

## A CONSTRUCTION METHOD OF COMPLEX DISCRETE GRANULAR MODEL

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### ABSTRACT

Discrete granular model in a low stress state as well as retaining homogeneous porosity is the premise and foundation to obtain good results with particle flow method. AutoCAD is used as the processing software in which Polyline is used to render the scope of modeling and the 3dface is used to set delivery area of particles, then the improved expansion-particle method is prepared for generating boundary Wall and command flow of PFC2D automatically. Next, servo boundary is used to adjust stress state among particles and the porosity of model. Finally, the required numerical model will be obtained by using the arbitrary polygon material borderline to control and divide materials into parts. It is proved that a method with visualization is useful for the construction of complex discrete granular model. It can also be used in any 2D model building. What's more, it is less overlap and none overflow after the damage of bond strength between particles.

**Keywords:** *Discrete Granular; Complex Boundary; Low Stress; Homogeneous Porosity; Servo Boundary.*

### 1. INTRODUCTION

Discrete granular model in a low stress state as well as retaining homogeneous porosity is the premise and foundation to obtain good results with particle flow method. Therefore, its top condition is how to get the computational model when granular discrete element is used in the analysis of engineering numerical modeling. At present, the most common method of modeling using PFC2D is expansion-particle method. It generates small particles firstly within designated areas, and then expands particle radius gradually until the particles are full of whole model area. However, expansion coefficient is difficult to be determined so that a larger amount of overlap and higher internal force between particles are produced. When it is directly used to analyze slope landslide, material failure and other dynamic mechanical behaviors' inaccurate results will be caused and a large number of particles will overflow the boundary because of the release of strain energy or no enough bonding strength derived from overlaps. An improved expansion method is proposed in this study. It develops an automatic generation technique to achieve command flow of complex discrete granular model with visualization operation of AutoCAD, and can also greatly simplify the

construction process of complex 2D discrete granular model.

### 2. PRINCIPLE OF PARTICLE DISCRETE ELEMENT

Discrete element method, which was put forward firstly by Cundall in 1971, was a numerical method for analyzing mechanical behaviors of discontinuous medium. Cundall and Strack developed the program of disk and sphere which established the basis of discrete element method in 1979. It uses the law of force-displacement and Newton's laws of motion alternately and updates contact force between contact elements continuously. The new element contact based on element position is set up constantly in order to simulate the movement of granular material and the process of interaction as shown in Figure 1.

Granular discrete element is an explicit numerical calculation method. Contact force between particles is in proportion to the overlap and stiffness of particles. Particle normal stiffness presents the relationship between total normal force and total displacement, and particle shear stiffness presents the relationship of shear stress-displacement increment as shown in Figure 2.

$$F_i^n = K^n U^n n_i, \Delta F_i^s = -k^s \Delta U_i^s \quad (1)$$

From Eq. (1), it shows that contact force is proportional to overlap displacement when particle stiffness has been decided. Especially when the stiffness is larger, if boundary constraint or bonding strength between particles disappears, it will lead to particles flowing over the boundaries because of the release of strain energy, which should be avoided during process of modeling.

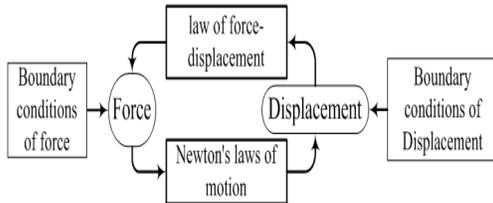


Figure 1 Calculation Cycle

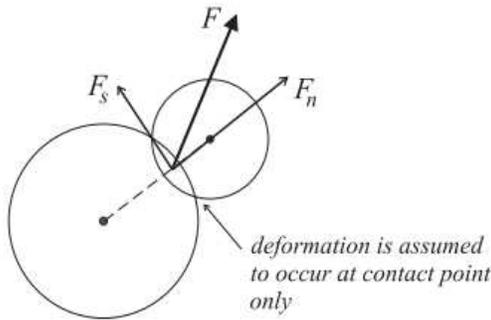


Figure 2 Particle Contact Force

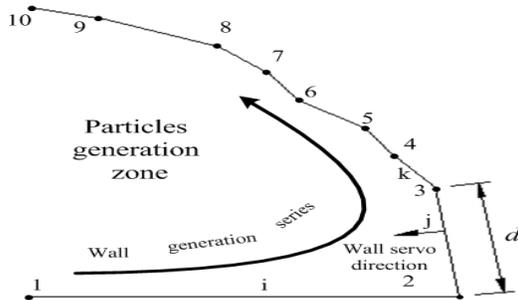


Figure 3 Boundary Servo Schematic Diagram

### 3. CONSTRUCTION OF COMPLEX DISCRETE GRANULAR MODEL

For discontinuous medium, material distribution and boundary conditions are often more complex. A model built rapidly and efficiently that can get reliable results will have an important significance to practical engineering.

**Estimate particle number.** For 2D model, if conceptual model area is  $S$ , designed porosity is  $n$ , the largest and smallest particle radius are  $R_{max}$

and  $R_{min}$ , then particle number  $N$  can be estimated as follows.

$$N = 4S(1-n) / [\pi(R_{min} + R_{max})^2] \quad (2)$$

If particles whose radius range is  $[R_{min}, R_{max}]$  are generated randomly, it would be difficult to reach mechanics balance because of interactions between particles. Therefore, scaling-inflation method is usually used to realize mechanics balance when particle number is determined. If  $R_{min}$  and  $R_{max}$  are zoomed using the same proportional scaling, the zoom factor is set as  $m$  (the value of  $m$  can be taken as a large number), then

$$R_{min0} = R_{min} / m, R_{max0} = R_{max} / m \quad (3)$$

Supposed  $i$  ( $i \in N^+$ ) is number of particle delivery regions, particle number  $N_k$  in each region can be assigned respectively according to the ratio of single area to the total delivery region area.

$$N_k = (s_k / \sum s_i) N \quad (4)$$

Where  $s_k$  is area of  $k$ th ( $k \leq i$ ) particle delivery region,  $\sum s_i$  is total area of particle delivery region.

**Expansion mechanism.** After  $N_k$  is assigned, particles can be expanded gradually. It will not stop expanding until particles are full of the whole model area and the designed porosity is achieved.

When particles are delivered randomly within a complex geometric model, if the angle of adjacent boundaries is too small or too large, it may lead to part of particles escaping from the model area. Therefore, model boundaries are controlled by arbitrary polygon in this study, and its vertex sequence is arranged counterclockwise in order to ensure that all particles are at the left of boundaries as shown in Figure 3.

After particles are generated randomly, if all particle area is  $s$ , then theoretical expansion coefficient can be defined as follows.

$$\eta = \sqrt{(1.0-n) / (1.0-p)} \quad (5)$$

Where  $n$  is designed porosity of model,  $p=1.0-s/S$ ,  $\eta$  is theoretical expansion coefficient.

It can be found easily in the process of expanding that if particle expansion time is only once particle arches emerging in great numbers will cause particles splashing and escaping from the model area, which will directly influence on test results. Consequently, particle expanding should be adopted in a stepwise manner. For example, it can be achieved in exponential form, such as  $\eta 0.6, \eta 0.2, \eta 0.1, \eta 0.05, \eta 0.05$ .

**Boundary servo.** After initial particle model is built based on above principles, stress state between particles will have a great difference because of stress concentration and particle overlap at the steering parts of fixed model boundary. If the particle model is used directly at this moment, strain energy between local stress concentration particles will release rapidly and particles will escape when boundary conditions are imposed such as removing boundary constraint. Therefore, the established model must be guaranteed that the particle stress state is in a low state when boundary conditions are applied. Boundary servo can be used to realize the construction of complex low stress model in this study. Strain energy can be most possibly released while particle positions are adjusted continuously in order to promote model porosity reached the designated value. The normal velocity of boundary Wall is defined as follows.

$$\dot{u}_n^w = G(\sigma^{measured} - \sigma^{required}) = G\Delta\sigma \quad (6)$$

Among which

$$\sigma^{measured} = \sqrt{f_{wx}^2 + f_{wy}^2} / A \quad (7)$$

Where  $f_{wx}, f_{wy}$  are contact force between boundary and particles in the direction of x and y, A is area of Wall. For 2D model, particle thickness is taken as '1', and A equals to D, D is the length of boundary Wall.

If normal velocity  $\dot{u}_n^w$  of boundary Wall is decomposed into  $v_{ix}$  and  $v_{iy}$  in horizontal and vertical direction, then boundary servo velocity is defined as followed.

$$v_{ix} = \dot{u}_n^w \cdot n_x, v_{iy} = \dot{u}_n^w \cdot n_y \quad (8)$$

Where  $n_x, n_y$  are representative of the ith boundary Wall's unit vector in the direction of x and y, whose direction of normal vector points to inside of model.

When the horizontal and vertical servo velocities are adjusted continuously the contact numbers between boundary Wall and particles are accounted at the same time. Then servo parameter G for next time step is obtained. Finally a new normal velocity of boundary Wall is also calculated. Continuous cycle and constant adjustment will be implemented until mean contact force reached the specified requirements.

If  $N_c$  is contact numbers between boundary Wall and particles,  $k_n^{(w)}$  is average contact stiffness, the variation of contact force in the unit time step caused by boundary movement can be defined as followed.

$$\Delta F^{(w)} = k_n^{(w)} N_c \dot{u}_n^{(w)} \Delta t \quad (9)$$

The variation of boundary average contact stress is

$$\Delta\sigma^{(w)} = k_n^{(w)} N_c \dot{u}_n^{(w)} \Delta t / A \quad (10)$$

Due to boundary stress must be smaller than the absolute value between testing stress and theoretical stress, if  $\alpha$  is taken as a factor of stress release, then

$$|\Delta\sigma^{(w)}| < \alpha |\Delta\sigma| \quad (11)$$

Simultaneous Eq. (9), Eq. (10) and Eq. (11), it can be seen

$$k_n^{(w)} N_c G |\Delta\sigma| \Delta t / A < \alpha \quad (12)$$

Thus, servo adjustment coefficient can be defined as follows.

$$G = \alpha A / (k_n^{(w)} N_c \Delta t) \quad (13)$$

Optimal expansion rate. After boundary servo, the increment of the ith boundary Wall's normal displacement is defined as follows.

$$\Delta l_i = k \cdot \sqrt{\Delta x_i^2 + \Delta y_i^2} \quad (14)$$

Where

$$k = \begin{cases} -1.0, & \text{boundary wall moves inward} \\ +1.0, & \text{boundary wall moves outward} \end{cases}$$

The area variation between boundary servo particle model and original model region is

$$\Delta S = \sum \Delta l_i \cdot d_i \quad (15)$$

Where  $d_i$  is the length of ith boundary Wall.

The expansion process is implemented in exponential form in this study. If the expansion process is divided into n levels, the ith level's amount of particle radius' expansion is  $\xi_i$  ( $i \leq n$ ), and then the ith level's actual particle radius expansion coefficient m is

$$m' = \eta^{\xi_1 + \xi_2 + \dots + \xi_i}, (i \leq n) \quad (16)$$

If the expansion coefficient was defined as  $\lambda = \xi_1 + \xi_2 + \dots + \xi_i$ , then it is easy to find that if  $\lambda < 1.0$ ,  $\eta$  is too large, and on the contrary if  $\lambda > 1.0$ ,  $\eta$  is too small, only  $\lambda = 1.0$ ,  $\Delta S = 0$ . And the actual particles radius expansion coefficient  $m'$  is the optimal expansion rate at this time. However, It is hard to make  $\Delta S$  equal to zero because of existence of servo mechanism, and the problem is transformed into problem about  $\lambda$  when  $\Delta S$  tends to zero. If  $\delta$  ( $\delta \in [0, 1]$ ) is defined as the percentage of area variation between boundary servo particle model and original model region, then

$$\delta = \left| \frac{\Delta S}{S} \times 100\% \right| = \left| 1 - \frac{1-n}{m^2} \cdot m'^2 \right| \quad (17)$$

$$= \left| 1 - \frac{1-n}{m^2} \cdot (\eta^{\xi_1 + \xi_2 + \dots + \xi_i})^2 \right| = f(\xi_i)$$

**Set particle materials.** According to above method, the discrete granular model in a low stress state as well as retaining homogeneous porosity can determine the characteristics of model particle geometry. Then different materials attribution can be controlled with one or more polygon areas according to the actual material distribution. All the particles can be assigned at this moment through

the judgment whether the center of every particle is located in the area of specified material or not.

For an arbitrarily shaped polygon, if edge number and nodes are NP, the *i*th particle's center coordinates is (xc, yc), then a ray  $y=yc$  ( $x \geq xc$ ) can be made. Whether the point (xc, yc) is in the polygon or not can be judged by the number of intersection  $n_p$  between the ray and the polygon edges. If the number '0' represents that the particle is located outside of the polygon, the number '1' represents that the particle is located within the polygon, then decision function that the point (xc, yc) in or out of the polygon can be defined as followed.

$$G(n_p) = \begin{cases} 0, & n_p \text{ is a even number} \\ 1, & n_p \text{ is a odd number} \end{cases} \quad (18)$$

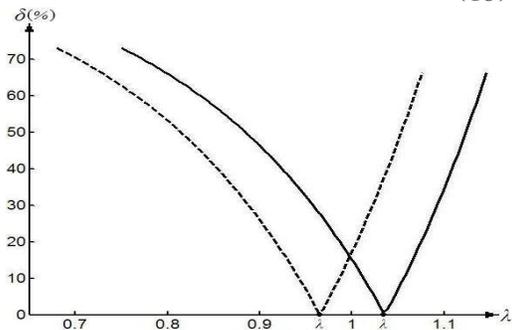


Figure 4 The Curve About  $\lambda \sim \Delta$

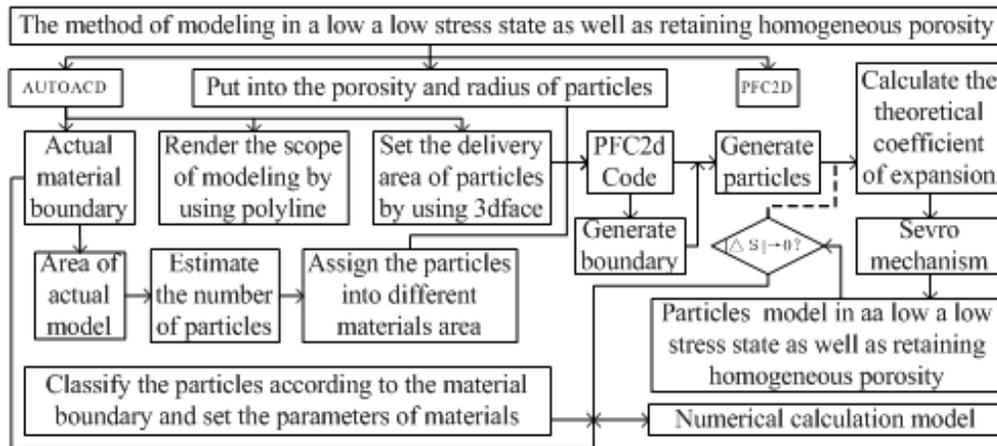


Figure 5 Flow Chart Of Complex Discrete Granular Model Construction Method

#### 4. THE MODELING PROCESS BASED ON PFC2D AND AUTOCAD

Using PFC2D and AutoCAD to build complex discrete granular model has a high degree of visualization and is very convenient to generate particles, to carry out boundary servo. An interface about AutoCAD and PFC is used to set the particle

delivery area, to identify the boundary normal direction, length and other information and to generate the PFC2D code automatically. Then the discrete granular model in a low stress state as well as retaining homogeneous porosity can be built by PFC. The specified process can be shown in Figure 5.

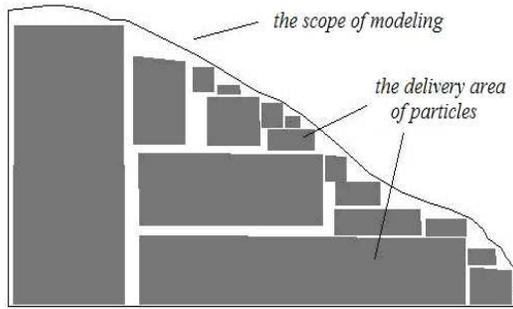


Figure 6 Schematic Diagram Of Calculation Model

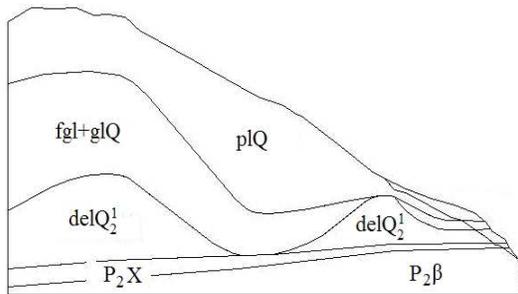


Figure 7 Arbitrary Polygon Material Borderline

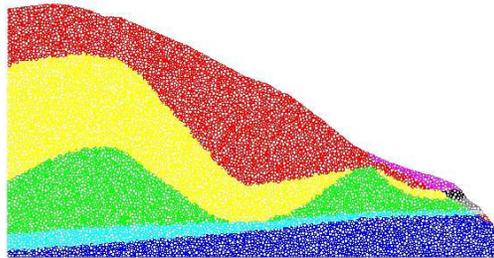


Figure 8 Particle Flow Model

## 5. CONCLUSION

An interface about AutoCAD and PFC is developed to generate PFC2D code automatically, which has a high degree of visualization, and can be applied into the analysis of slope stability. The proposed method has a high degree of visualization to build and modify model by using AutoCAD to control model area and particle delivery area. The developed expansion-particle method can achieve a good model that is in a low stress state as well as retaining homogeneous porosity by boundary servo mechanism. It simplifies the process of material classification by using the arbitrary polygon, and to control the material boundary by judging whether the particle is within polygon or not.

When the discrete granular model is built by this proposed method, there must be a small overlap between particles and almost no particle overflowing the boundaries. Therefore, the

construction of complex discrete granular model can be used excellently.

## 6. A COMPLEX BOUNDARY MODEL

A PFC slope model is built according to above modeling process. Figure 6 is schematic diagram of arbitrary polygon area, where Polyline is used to render model area and 3Dface is used to set particle delivery area. After using servo mechanism the model in a low stress state as well as retaining homogeneous porosity is achieved. The arbitrary polygon material borderline is shown in Figure 7 and the particle flow calculation model is shown in Figure 8.

## 7. ACKNOWLEDGMENT

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