

INFORMATION FLOW ANALYSIS BETWEEN TRAFFIC FLOW TIME SERIES BASED ON THE TRANSFER ENTROPY

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ABSTRACT

In this paper, we employ transfer entropy as our analysis tool of traffic flow. We select the passenger flow volume of Beijing Subway Line 1 as our data source. The research aims at finding out the strength and the direction of information transfer of the passenger flow volume.

Keywords: *Traffic flow, Passenger flow volume, Transfer entropy, Information flow, Beijing subway.*

1. INTRODUCTION

Economists are employing more and more tools for physics, to solve the economic problem [1, 2, 3]. In the system Theory, entropy [4, 5] is a significant parameter to quantify the degree of irregularity or uncertainty of the system. The system tends to be more chaotic when the entropy increases. Information is a decrease in uncertainty. This concepts origins from Physics and is now applied to time series analysis. If a time series is regular, the previous value(s) may contain the information of the next adjacent value(s), and decrease the uncertainty of the next value. This is widely used in statistics.

When investigating interacted time series, transfer entropy [6] would be an effective method. Using transfer entropy to measures the interaction between time series has become popular in economy researches [7, 8], commonly between financial indexes, e.g. stock price. The transfer entropy not only measures the information flow of the system's internal structure, but also captures the direction of the information flow [6, 9, 10]. In this paper, the research would discover the internal relationship of the Traffic flow time series, and further to evaluate the traffic network.

In recent years, Beijing is building more and more subway lines. Currently, 15 subway lines is operating (up to Apr. 2012), 11 of them are newly accomplished in the last 5 years. But few researches of the passenger flow volume have been done. Studying the passenger flow volume (PFV) to obtain the PFV between individual stations; find the

“key stations” of PFV; quest the possible irrelevant structures of the subway networks; help us to make further decision of subway line construction.

Through transfer entropy, this paper focuses on information flow between 7 major stations of the Beijing Subway Line 1 (BSL1), directionally and quantitatively, to find out the relationship of the PFV; in the section 2, a brief introduction of the transfer entropy would be given; in the section 3, an overview of BSL1 is provided; in the section 4, the result by calculate the transfer entropy is obtained; in section 5, conclusion is summarized, and future works are mentioned.

2. METHOD OF ANALYZING

First of all, we will explain the definition of Shannon entropy.

Shannon entropy is a classic definition of entropy. For a stochastic discrete variable x , the Shannon entropy $H(x)$ is defined as:

$$H(x) = - \sum_{i=1}^n p_i \log p_i \quad P(x = x_i) = p_i \quad (1)$$

The $H(x)$ describes the uncertainty of the stochastic variable x . When x equals to a particular value as surely, the $H(x)$ equals zero. Reversely, $H(x)$ reaches the maximum when $p_i = 1/n$.

Consider two processes I, J , the definition of transfer entropy from J to I is:

$$T_{J \rightarrow I} = \sum p(i_{t+1}, i_t^{(I)}, j_t^{(J)}) \log \frac{p(i_{t+1}, i_t^{(I)}, j_t^{(J)})}{p(i_{t+1}, i_t^{(I)})} \quad (2)$$

Where i_t and y_t represent the states at time t of I and J , respectively. In terms of relative entropy, it can be rephrased as the distance from the assumption that J has no influence on I , when:

$$P(i_{t+1} | i_t^{(k)}, j_t^{(l)}) = P(i_{t+1} | i_t^{(k)}) \quad (3)$$

One may rewrite Equation (2) as:

$$\begin{aligned} T_{I \rightarrow J} &= H(i_{t+1}^{(k+1)}, j_t^{(l)}) - H(i_t^{(k)}, j_t^{(l)}) \\ &\quad - H(i_{t+1}^{(k+1)}) + H(i_t^{(k)}) \\ &= hI(k; t) - hI(k, l; t). \end{aligned} \quad (4)$$

3. OVERVIEW OF THE SUBWAY LINE 1

Beijing Subway line 1(BSL1) is a west-east direction subway, 31 kilometers long, west to the Pingguoyuan Station, which is outside Beijing Fifth Ring road, east to the Sihui East Station. Now a sketch map of BSL1 is given below (up to Apr. 2012).

