of Technology, Guangzhou 510640, China

ABSTRACT

This paper presents a novel adaptive algorithm using mean reversion strategy without transaction cost. The antocior algorithm only exploits the property of “reversal to the mean” and its performance not only significantly depends on the size of window but also fluctuates wildly according to the different size of window. To overcome these limitations, this proposed algorithm is designed to deal with the portfolio selection problem by fully exploiting both the price momentum and the price reversal in Chinese stock markets. Equipped with several parameters, the proposed mean reversion strategy can better track the changing stock market. Extensive experiments on real stock data from Chinese markets demonstrate the effectiveness of our strategies in comparison with the anticior algorithm without knowledge of the investment duration.

Keywords: Online Portfolio, Universal, Mean Reversion, Momentum, Reversal

1. INTRODUCTION

Portfolio selection (PS) problem, which has recently attracted increasing interests in machine, online algorithms and computational finance, is a practical financial engineering problem to seek the best allocation of wealth among several stocks in the long run by maximizing cumulative wealth or risk-adjusted return. In this article, we investigate sequential portfolio selection (also termed on-line portfolio selection) strategies, which sequentially determine portfolios based on past price of stocks [1].

Classical approaches for portfolio selection have been based on the assumption of knowing probability distribution of the stocks. In addition, risk references of investors also play an important role in portfolio optimization. Different ways to model these reactors result in different approaches in traditional portfolio optimization, such as the mean-variance analysis pioneered by Markowitz [2]. Markowitz’s model concerns distributional assumptions about the behavior of stock prices and depends on some objective function and/or utility function defined according to the investor’s goal. This conceptual model has proved in the past to be useful by finance practitioners, private investors and researchers. However, this approach about distributional assumptions encounters many difficulties because the future evolution of stock prices is notoriously difficult to predict, while the selection of a distribution class inevitably brings a measure of arbitrariness.

Cover [3] has proposed a different approach to overcome the problems related to the necessity of making statistical assumptions about the stock prices behavior, which is completely based on their past. Within Cover’s investment framework, portfolio selection is based completely on the sequence of past prices, which is taken as is, with little, if any, statistical processing. To emphasize this independence from statistical assumptions, such portfolios are called universal portfolios [3-9]. The optimal growth rate of wealth is achieved by a best constant rebalanced portfolio (BCRP) that is an asset allocation algorithm which keeps the same distribution of wealth among a set of stocks from period to period. Later, this problem has also been actively studied from a learning to select portfolio perspective, with roots in the fields of machine learning, data mining, information theory and statistics.

In contrast to universal portfolios which have focused on finding a good portfolio vector that it is fixed, switching algorithms have provided instead an investment regime that switches form one weight vector to another with the market changing, according to a prior distribution [10-14].

Due to the sequential nature, recent on-line portfolio selection techniques often design algorithms using the mean reversion property [1,
The remainder of this paper is organized as follows. Section 2 describes formally defines the problem of on-line portfolio selection. Section 3 reviews related work and highlights their limitations. Section 4 presents the methodology employed in this paper. Section 5 discusses data sources and presents the results of a set of numerical experiments. Section 6 concludes the study.

2. PROBLEM SETTING

One approach for studying sequential investment strategies consists of the following market model and investment. We model the market as a sequence of price relative vectors \( x_t = (x_t(1), x_t(2), \cdots, x_t(m)) \), where \( x_t(i) \) represents the portion of the wealth invested in the stock \( x_t(j) \) at day \( t \). Typically, we assume the portfolio is self-financed and no margin/short is allowed, therefore each entry of a portfolio is non-negative and adds up to one, that is, 

\[ b_t \in \Delta_m, \text{ where } \Delta_m = \{ b_t \in \mathbb{R}_+^m, \sum_{i=1}^{m} b_t(i) = 1 \}. \]

The investment procedure is represented by a portfolio strategy, that is, \( b_t = \frac{1}{m} \) and following sequence of mappings \( b_t : \mathbb{R}_+^{m(t-1)} \to \Delta_m, \text{ where } b_t = b_t(x^{t-1}) \) is the \( t \)-th portfolio given past market sequence \( x^{t-1} = (x_1, x_2, \cdots, x_{t-1}) \). We denote by \( b^n = (b_{1}, b_{2}, \cdots, b_{n}) \) the strategy for \( n \) periods.

