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A WS-REWIRING EVOLVING MODEL OF SCALE-FREE NETWORK*

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ABSTRACT

Although the BA model mimics the growth and preferential connectivity of the real-world networks, there are still some limitations. By studying the small-world network model and the BA model, we propose a WS-rewiring evolving model of scale-free networks in this paper. Comparing to BA model, we found that the power-low tail of the degree distribution of the new model has increased.

Keywords: *Scale-free Network, BA Model, the Small-word Network, Rewire*

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1. INTRODUCTION

We can describe the relationships of the world as a network, such as molecular interactions, the interaction between the protein and the organism, the energy conversion between objects, dating relationships between people and food chain relations. The real-world complex system can be abstracted into a complex network whose vertices are the elements of the system and whose edges represent the interactions between them. Then we study and analyze the structural characteristics of complex system by the network model of the system.

Since the ER random network [5] proposed in 1960s, researchers have begun to increasingly focus on complex networks. In the ER model, the scale of the network has been given at the start and any two vertices of the network are connected with the same probability. However, the real network is not totally random and its scale is not specified initially; for example, the vertices in most of the physical and biologic system don't distribute randomly, but are organized according to some structures. The ER model is just an ideal one in which the average length of paths is rather lower, the clustering coefficient is low, and the degree follows the Poisson distribution. Contrary to the random network, the regular network has a greater clustering coefficient, but with the scale of the network increasing, its average path length becomes greater. Therefore, the random network lack of clusters whereas the regular network hasn't short paths, and the two networks represent two extremes. But many biological, technological and social networks lie somewhere between these two extremes.

After analyzing the ER random network and regular network, Watts and Strogatz issued an innovative paper in Nature in 1998, which proposes a famous small world network model [2]-[3]. The WS model is formed by rewiring a small amount of edges of regular networks—that is, a few random factors is added to regular networks, which makes the average path length between vertices lower and the average clustering coefficient greater , networks meet these two properties all have small-world properties. Many real networks have the small-world phenomenon, such as the actors' networks, electricity networks and caenorhabditis elegans networks, etc.

Following the small-world network proposed, analyzed the world-wide-web, Barabási and Albert found that the degree distribution of WWW was different from ER random network. Barabási and Albert firstly research on the formation mechanism distributed in power-law of complex networks from a dynamic and growth perspective, and proposed scale-free networks [1]. And they give a reasonable explanation for the formation of the Internet and the World Wide Web through evolution rules combined the growth and the preferential attachment mechanisms.

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The WS model and BA model are two important achievements of the complex network, and promote the study of complex networks to enter a period of vigorous development. The WS small-world network model depicts the statistical characteristics of large clustering coefficient and short average path for real networks. The BA scale-free network model explains the phenomenon that the rich become richer which is generally found in complex networks. The WS network model is familiar with random network in degree distributions, they are uniform networks which degrees of all vertices are approximately equal, and so the topology of WS model is homogeneous. While the degree distribution of BA model follows a power-law distribution, the network connections are very uneven, vertices with higher degree account for a very small scale, and most vertices have only a small number of links. In summary, the scale-free network has the small-world phenomenon, but the small-world network is not equal to the scale-free network.

The small world nature and the scale-free nature are two most essential characteristics of complex networks. The BA model is the most common type of dynamic model which is used to analyze the scale-free network, but it still cannot completely simulate the real-world networks. The edges are varying by time. Therefore in this study, we propose a new network evolution model introducing rewiring mechanism in the WS model into the BA model, and our proposed models both have small-world and scale-free properties.

2. WS MODEL AND BA MODEL

2.1 The Construction Algorithm Of WS Model[3]

The network generation algorithm of the WS small-world model is as follow:

(1) Starting from a ring lattice with N vertices and k edges per vertex, each vertex connected to its k nearest neighbors by undirected edges. Here k is even.

(2) We choose a vertex and the edge that connects to its nearest neighbor in a clockwise sense. With probability p, we reconnect this edge to a vertex chosen uniformly at random over the entire ring, with duplicate edges forbidden; otherwise we leave the edge in place.

(3) We repeat the second process by moving clockwise around the ring, considering each vertex and the edges of it, until each edge in the original lattice has been considered once.

The algorithm show, for p = 0, the original ring is unchanged; as p increases, the graph becomes increasingly disordered until for p = 1, all edges are rewired randomly. Therefore the WS small-world network is a transition of the regular network to random network, highly clustered like a regular graph, yet with small characteristic path length, like a random graph.

