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AN OPTIMUM ALGORITHM FOR CUT-OFF GRADE CALCULATION USING MULTISTAGE STOCHASTIC PROGRAMMING

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ABSTRACT

Cut-off grade is one of the very important technical and economic parameters, which affects the economic and social efficiency in a mine enterprise. It is also the basis of the deposits technical economy evaluation, feasibility study, mining planning and design and the foundation of the decision-making in mining investment. Thereby, cut-off grade optimization is one of the core contents and therefore the key parameter in mine mining, management and decision. The optimized determination of cut-off grade in mine mining involves complex analysis and scientific calculation, which is tightly related to economic management, ore dressing, mining, geology, applied mathematics and computer knowledge subjects. In this paper, we modeled the cut-off grade optimization problem as a multistage stochastic programming issue. In the proposed stochastic programming model, each ore grade in a given ore body is a random variable following a distribution. And a mathematical model for selecting an optimum cut-off grade function was constructed, which aimed at the maximization of total present value of an open pit. Experimental results show that the proposed algorithm for cut-off grade optimization is efficiency.

Keywords: *Cut-off Grade, Optimization, Net Present Value, Stochastic Programming*

1. INTRODUCTION

In the open-pit mining project of ore deposits, grade is the percentage of useful ore elements or compounds in the ore body. Cut-off grade or marginal grade is the ore grade when marginal cost is equal to the marginal benefit. Cut-off grade is one of the very important technical and economic parameters which affect the economic efficiency and the social efficiency in a mine enterprise. It is also the basis of the deposits technical economy evaluation, feasibility study, mining planning and design and also the foundation of the decision-making in mining investment. Thereby, cut-off grade optimization is one of the core contents and therefore the key parameter in mine mining, management and decision. The optimized determination of cut-off grade in mine mining involves complex analysis and scientific calculation, which is tightly related to economic management, ore dressing, mining, geology, applied mathematics and computer knowledge subjects. Taylor[1, 2] presents one of the best definitions of cut-off grade. He defined cut-off grade as "any grade that, for any specific reason, is used to separate two courses of action, e.g. to mine or to leave, to mill or to dump".

In open-pit mines, ores in general are defined operationally by a cut-off grade. Material with a mineral content above the cut-off is scheduled for further processing. Other materials are left, or dumped as waste. Depending upon the mining method, waste is either left in situ or sent to the waste dumps, whereas ore is sent to the treatment plant for further processing and eventual sale.

An essential preliminary to an analysis of cut-off grade strategy is an examination of net present value (NPV) maximization. There are many approaches for the determination of cut-off grades. But most of the research that has been done in the last three decades shows that determination of cut-off grades with the objective of maximizing Net Present Value (NPV) is the most acceptable method, which is based on a finite resource. Most of these works have been done to devote to the cut-off grade determination in 1980's and 1990's. In the-state-of-art literatures, many researchers developed methods and algorithms with different focuses and merits. Bascetin [3] proposed a new method to make the determination of a cut-off grade strategy based on Lane's algorithm adding an optimization factor based on the generalized reduced gradient algorithm to maximize the project's NPV. Osanloo

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[4] used an equivalent grade factor to find optimum cut-off grade of multiple metal deposits. First, the objective function is defined for multiple metal deposits and then objective function is converted to one variable function by using equivalent factors. The optimum equivalent cut-off grade of main metal can be found by the optimization techniques such as the Lane algorithm or elimination methods. At final step, the optimum cut-off grades will be determined by interpolation of grade-tonnage distribution of deposit. Osanloo [5] proposed a heuristic modern approach for cut-off grade optimization, which not only maximizes the profitability of projects, but also minimizes the adverse environmental impacts of the project simultaneously.

Until now, the optimized cut-off grade determination is still an unsolved problem. There are some aspects must be considered in solving this problem: 1) the cut-off grade is changeable with respect to time; 2) the cut-off grade distribution is different in the different given ore-body; and 3) the cut-off grade is stochastic and uncertain nature. But in the most previous work [3-9], the cut-off grade is determined by maximizing the NPV in a determinate context. In the real world, the maximum of NPV is not often practicable and to maximize the expectation of NPV is a good choose.