On the \( t \)-th period, a portfolio \( b_t \) produces a portfolio period return \( S_t \), that is, the wealth increases by a factor of 

\[ S_t = b_t^T \cdot x_t = \sum_{i=1}^{m} b_t(i) x_t(i). \]

Since we reinvest and adopt price relative, the portfolio wealth would multiplicatively grow. Thus, after \( n \) periods, a portfolio strategy \( b^n \) produces a portfolio cumulative wealth of \( S_n \), which increases the initial wealth by a factor of 

\[ S_n(b^n, x^n) = S_0 \prod_{t=1}^{n} b_t^T \cdot x_t. \]
where \( S_n \) is set to RMB 1 for convenience.

Finally, let us formulate the online portfolio selection problem. In this task, a portfolio manager is a decision maker, whose goal is to produce a portfolio strategy \( b^n \) aiming to maximize the cumulative wealth \( S_n \). He/she computes the portfolios sequentially. On each period \( t \), the manager has access to the sequence of previous price relative vectors \( x^{t-1} \). On the basis of this historical information, he/she computes a new portfolio \( b_t \) for next price relative vector \( x_t \), where the decision criterion varies among different managers. The portfolio \( b_t \) is scored based on portfolio period return \( S_t \). Note that without historical information, the initial portfolio is set to uniform. The resulting portfolio is evaluated by its portfolio daily return. This procedure is repeated until the end, and the portfolio strategy is finally scored according to portfolio cumulative wealth \( S_n \).

The step of on-line portfolio selection algorithm:

- Initialize \( S_0 = 1 \), \( b_t = (\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}) \)
- for each trading day \( t = 1, 2, \ldots, n \) do
  1. Portfolio manager learns the portfolio \( b_t \) based on historical information
  2. Market reveals the market price relative \( x_t \)
  3. Portfolio incurs a portfolio daily return \( s_t = b^T x_t \)

For a given trading day \( t \), consider the most recent past \( w \) trading days, where \( w \) (the window) is some integer parameter.

We define

\[
LX_1 = [\log(x_{t-w+1}), \ldots, \log(x_{t-w})]^T \\
LX_2 = [\log(x_{t-w+1}), \ldots, \log(x_{t})]^T
\]

where \( \log(x_t) \) denotes \( \log(x_{t-1}), \ldots, \log(x_t) \). Set \( w^j_t = [t-2w+1, t-w] \) and \( w^j = [t-w+1, t] \) be two consecutive windows. We denote the \( j \)th column of \( LX \) by \( LX_k(j) \). Moreover, let \( \mu_k = (\mu_k(1), \ldots, \mu_k(m)) \) be the vectors of averages of columns of \( LX \) and let \( \sigma_k = (\sigma_1(1), \ldots, \sigma_1(m)) \) be the vector of standard deviations of columns of \( LX \).

In the above portfolio selection model, we make several general assumptions as follows.

1. Transaction cost: we assume no transaction cost or taxes exist in this portfolio selection model;
2. Market liquidity: we assume that one can buy and sell required quantities at last closing price of any given trading period;
3. Impact cost: we assume that market behavior is not affected by a portfolio selection strategy in our study.

3. THE ANTICOR ALGORITHM

The Anticor algorithm [15] evaluates changes in stocks’ performance by dividing the sequence of previous trading days into equal-sized periods called windows, each with a length of \( w \) days. \( w \) is an adjustable parameter called the window size. The Anticor algorithm is based on a “mean reversion” assumption: both a stock’s high and low prices are temporary and that a stock’s price will tend to move to the average price over time, namely, stock growth rates are stable in the long term and occasional larger return rates will be followed by smaller rates (and vice versa).

Specifically, whenever the algorithm [15] detects that (i) a stock \( i \) outperformed a stock \( j \) during the last window, but (ii) \( i \)’s performance in the last window is anti-correlated to \( j \)’s performance in the second-to-last window, then it transfers wealth from \( i \) to \( j \).

For a given trading day, the growth rate of any stock \( i \) during a window of time is measured by the product of relative prices during this window. The cross-correlation matrix (and its normalization) between column vectors in \( LX_1 \) and \( LX_2 \) are defined as

\[
M_{\text{cov}}(i,j) = \frac{1}{w-1} (LX_1(i) - \mu_t(i))(LX_2(j) - \mu_t(j))^T
\]

\[
M_{\text{cov}}(i,j) =\begin{cases} 
\frac{M_{\text{cov}}(i,j)}{\sigma_1(i)\sigma_1(j)} & \sigma_1(i), \sigma_1(j) \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