Pingguoyuan - (6 more station between) - Gongzhufen (At the west third ring road) - (3 more station) -

Fuxingmen (Transfer station to the Line 2, which travels around the central part of Beijing) -

Xidan (Transfer station to the Line 4) - (3 more station) -

Dongdan (Transfer station to the Line 5) - Jianguomen (Transfer station to the Line 2 - the loop line) - (1 more station) -

Guomao (At east third ring road; Central business district; Transfer station to the line 10) - (1 more station) -

Sihui - Sihui East (Both of these two stations are Transfer station to Batong Line, Which extended to the east, leads to a satellite city - Tongzhou)



Figure 1 Overview of BSL1

4. DATA ANALYZING

The PFV 2011 of 7 major stations of BSL1: Pingguoyuan, Gongzhufen, Fuxingmen, Dongdan, Jianguomen, Guomao and Sihui East were picked as our data source. Every 15 minutes, we record the total number of passengers entering or exiting the station.

In order to eliminate the interference of the first train or the last train, the data before 6 a.m. and after 11 p.m. are eliminated. After 6 a.m. the first train from all direction (including the concerned line at transfer station) has arrived at the station and the last train to all direction has not departed before 11 p.m.

Data are obtained during Jul. 2011, 68 inflowing and 68 outflowing data every single day every station, 29512 totally.

For each station, the data of inflowing are divided into 3 category labeled high, medium and low. Here, the thresholds are the 1st quartile and the 3rd quartile.

To pursue the relationship between the increase and decrease of the PFV of the 7 major station, we also calculate the change of the PFV ($d_i = i_{t+1} - i_t$) and using the same method divide them into 3 category. The same work was done to process the data of outflowing PFV, and the Total PFV.

In the next part we would examine the result.

The transfer entropy between 7 stations, Pingguoyuan (PGY), Gongzhufen (GZF), Fuxingmen (FXM), Dongdan (DD), Jianguomen (JGM), Guomao (GM), Sihuidong (SHD), is given below. First we examine the transfer entropy of inflowing PFV, which is given by the Table 1:

Table 1 : Transfer entropy of Inflowing PFV

From \ to	PGY	GZF	FXM	DD	JGM	GM	SHD	Mean
PGY	0	0.0573	0.0452	0.0253	0.0214	0.0323	0.1076	0.0482
GZF	0.0364	0	0.0501	0.0675	0.0496	0.0377	0.0639	0.0509
FXM	0.0298	0.0194	0	0.0866	0.0903	0.0649	0.0461	0.0562
DD	0.0287	0.0088	0.0763	0	0.0595	0.0545	0.0512	0.0465
JGM	0.0301	0.0224	0.0606	0.0802	0	0.0522	0.034	0.0466
GM	0.0258	0.0317	0.088	0.0636	0.0759	0	0.0441	0.0548
SHD	0.0125	0.0489	0.0411	0.0157	0.0201	0.0409	0	0.0299
Mean	0.0272	0.0314	0.0602	0.0565	0.0528	0.0471	0.0578	

Noticing the value that is significantly big in this table, they are PGY to SHD, FXM to DD & JGM, JGM to DD, GM to FXM. Especially PGY, the transfer entropy from PGY to SHD is much higher than PGY to others. This can be explained by the geographical position difference: FXM, JGM and GM are considered to be the central district of Beijing, while the SHD and PGY are near to residential area.

Averagely, FXM station has the biggest transfer entropy to others. SHD has the lowest transfer entropy to others. PGY has the lowest transfer entropies.

The transfer entropies of outflowing PFV are given by Table 2. This exhibits some different properties. Averagely, the SHD has the highest transfer entropy from others, while the PGY is the second.

This also happens in the Transfer entropy of total PFV.