In this paper, we proposed a multistage stochastic programming based method for optimizing cut-off grade in open-pit mines that can get the optimum cut-off grade in a stochastic context.

2. TECHNICAL BACKGROUNDS

2.1 Stochastic Programming

Stochastic programming is a framework for modeling optimization problems that involve uncertainty [10-14]. Whereas the deterministic optimization problems are formulated with known parameters, real world problems almost invariably include some unknown parameters. When the parameters are known only within certain bounds, one approach to tackling such problems is called robust optimization. Here the goal is to find a solution which is feasible for all such data and optimal in some sense. Stochastic programming models are similar in style but take advantage of the fact that probability distributions governing the data are known or can be estimated. The goal here is to find some policy that is feasible for all (or almost all) the possible data instances and maximizes the expectation of some function of the decisions and the random variables. More generally, such models are formulated, solved analytically or numerically, and analyzed in order to provide useful information to a decision-maker.

As an example, consider two-stage linear stochastic programs. Here the decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision. The optimal policy from such a model is a single first-stage policy and a collection of recourse decisions (a decision rule) defining which second-stage action should be taken in response to each random outcome. Stochastic programming has applications in a broad range of areas ranging from finance to transportation to energy optimization.

2.2 Multistage stochastic programming

Stochastic programming is a mathematical program where some of data incorporated into the objective or constraints is uncertain. A mathematical program can be shown as follow:

$$\begin{aligned} \min\quad & f(x) \\ s.t.\quad & g_i(x) \le 0,\ i=1,\ldots,m. \end{aligned} \tag{1}$$

If we introduce stochastic variable $\hat{\xi}$ into Eq. (1), where $\hat{\xi} \in (\Omega, F, P)$, then the problem turns into a stochastic problem,

$$\begin{aligned} z(\xi) = & \min f(x, \hat{\xi}) \\ & s.t.\ g_i(x, \hat{\xi}) \le 0,\ i=1,\ldots,m. \end{aligned} \tag{2}$$

A two-stage stochastic program is defined as

$$\begin{aligned} \min\quad & c_1 x + Q(x) \\ s.t.\quad & W_1 x = h_1 \\ & x \ge 0 \end{aligned}, \tag{3}$$

where Q is a penalty function,

$$Q(x) = \sum_{\xi\in\Xi} prob(\xi) Q(x,\xi). \tag{4}$$

For each realization of $\xi \in \Xi$, we get,

$$\begin{aligned} Q(x,\xi) = \min\quad & c_2(\xi) y \\ s.t.\quad & W_2 y = h_2(\xi) - T(\xi) x \\ & y \ge 0, \end{aligned} \tag{5}$$

$$\begin{aligned} Q(x,\xi) = \min\quad & c_1 x + \sum_{\xi\in\Xi} prob(\xi) c_2(\xi) y(\xi) \\ s.t.\quad & W_1 x = h_1 \\ & T(\xi) x + W_2 y(\xi) = h_2(\xi), \quad \xi = 1,\ldots,\Xi \\ & y(\xi) \ge 0, \quad \xi = 1,\ldots,\Xi \\ & x \ge 0, \end{aligned} \tag{6}$$

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where $d(\xi) \in R^m$, $T(\xi) \in R^m$ and Ξ is the distribution of ξ .

The two-stage stochastic programming models can be naturally extended to a multistage setting, and can be formulated as,

$$\begin{aligned} \min \quad & c_1 x_1 + Q_2(x_1) \\ s.t. \quad & W_1 x_1 = h_1 \\ & x_1 \geq 0, \end{aligned} \tag{7}$$

$$Q_t\left(x_{t-1,a(k)}\right) = \sum_{\xi_{t,k} \in \Xi_t} prob\left(\xi_{t,k}\right) \mathrm{Q}_{t,k}\left(x_{t-1,a(k)}, \xi_{t,k}\right), \tag{8}$$

$$\begin{aligned} \mathrm{Q}_{t,k}\left(x_{t-1,a(k)}, \xi_{t,k}\right) = \min \quad & c_t\left(\xi_{t,k}\right) x_{t,k} + Q_{t+1}\left(x_{t,k}\right) \\ s.t. \quad & W_t x_{t,k} = h_t\left(\xi_{t,k}\right) - T_{t-1}\left(\xi_{t,k}\right) x_{t-1,a(k)}, \\ & x_{t,k} \geq 0. \end{aligned} \tag{9}$$

where $Q_N + 1(x_N) = 0$, for all x_N .