where \( M_{\text{cov}}(i,j) \) measures the correlation between log-relative prices of stock \( i \) over the first window and stock \( j \) over the second window. Moreover, if \( \mu_t(i) \geq \mu_t(j) \) and \( M_{\text{cov}}(i,j) > 0 \), then A. Borodin et al. set

\[
\text{claim}_{i\rightarrow j} = M_{\text{cov}}(i,j) + M_{\text{cov}}^{-}(i,j) + M_{\text{cov}}^{-}(j,j)
\]

\[
\text{transfer}_{i\rightarrow j} = b_{i\rightarrow j}\cdot \text{claim}_{i\rightarrow j}/\sum_j \text{claim}_{i\rightarrow j}
\]

where \( M_{\text{cov}}^{-}(\cdot,\cdot) = \max\{-M_{\text{cov}}(\cdot,\cdot), 0\} \). They will shift their investment from stock \( i \) to stock \( j \) by (3)
21

price reversal path and price momentum

profitable for short-term (weekly, monthly) and
investigated. Specifically, the contrarian strategy is
U.S. market, depending on the time horizon
find both momentum and contrarian profits in the
comprehensive investigation, Conrad and Kaul [18]
profitability in most developed markets. In a
confirmed the existence and stability of momentum
exploit the price fluctuation.
Thus, these three problems in the anticor algorithm
real world. Finally, in the short term, the strategy
fully exploit the potential of mean reversion in the
problems. Firstly, the performance of the anticor algorithm
size of window depends on changing of the window size.
Second, it cannot the anticor algorithm causes three potential
empirically effective on most datasets, but unfortunately, this method brings
about large scale computation. So it is difficult to apply for the anticor algorithm in practice, especially for multiple stocks in the long term.

4. THE ANT-COLONY ALGORITHM

4.1 Motivation And Overview

The anticor algorithm exploiting the mean reversion property can achieve good results on most datasets at the time and may better fit the markets. However, they rely on the assumption of price path. Though it is empirically effective on most datasets, the anticor algorithm causes three potential problems. Firstly, the performance of the anticor algorithm changes dramatically along with the changing of the window size. Second, it cannot fully exploit the potential of mean reversion in the real world. Finally, in the short term, the strategy fails when it merely adopt the philosophy of the mean reversion, but does not the price momentum. Thus, these three problems in the anticor algorithm call for a more powerful approach to effectively exploit the price fluctuation.

Momentum has received substantial attention in the finance literature since firstly discovered by Jegadeesh and Titman [17]. Many later studies have confirmed the existence and stability of momentum profitability in most developed markets. In a comprehensive investigation, Conrad and Kaul [18] find both momentum and contrarian profits in the U.S. market, depending on the time horizon investigated. Specifically, the contrarian strategy is profitable for short-term (weekly, monthly) and long-term (2-5 years, or longer) intervals, while the momentum strategy is profitable for medium-term (3-12-month) holding periods. Kang et al. [19] find statistically significant short-term reversal and intermediate-term momentum profits in Chinese stock markets over the period 1993-2000. Balvers and Wu [20] demonstrate that mean reversion and momentum can simultaneously occur to the same set of assets in 18 developed countries. They also report momentum persists longer than previously found in isolation and mean reversion takes place quicker. Lu et al. [21] find that there exist an ultra-short-term momentum and a short-term reversal besides the intermediate-term momentum and the long-term reversal in Chinese stock markets. Pan et al. [22] find economically significant momentum profits in weekly returns in Chinese A-share market and generate robust momentum profit in weekly returns using return interval ranking strategy. They argue that the weekly momentum lasts for about 1 year and more than half of the profit is realized in the first 3 weeks. Yan et al. [23] report that the portfolio with the formation period of 1 week and the holding period of 1-3 weeks demonstrates significant momentum effect, the portfolio with the formation period beyond 1 week and the holding period over 3 weeks begins to display return reversal effect, and the portfolio with formation period and holding period extend to 12-26 weeks shows no remarkable momentum effect. Pan et al. [24] find the ultra-short-term momentum, the short-term mean reversion exist in Chinese stock markets. In summary, ultra-short-term momentum, the short-term price momentum and reversals coexist in Chinese stock markets. In this paper, we adopt the properties of not only the price reversal but also the price momentum in the ultra-short term and the short term.

To investigate the price fluctuation in stock markets, we sort each pair of stocks into two categories based on the past ultra-short-term, short-term price changing (from day $t - 2w + 1$ to day $t - 1$): price reversal path and price momentum path. Price reversal path is presented in paper [15], and price momentum path is introduced in this paper to exhibit price continuation during these two windows. We will apply the contrarian strategy to both price paths at day $t + 1$.