2.2 The construction algorithm of BA model[1]

(1) Growth: Starting with a small number m_0 of vertices, at every time step we add a new vertex with m ($\le m_0$) edges that link the new vertex to m different vertices already present in the system.

(2) Preferential attachment. When choosing the vertex to which the new vertex connects, assume that the probability $\prod_i$ that a new vertex is connected to vertex i depends on the degree k_i of vertex i , in such a way that

$$\prod_i = \frac{k_i}{\sum_j k_j}$$

After t time steps this procedure results in a network with $N = t + m$ vertices and mt edges. Numerical simulations and theoretical analysis based on the continuum theory [1], master-equation [7] and rate-equation [6] approaches, all indicate that this network evolves into a scale-invariant state, for which the probability $P(k)$ that a vertex has k edges satisfies a power law with an exponent $\gamma_{BA} = 3$, i.e., $P(k) \propto 2m^2 k^{-3}$ [8]. A small amount of nodes in the BA model accounted for a large number of network connections and played a critical role in maintaining the whole network's synchronization stability. Because of this significant heterogeneity, the scale-free network is robust against random vertex failures but is fragile to a purposeful attack of vertex removal [4]. We call the new model as R-model.

3. A WS-REWIRING EVOLVING MODEL OF SCALE-FREE NETWORK

The WS-rewiring evolving model of the scale-free network based on the BA model differs from the BA model in taking the rewiring mechanism in the model in the same way as WS network. According to the different rewiring mechanisms introduced in different time, the WS-rewiring evolving model can be considered in three cases: rewiring each edge in the model at every time step; rewiring each edge in the model after a special time step t; rewiring each edge in the model after the whole network model has formed. This study only

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discusses the third case. The network generation algorithm of the R-model is as follow:

(1) **Growth.** Starting with a small number m_0 of vertices, at every time step we add a new vertex with m ($\le m_0$) edges that link the new vertex to m different vertices already present in the system.

(2) **Preferential attachment.** When choosing the vertex i to which the new vertex connects, assume that the probability $\prod_i$ in the same way as the BA model.

(3) **Rewiring.** After the end of the first two steps, the network vertex is no longer increase. We choose a vertex and the edge that connects to its nearest neighbor in a clockwise sense. With probability p, we reconnect this edge to a vertex chosen uniformly at random over the entire ring, with duplicate edges forbidden; otherwise we leave the edge in place. We repeat this process, considering each vertex and the edges of the network, until each edge in the model has been considered once.

Expression in its presentation of papers on the initial conditions of the BA network is not very clear. There are three initial conditions: starting with a small number m_0 of isolated vertices; starting with a small number m_0 of fully connected graph; starting with a small number m_0 of vertices and small number e_0 of edges. Due to limited network size of this experiment, the initial network is set to a small number m_0 of fully connected graph.

4. SIMULATION

From the generation algorithm of the R-model, we can know: for p = 0 the R-model is a BA model; as p increases, the R-model becomes increasingly disordered until for p = 1, all edges are rewired randomly. In the simulation experiment, we take p = 0.1, p = 0.8, p = 1 three sets of data to observe the difference between the R-model and the BA model.

Comparing the four pictures in Figure 1, it can be seen: for p = 0.1, the size of each node is similar to the BA model; for p = 0.8 and p = 1, the size of each node in these two R-models are similar; as p increases, the number of the small size vertices has decreased a lot, by the contrary, the number of the big nodes has increased.

From Figure 2, we observe that the degree distribution of the four networks still satisfy the power-law distribution, but the power-low tail of the degree distribution has changed from a to d. From a to d, p increases, the degree of the small size vertices increases. Compare a and d, when k closes to 0 the maximum of $P(k)$ is over 0.4 in the a and down to 0.12 in the d. Apparently the degree of a part of small size vertices has increased through the rewiring of edges in the network.

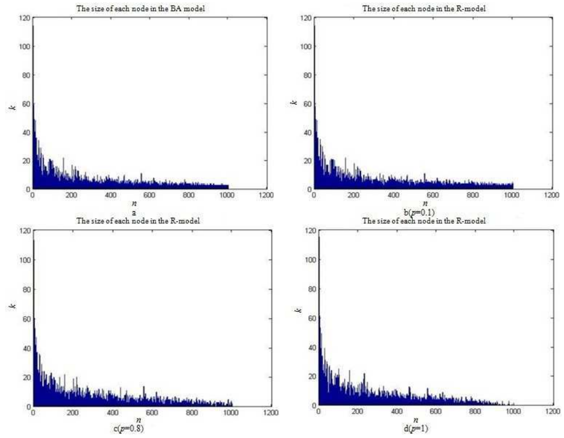

Figure 1. The Size Each Node In The BA Model And R-Model, With N = 1000, M_0 = 5, M = 3, P = 0.1,0.8,1, Respectively.

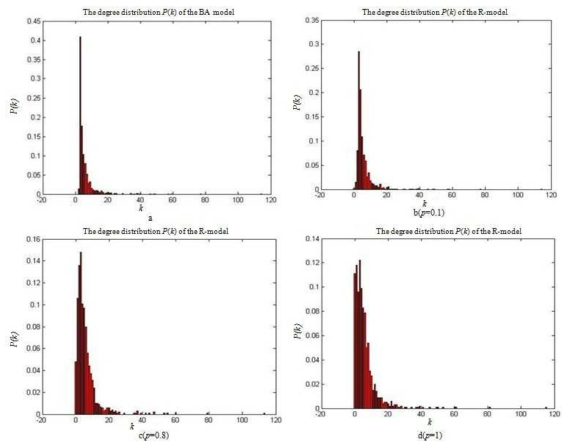

Figure 2. The Degree Distribution P(K) Of The BA Model And R-Model, With N = 1000, M0 = 5, M = 3, P = 0.1,0.8,1,Respectively.

In summary, the BA model allocates some wealth to the nodes with less wealth by rewiring the edges of the model; this characteristic plays an important role in the long tail theory. The long tail theory holds that as long as the channel of product storage and distribution is large enough, the market share occupied by the products of not strong demands and poor sales can reach and even be larger than the market shares occupied by the few hot products. That is, many small markets converge produce a match with the mainstream market shares. From the perspective of the long tail theory, the nodes with less wealth in the tail gain a certain amount of wealth by the R-model, and also prompting the entire efficiency. The cases of the long tail theory are the Google Ad words, Amazon,

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I tune, etc. Take the Google for example, a large number of small value of the media and advertisers converged by the Google and form the considerable benefits. If degrees of these small groups be improved higher, the effectiveness also will increase.

From the experiment, we also found that the average degree of four networks is in [5.8, 6], the clustering coefficient is about 0.03, the difference is that the clustering coefficient of R- model corresponding decreases with *p* increases.

5. SIMULATION

The WS-rewiring evolving model of the scale-free network, based on the BA model, differs from the BA model in taking the rewiring mechanism in the model in the same way as WS network. Rewiring edge means the edges is changing in the R-model., so that the R-model can reflect the real-world network better. We found that, although the degree distribution of R-model satisfied power-low distribution, the power-low tail has a shifting that the degree of a number of small size vertices has been increased.

REFRENCES:

[1] A.L. Barabási, R. Albert. “Emergence of Scaling in Random Networks”, Science, 286, 5439 (1999) , pp. 509-512.

[2] D.H. Shi, “Theory of Network Degree Distribution”, Higher education press, Beijing , 2011, pp. 2-4.

[3] D.J. Watts, S.H. Strogatz, “Collective dynamics of small-world networks”, Nature, 393 (1998) , pp. 440-442.

[4] L. Wang, G. Z. Dai, “The scale-free, scale-free phenomenon and the control of complex network”, science press, Beijing, 2009, pp. 147-150.

[5] P. Erdōs, A. Rényi, “On the evolution of random graphs”, Publ. Math. Inst. Hung. Acad. Sci. ,1960,5, pp. 17-60 .

[6] P.L. Krapivsky, S. Redner, F. Leyvraz. “Connectivity of Growing Random Networks”, Physical Review Letters, 85 (2000), pp. 4629-4632.

[7] S. N. Dorogovtsev, J. F. F. Mendes, A. N. Samukhin, “Structure of Growing Networks with Preferential Linking”, Physical Review Letters, 85 (2000) , pp. 4633-4636.

[8] X. Li, G. A. Chen. “local-world evolving network model”. *Physica A: Statistical Mechanics and its Applications*, 328 (2003) , pp. 274-286.