Table 2: Transfer Entropy Of Outflowing PFV

From \ to	PGY	GZF	FXM	DD	JGM	GM	SHD	Mean
PGY	0	0.0175	0.0325	0.0328	0.0156	0.0178	0.089	0.0342
GZF	0.0726	0	0.0193	0.0266	0.0407	0.0596	0.1054	0.0541
FXM	0.0738	0.0771	0	0.0374	0.0901	0.0839	0.0908	0.0755
DD	0.0709	0.0836	0.0859	0	0.1039	0.0754	0.0724	0.082
JGM	0.0798	0.0647	0.0366	0.0308	0	0.063	0.0841	0.0598
GM	0.0877	0.0518	0.035	0.0344	0.0535	0	0.1007	0.0605
SHD	0.0616	0.0395	0.0393	0.0392	0.0207	0.0278	0	0.0380
Mean	0.0744	0.0557	0.0414	0.0335	0.0541	0.0546	0.0904	

Table 3: Transfer Entropy Of Total PFV

From \ to	PGY	GZF	FXM	DD	JGM	GM	SHD	Mean
PGY	0	0.036	0.0369	0.0526	0.0349	0.0428	0.1031	0.051
GZF	0.0791	0	0.0645	0.0514	0.0697	0.075	0.0695	0.0682
FXM	0.0489	0.0158	0	0.0409	0.1006	0.1022	0.0398	0.058
DD	0.0486	0.0338	0.0716	0	0.0849	0.0922	0.0342	0.0609
JGM	0.0414	0.0098	0.0377	0.0344	0	0.0528	0.0346	0.0351
GM	0.0441	0.0094	0.0463	0.0308	0.0426	0	0.028	0.0335
SHD	0.0388	0.0436	0.0375	0.0518	0.0484	0.0564	0	0.0461
Mean	0.0501	0.0247	0.0491	0.0437	0.0635	0.0702	0.0515	

This time GZF have the biggest transfer entropy of Total PFV to others.

For each cell of the “from A station to B station” figure, we calculate the average of the 3 data of transfer entropy (PFV inflowing, PFV outflowing, total PFV inflowing, from station A to station B), and express them by brightness of the color in the corresponding cell. Then, we derive the Fig.2.

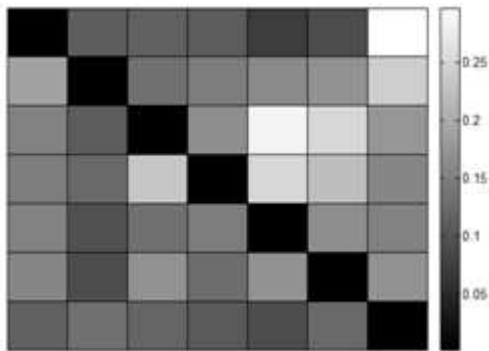


Figure 2 Transfer Entropy Of PFV

In Fig. 2, from top to bottom, and left to right, the corresponding stations are PGY GZF FXM DD JGM GM SHD. The brighter part represents higher transfer entropy,

Then we turn to check the transfer entropy between the change (increase /decrease) of PFV, Which is given by the Table 4, Table 5, Table 6.

Table 4: Transfer Entropy Of Inflowing PFV Change

From \ to	PGY	GZF	FXM	DD	JGM	GM	SHD	Mean
PGY	0	0.0296	0.0183	0.0202	0.0132	0.0199	0.054	0.0259
GZF	0.0239	0	0.039	0.0237	0.0646	0.0398	0.0381	0.0382
FXM	0.0222	0.0515	0	0.0474	0.0963	0.0747	0.0336	0.0543
DD	0.0109	0.0242	0.0395	0	0.0388	0.0304	0.0129	0.0261
JGM	0.011	0.0508	0.0561	0.0399	0	0.0514	0.0329	0.0403
GM	0.0292	0.0368	0.0452	0.0323	0.0676	0	0.0411	0.042
SHD	0.0347	0.0428	0.0214	0.0217	0.0315	0.0199	0	0.0287
Mean	0.022	0.0393	0.0366	0.0309	0.052	0.0393	0.0354	

Table 5: Transfer Entropy Of Outflowing PFV Change

From \ to	PGY	GZF	FXM	DD	JGM	GM	SHD	Mean
PGY	0	0.026	0.0205	0.0205	0.0081	0.0104	0.0362	0.0203
GZF	0.0349	0	0.0614	0.0386	0.0555	0.063	0.0411	0.0491
FXM	0.0242	0.0658	0	0.0696	0.0802	0.0809	0.0415	0.0604
DD	0.0186	0.0559	0.0609	0	0.0431	0.0563	0.0374	0.0454
JGM	0.0277	0.0683	0.0721	0.0469	0	0.0799	0.0294	0.054
GM	0.0277	0.0621	0.0571	0.0504	0.0889	0	0.0445	0.0551
SHD	0.0415	0.0339	0.022	0.0279	0.0243	0.0331	0	0.0304
Mean	0.0291	0.052	0.049	0.0423	0.05	0.0539	0.0383	

Table 6: Transfer Entropy Of Total PFV Change

From \ to	PGY	GZF	FXM	DD	JGM	GM	SHD	Mean
PGY	0	0.0533	0.0154	0.0293	0.0309	0.0181	0.0648	0.0353
GZF	0.0696	0	0.093	0.0713	0.1005	0.0703	0.0831	0.0813
FXM	0.0445	0.0984	0	0.0972	0.125	0.1069	0.091	0.0938
DD	0.0502	0.061	0.0909	0	0.0741	0.069	0.0713	0.0694
JGM	0.0526	0.0691	0.094	0.0741	0	0.0877	0.0582	0.0726
GM	0.0777	0.0823	0.0991	0.0861	0.1167	0	0.0941	0.0927
SHD	0.0812	0.065	0.0581	0.0574	0.0524	0.0492	0	0.0606
Mean	0.0626	0.0715	0.0751	0.0692	0.0833	0.0669	0.0771	

From Table 4-6, we found that FXM is dominating. The transfer entropy of PFV change from FXM is the biggest one. So we can esteem FXM as a information source as BSL1.

Similarly, we sum the Transfer entropy of the PFV inflowing change, PFV out flowing change and total PFV change, and depict the transfer entropy by the brightness of the color. The consequence is given at Fig.3. Then we sum the six transfer entropy together, to see the total situation of transfer entropy.

From Fig.4 we can also extract FXM station from others. The averagely have the biggest transfer entropy to others, so we can esteem the FXM as an information source as the PFV of the BSL1, especially to the nearby urban stations.

5. CONCLUSION AND FUTURE WORKS

In this passage, we employed transfer entropy as our tool, to analyze the passenger flow volume between 7 major stations. We derived the

information source of the Beijing Subway Line 1. However, Beijing is constructing more and more subway lines, in this paper we did not discuss the transfer entropy between stations that located on different subway lines. Only by calculating the transfer entropy between different lines can we correctly evaluate the Whole subway network. This would be a potential future work direction. In addition, whether the transfer entropy is effective in other kind traffic flow analysis is still to be tested.

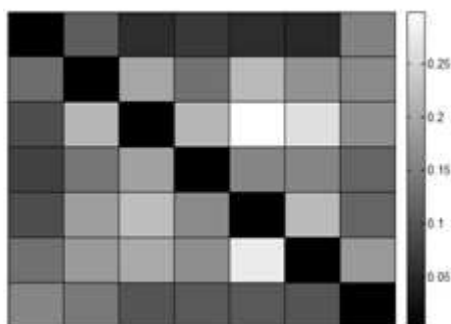


Figure 3 Transfer Entropy Of PFV Change

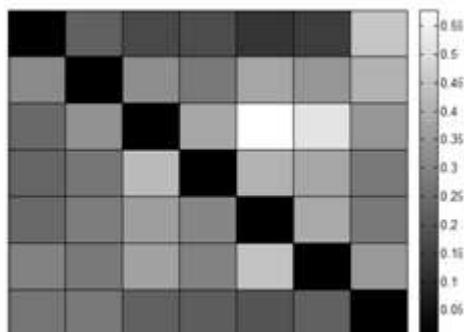


Figure 4 Transfer Entropy Of PFV & PFV Change

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