4.2 Motivation And Overview

This section presents a new type of algorithm, named the ant-colony algorithm, to capture the ultra-short-term and short-term dynamics of the momentum and reversal of Chinese stock prices. Firstly, we suppose that the price reversal of a pair
of stock will take place for all price paths at the next period. So, we will take the contrarian strategy at day \( t+1 \). Secondly, we introduce the parameters \( \alpha, \beta \in [0,1], \alpha, \beta \in (-1,0] \) to describe the strength of this correlation as well as the strength of the “self-anti-correlations”. If \( \mu_i (i) \geq \mu_j (j) \) and \( M_{cor}(i, j) > \alpha \) (price reversal path), then we improve the formulation (3) as follows,

\[
\text{claim}_{t \rightarrow s} = M_{cor}(i, j) + M_{cor,-}(i, i) + M_{cor,-}(j, j)
\]

where

\[
M_{cor,-}(\cdot, \cdot) = \begin{cases} -M_{cor}(\cdot, \cdot) & M_{cor}(\cdot, \cdot) < \alpha \\ 0 & \text{else} \end{cases}
\]

For the price momentum path, we describe the algorithm as follows: if \( M_{cor}(i, j) < \beta \) and \( \mu_i (i) \geq \mu_j (j) \), then the formulation (3) or (6) becomes,

\[
\text{claim}_{t \rightarrow s} = M_{cor}(i, j) + M_{cor,+}(i, i) + M_{cor,+}(j, j)
\]

where

\[
M_{cor,+}(\cdot, \cdot) = \begin{cases} M_{cor}(\cdot, \cdot) & M_{cor}(\cdot, \cdot) > \beta \\ 0 & \text{else} \end{cases}
\]

Combining with equations (4) (5) (6) (7), we depict the ant-colony algorithm in table I.

### 5. SELECTION AND SIMULATION RESULTS

In this section, we present numerical experiments which illustrate the performance of the ant-algorithm. For each experiment, the data are obtained from Shanghai Stock Exchange (SHSE) and Shenzhen Stock Exchange (SZSE) in Chinese Stock Markets and Accounting Research Database(2012), published by Wind Information Co., Ltd (Wind Info). We collect daily individual stock prices (adjusted in the usual manner for dividends) for all “A” share stocks traded at the SHSE and SZSE, which started on January 30, 1991 and ended on January 11, 2012 with 5,132 daily return observations. Those data in the first 3 months are excluded because it was not stable in this time when “A” shares began trading after the initial setup of the two exchanges. In this paper, we randomly pick four datasets to constitute four portfolios in the above data. Table II details the four experimental datasets.

#### TABLE I: The Ant-Colony Algorithm

<table>
<thead>
<tr>
<th>Ant- colony ((w, t, x, \hat{b}_t))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> ( w, \alpha, \beta, \alpha, \beta )</td>
</tr>
<tr>
<td><strong>Initialize</strong> ( S_0 = 1, b_i = (\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}) )</td>
</tr>
<tr>
<td>Update the portfolio selection: For ( t = 1, 2, \cdots, n ) do</td>
</tr>
<tr>
<td>1. Receive stock price relatives: ( x_t )</td>
</tr>
<tr>
<td>2. If ( t &lt; 2w ), then ( \hat{b}_{t+1} = \hat{b}_t \cdot t = 1, 2, \cdots, n )</td>
</tr>
<tr>
<td>3. If ( t \geq 2w ), then</td>
</tr>
<tr>
<td>( \text{If} M_{cor}(i, j) &gt; \alpha ) and ( \mu_i (i) \geq \mu_j (j) ), then</td>
</tr>
<tr>
<td>( \text{Else if} M_{cor}(i, j) &lt; \beta ) and ( \mu_i (i) \geq \mu_j (j) ) then</td>
</tr>
<tr>
<td>( \text{Else if} M_{cor}(i, j) &lt; \beta ) and ( \mu_i (i) &lt; \mu_j (j) ) then</td>
</tr>
<tr>
<td>( \text{Else if} M_{cor}(i, j) &lt; \beta ) and ( \mu_i (i) &lt; \mu_j (j) ) then</td>
</tr>
<tr>
<td>4.</td>
</tr>
<tr>
<td>( b_{t+1}(i) = \hat{b}<em>t(i) + \sum</em>{j=1}^{n} (\text{transfer}<em>{j \rightarrow s} - \text{transfer}</em>{s \rightarrow j}) )</td>
</tr>
<tr>
<td>5. end for</td>
</tr>
</tbody>
</table>