Through the equivalent substitution and derivation, stochastic programming problem is equivalent to a deterministic mathematical programming. In practice, we cannot reach its equivalent deterministic programming forms in most cases. Stochastic programming has its own theory and solution methods. Stochastic programming solving methods, which often used in the literatures, include such as random quasi gradient method, the sample mean value method, interior point method, solving compensation stochastic programming problems in type Newton algorithm *etc*.

3. ALGORITHM AND ITS APPLICATION

3.1 Cut-Off Grade Optimization Under NPV Maximum

For an operating mine, there are typically three stages of production:

- The mining stage, where units of various grade are extracted up to some capacity;
- The treatment stage, where ore is milled and concentrated, again up to some capacity constraint;
- The refining stage, where the concentrate is smelted and/or refined to a final product which is shipped and sold.

The latest stage is also subject to capacity constraints. For simplicity, assume a two-metal deposit. In this deposit, ore is sent to a concentrator and the concentrator will produce two concentrates. Each concentrate for smelting and finally refining is sent to a refinery plant. Each stage has its own associated costs and a limiting capacity. The operation as a whole will incur continuing fixed cost. The notations of the mining operation in a two metal deposit are shown in table 1. By considering costs and revenues in this operation, the profit is determined by using the following equation:

$$P = (s_1 - r_1)Q_{r1} + (s_2 - r_2)Q_{r2} - mQ_m - cQ_c - fT, \tag{10}$$

where m is the mining cost ($/tone of material mined), c is the concentrating cost ($/tone of material concentrated), r_1 is the refinery cost ($/unit of product 1), r_2 is the refinery cost ($/unit of product 2), f is the fixed cost, s_1 is the selling price($/unit of product 1), s_2 is the selling price ($/unit of product 2), T is the length of the production period to be considered, Q_m is the quantity of material to be mined, Q_c is the quantity of material to be concentrated, Q_{r1} is the amount of product 1 actually produced over this production period, Q_{r2} is the amount of product 2 actually produced over this production period.

Table 1: Notations Of The Mining Operation In A Deposit

Stage	Capacity Symbol	Cost Symbol
Mine	M	m
Concentrator	C	c
Refinery of metal 1	R_1	r_1
Refinery of metal 2	R_2	r_2

If d is the discount rate, the difference v between the present values of the remaining reserves at time $t = 0$ and $t = T$ is,

$$v = P - VdT, \tag{11}$$

where V is the present values at time $t = 0$. Substituting Eq. (10) into Eq. (11) yields

$$\begin{aligned} v = {} & (s_1 - r_1)Q_{r1} + (s_2 - r_2)Q_{r2} \\ & - mQ_m - cQ_c - (f + Vd)T. \end{aligned} \tag{12}$$

The quantities of refined metals Q_{r1} and Q_{r2} are related to that sent from the mine to concentrator (Q_c), therefore,

$$Q_{r1} = \bar{g}_1 y_1 Q_c, \tag{13}$$

$$Q_{r2} = \bar{g}_2 y_2 Q_c, \tag{14}$$

where $\bar{g}_1$ is the average grade of metal 1 sent for concentration, $\bar{g}_2$ is the average grade of metal 2 sent for concentration, y_1 is the recovery rate.

According to Eq. (12), (13) and (14), we get

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$$\begin{aligned} v &= (s_1 - r_1)\bar{g}_1 y_1 Q_c + (s_2 - r_2)\bar{g}_2 y_2 Q_c \\ &\quad - cQ_c - mQ_m - (f + Vd)T \\ &= [(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c]Q_c \\ &\quad - mQ_m - (f + Vd)T. \end{aligned} \quad (15)$$

We would now like to schedule the mining in such a way that the decline in remaining present value takes place as rapidly as possible. This is because later profits get discounted more than those captured earlier, so the value of v should be maximized. Eq. (15) is the fundamental formula and all the cut-off grade optimum can be developed from it. The time taken T is related to the constraint capacity. Four cases arise depending upon which of the four capacities are actually limiting factors. Limiting economic cut-off grades may be limited individually by mining, milling facilities (crusinging and concentrator plants, etc.) or marketing throughputs.

If mining throughput or the mining rate is the governing limitations, then the time T is given by:

$$T = Q_m / M. \quad (16)$$

If milling throughout or the concentrator rate is the governing limitation, then the time T is controlled by the concentrator:

$$T = Q_c / C. \quad (17)$$

If the refinery output of metal 1 is the limiting factor the time T is controlled by the refinery of metal 1:

$$T = Q_{r1} / R_1 = \bar{g}_1 y_1 Q_c / R_1 \quad (18)$$

If the refinery output of metal 2 is the limiting factor and the time T is controlled by the refinery of metal 2:

$$T = Q_{r2} / R_2 = \bar{g}_2 y_2 Q_c / R_2 \quad (19)$$

Substituting Eq. (16), (17), (18) and (19) into Eq. (15), we can get:

$$v_m = [(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c]Q_c - (m + \tfrac{Vd+f}{M})Q_m, \quad (20)$$

$$v_c = \left[(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - (c + \tfrac{Vd+f}{C})\right]Q_c - mQ_m, \quad (21)$$

$$v_{r1} = \left[(s_1 - r_1 - \tfrac{f+Vd}{R_1})\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c)\right]Q_c - mQ_m, \quad (22)$$

$$v_{r2} = \left[(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2 - \tfrac{f+Vd}{R_2})\bar{g}_2 y_2 - c)\right]Q_c - mQ_m. \quad (23)$$

Then for any pair of cut-off grades, it is possible to calculate the corresponding V_m, V_c, V_{r1}, V_{r2}. The controlling capacity is always the one corresponding to the least of these four equations. Therefore:

$$\max\{v_e\} = \max[\min(v_m, v_c, v_{r1}, v_{r2})] \quad (24)$$

From the above formulae, we can see if the present value V is obtained, the optimization cut-off grades g_1^* of metal 1 and g_2^* of metal 2 are determined. But, before we get g_1^* and g_2^* we cannot compute the net present value V. To solve this problem, we suppose an initial value of V_0, and then use V_0 to optimize the cut-off grades g_1^* and g_2^*. After the optimal cut-off grades are gotten, we can use the new cut-off grades to get the new net present value. We will do this iteration until we get the optimal cut-off grades.

4. EXPERIMENTAL RESULTS

Based on the above discussed cut-off grade optimization model and solving methods, we optimized the cut-off grade in an open-pit mine in China, which is a very famous large enterprise, located at the Luan chuan county in the city of Luoyang, Henan Province. The open pit mine now faces the following problems. First, the cut-off grade scheme was established according to the mining technique, milling technology and price of the concentrates in 2000s, but now whether the scheme is reasonable need to be studied. Second, the geological condition, mining and milling process technology, it is urgent to optimize cut-off grade in order to guide mining and milling production.

In the open pit mine main technical parameters are shown in Table 3. All the parameters are obtained from the open pit enterprise in 2011.

Table 2: Model Parameters For An Open Pit Mine In 2011

#	Operation component	Unit	Value
1	Mining variable cost	yuan/ton	55.56
2	Mining recovery rate		96.7%
3	mining dilution rate		3.3%
4	Proportion of ore	m^3	3.2
5	proportion of waste ore	m^3	2.9
6	Stripping cost	yuan/ton	21.21
7	Price of concentrate metal 1	yuan/ton	244600
8	Price of concentrate metal 2	yuan/ton	212300
9	Treatment Cost	yuan/ton	51.02
10	Recovery rate of metal 1		84%
11	Recovery rate of metal 2		67%
12	Discount rate		8%
13	Average stripping coefficient	t/t	0.56
14	Mining capacity	ton/day	30000

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According to the above multistage stochastic programming model and calculation method we get the simulation program to perform the optimization calculation. The hardware environment: desktop PC, Core i7 quad-core processor (i7 2600K), 8G memory; and the software environment for: Windows 764, python-2.7.2, Coopr_3.1.5409, pySP, eclipse+pyDev.

Based on the above decision stage, we get the optimum cut-off grade, mining time, operation cost net income and net present value, which are shown in Table 3.

Table 3: Economic Optimization Results By Using Muti-Stage Stochastic Programming Methods

# Stage	g_1^* (Percentage)	g_2^* (Percentage)	Mining time (Year)	Total mines (t)	Total Time (year)	Costs （yuan）	Net income （yuan）	NPV (yuan)
1	0.0311	0.0324	0.03	254029	0.03	1464358747	800582541	798736264
2	0.0372	0.0354	0.98	9669839	0.23	11814302692	7401146178	7269812630
3	0.0407	0.0349	0.39	3863399	0.62	22275845564	14583901898	13901274105
4	0.0485	0.0259	0.62	6132756	1.24	35359842142	23146713878	21036098334
5	0.0304	0.0412	0.84	8339775	2.08	48082309788	29325661558	24978562732
6	0.0339	0.0316	1.01	9972823	3.09	57490806053	32521419678	25634184553
7	0.0415	0.0330	1.33	13199099	4.43	76102660202	49235286902	35023901199
8	0.0258	0.0417	1.41	13985839	5.84	80627929872	46103375100	29417289838
9	0.0405	0.0351	1.78	17653077	7.62	101785228317	66581029684	37035758242
10	0.0235	0.0464	2.01	19862122	9.63	114509807216	67353404823	32105027449
11	0.0413	0.0235	2.24	22151096	11.86	127693752957	72670185928	29159805900
12	0.0367	0.0312	2.28	22617260	14.15	130388810294	76704863748	25816198637
13	0.0466	0.0307	2.49	24646085	16.64	142110708397	95990981553	26674191644
14	0.0317	0.0285	1.62	16016548	18.26	92321936149	48064858874	11792732517
15	0.0327	0.0274	1.42	14095509	19.68	81248606241	42333838914	9308636275
16	0.0349	0.0304	1.42	14099679	21.10	81280761846	45940476879	9052977869
17	0.0412	0.0345	1.42	14050929	22.52	81015953894	53154879486	9390770700
18	0.0401	0.0423	1.66	16444258	24.19	94827872467	67192983493	10446335175
19	0.0373	0.0325	1.40	13856818	25.58	79887587927	48231065533	6732624179
20	0.0471	0.0298	1.54	15202631	27.12	87658548374	58983563664	7315809040
21	0.0398	0.0310	1.48	14651032	28.60	84468113266	51967785438	5751772118
22	0.0321	0.0411	1.65	16350076	30.25	94268022793	58921374211	5743018723

In addition, in the mining enterprises, cut-off grade is often a fixed economic and technical parameter. Therefore, the average of the optimum cut-off grade of each stage is taken as the final optimum cut-off grade in the mining enterprises. The cut-off grade of metal 1 is 0.0370%, and the cut-off grade of metal2 is 0.0337%. Table 3 also shows that, based on the proposed methods, the mine can produce about 30.25 years, cost 172668000000 yuan, and can gain net income 105721000000 yuan, which folded into the current net present value 384386000000 yuan.

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5. CONCLUSIONS

One of the most difficult problems in a mining operation is how to determine the optimum mineral cut-off grade over the lifespan of the mine, which will maximize the operation net present value (NPV), subject to different constraints. The determination of cut-off grade must take into account the dynamic characteristics of grade in time and space and the randomness of grade distribution. We formulated a new cut-off optimization model using stochastic programming to maximize the net present value function of a mineral enterprise, by taking into account the spatial and temporal characteristics in mining of the ore body, the randomness and uncertainty of grade distribution in different section, and production capacity of a mineral enterprise in an open-pit. And we also set up a new cut-off grade optimization model for a single mineral and a new cut-off grade optimization model for multiple minerals. This paper describes the process to determine the cut-off grade strategy used in a mining operation based on multistage stochastic programming recursively calculated for every production year, which dynamically adjusts the remaining reserves and thus the total life of the mine to maximize the project NPV. The benefits of the methodology are demonstrated using a hypothetical case study. The authors have observed an improvement of the total NPV using the general reduced gradient approach to iteratively calculate the optimization factor of every production year.

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