#### TABLE II: Summary Of 4 Real Datasets From Chinese Markets

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Market</th>
<th>Time frame</th>
<th>#Days</th>
<th>#Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHSE (9)</td>
<td>SHSE</td>
<td>July 26(^{th}) 2000- Jan 11(^{th}) 2012</td>
<td>2776</td>
<td>9</td>
</tr>
<tr>
<td>SHSE (18)</td>
<td>SHSE</td>
<td>Nov 2(^{nd}) 1999- Jan 11(^{th}) 2012</td>
<td>2949</td>
<td>18</td>
</tr>
<tr>
<td>SZSE (10)</td>
<td>SZSE</td>
<td>Dec 8(^{th}) 1997- Jan 11(^{th}) 2012</td>
<td>3410</td>
<td>10</td>
</tr>
<tr>
<td>SZSE (26)</td>
<td>SZSE</td>
<td>Jan 19(^{th}) 2001- Jan 11(^{th}) 2012</td>
<td>2655</td>
<td>26</td>
</tr>
</tbody>
</table>

2 There are two types of shares traded at the Chinese stock markets: “A” shares and “B” shares. “A” shares are quoted in the domestic currency unit (RMB) and are traded only by domestic Chinese citizens, while “B” shares are quoted in US dollar terms and can be traded only by foreigners.
In our experiments, we implement the proposed ant-colony algorithm and the anticor algorithm with the window size $w$ from 2 to 60. The experiment evaluates the cumulative wealth at the end of the trading period during different windows, respectively.

If $0 < \alpha, \beta < 0.2, -0.2 < \alpha, \beta < 0$, then the strength of the correlation or the strength of the self-anti-correlation is too weak and further lead to obvious nonsense for the ant-colony algorithm. Hence, we set $\alpha = \beta = 0.2, \alpha, \beta = -0.2$ for the ant-colony algorithm. We plot the wealth curve of cumulative wealth and compare the final wealth of cumulative wealth in the figure 1, where Antior represents the anticor algorithm, Ant-colony represents the ant-colony algorithm where colony algorithm where $\alpha = \beta = 0.2, \alpha, \beta = -0.2$ and Ant-Colony* represents the ant-colony algorithm where $\alpha = \beta = 0, \alpha, \beta = 0$. The $x$-axis represents the size of the time windows within the coordinate and the $y$-axis represents the wealth achieved by these algorithms for an initial investment of RMB 1 within the logarithmic coordinate.

From the experimental results shown in Figure 1, we can draw several observations below. First of all, we observe that Ant-Colony* perform better than Antior on some periods and occasionally worse, which shows that Ant-Colony* is sensitive to the window size for portfolio selection, but on the other hand, validates the importance of exploiting the price momentum property in Chinese financial markets by an effective online learning strategy. Second, Ant-Colony performs better than Ant-Colony* and further better than Anticor (with a few point exceptions). Finally, the cumulative wealth achieved by Ant-Colony has little to do with the window size in the short term, which demonstrates that Ant-Colony doesn’t depend significantly on the window size. Moreover, this also shows that to achieve better investment return, it is more powerful and promising to not only exploit the price momentum and mean reversion property but also capture the strength of the correlation or self-anti-correlations for portfolio selection.

Theoretically, our proposed algorithm does not need to smooth the parameter by compounding, so it enjoys linear computational time complexity. Empirically, it takes the least times on all datasets. Such time efficiency supports its large-scale real applications.
6. CLONCLUSION

In this paper, we propose a novel on-line portfolio selection strategy named “ant-colony algorithm”, which successfully applied machine learning techniques for on-line portfolio selection by exploiting the mean reversion property of the financial markets. Unlike the anticor algorithm using only price reversal, the ant-colony algorithm exploits both price reversal and price momentum in Chinese stock markets in the short term. Empirically the ant-colony algorithm surpasses the anticor technique from real markets. We also find the ant-colony algorithm is hardly affected by the size of window. Moreover, this proposed mean reversion strategies can better track the changing stock market.

ACKNOWLEDGEMENTS

This work was supported by the Major Program of the National Social Science Foundation of China (Grant No.11&ZD156) and the General Project of Program of the National Natural Science Foundation of China (Grant No. 71171086).

REFERENCES:

