Journal of Theoretical and Applied Information Technology
31 August 2012. Vol. 42 No.2

ISSN: **1992-8645** www.jatit.org E-ISSN: **1817-3195**

DELAY ANALYSIS OF THREE ARQ PROTOCOLS IN GEOM/G/1 QUEUE MODEL

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ABSTRACT

In this paper, we study the performance of the classical SW-ARQ, multichannel SW-ARQ and GBN-ARQ in discrete-time Geom./G/1 queue with setup mechanism. Based on the method of embedded Markov chain, the expressions of the packet average waiting delay, system average delay and channel utilization are respectively obtained. Finally, by numerical examples, we analyze the influences of packet length, the successful transmission probability and packet arrival rate on system average delay. The numerical simulation results show that the system delay could be lower by choosing an appropriate ARQ protocol.

Keywords: *Geom./G/1 Queue Model, Packet Average Waiting Delay, System Average Delay, Channel Utilization*

1. INTRODUCTION

Automatic Repeat reQuest (ARQ) protocols [1] have been widely used because of their high reliability, and there are three classical ARQ schemes: stop-and-wait (SW), go-back-N (GBN) and selective-repeat (SR). Among them, GBN-ARQ protocol is extremely popular because its implementation is simpler than SR-ARQ since it doesn't need buffering and resequencing at the receiver, and its high throughput performance. SW-ARQ protocol is lower than GBN-ARQ and SR-ARQ in the throughput, but SW-ARQ requires the least overhead and is the simplest to implement. Multichannel SW-ARQ offers better reliability than classical SW-ARQ.

In order to obtain the best system performance of ARQ protocols, Benelli and Garzelli [2] presented some new SW-ARQ protocols that significantly improve the throughput, while retaining the simple implementation of the classical SW-ARQ. Literature [3] studied the average transmission delay for SW-ARQ and GBN-ARQ with sliding window in the IEEE 802.11 standards. Literature [4] analyzed the performance of the three classical ARQ protocols for a multichannel system. In [5], Li and Zhao studied the resequencing delay and packet delay of SW-ARQ over a communication link consisting of parallel multichannel with same transmission rates and possibly different error rates. In [6], the authors investigated the performance of an adaptive GBN-ARQ protocol in time-varying channel environments with random or correlated feedback errors. The throughput of the three-mode GBN (TM-GBN) (that includes standard GBN (SGBN), n -copy GBN (n GBN) and continuous GBN (CGBN)) were researched in [7]. All of the above referenced work considers the throughput and algorithm of ARQ protocols. However, their queueing performance and random vacation have been seldom studied.

In this paper, we compute the packet average waiting delay, system average delay and channel utilization under the classical SW-ARQ, multi channel SW-ARQ and GBN-ARQ. For the purpose, we devise the discrete-time Geom./G/1 queue model sententiously based on equivalent service delay. The first moment and second moment of the equivalent service delay are solved. The expressions are used to derive the packet average waiting delay, system average delay and channel utilization.

This paper is organized as follows. Section 2 presents the analytical model based on the discrete-time. In Section 3, we derive the packet equivalent service delay, the system average delay and channel utilization. Numerical results and the comparison of the three ARQs delay performance are presented in Section 4. Finally, conclusions are drawn in Section 5.

2. SYSTEM MODEL

In order to study the behaviour of the three ARQ protocols farther by modeling, our analysis is based on the following assumptions.

The acknowledgement (ACK/NACK) is assumed to be error free; packets are transmitted on FCFS (First Come First Service) basis; the packet service delay is generally distributed and not dependent on the arrival process; l denotes one packet length; p denotes the successful transmission probability; t_T denotes the delay of transmission and propagation; packet arrival randomly occurs according to a Bernoulli process with rate λ , $0<\lambda<1$; μ denotes the packet service rate.

The packet service delay S which is the interval from the beginning of transmission to the time of the packet is successfully received at the receiver, which can be considered an equivalent service delay, which follows general distribution and can be described as a Geom./G/1 queuing model.

In this paper, the discrete-time Geom./G/1 queuing model with setup is presented. Once the system without packets, the service facilities are shut down until a new packet arrival. However, after a period of setup time V , the system begins service for the data packets, called Geom./G/1 (ES, SU). Assume that the setup time $V_i = l$, $E(V)=l$.

In the steady state, L_n is the remaining number of packets after the n th packet leaves, and $\{L_n, n\geq 1\}$ is the embedded Markov chain of the queue length process.

$$L_{n+1} = \begin{cases} L_n - 1 + C, & L_n \geq 1, \\ Q_b + C - 1, & L_n = 0. \end{cases} \tag{1}$$

Where Q_b denotes the number of packets in the system at the busy period begins. C is the number of packets entering the system in a service interval.

When $\rho = \lambda/\mu < 1$, the average waiting time of Geom./G/1 (ES, SU) can be obtained by [8], as follows:

$$E(W) = \frac{\lambda E(S(S-1))}{2(1-\rho)} + \frac{2E(V)+\lambda E(V(V-1))}{2[1+\lambda E(V)]} \tag{2}$$

Therefore, the average delay of the system is as follows

$$E(T) = E(W) + E(S)$$
$$= \frac{\lambda E(S(S-1))}{2(1-\rho)} + \frac{2E(V)+\lambda E(V(V-1))}{2[1+\lambda E(V)]} + E(S) \tag{3}$$

At the discrete-time Geom./G/1 queuing model with setup, the busy period of the model can be obtained as follows by [8]

$$B_v(z) = Q_b[B(z)] \tag{4}$$

Where $B(z)$ denotes the PGF of the busy period in the classical system. Hence, the mean value of busy period is given by

$$E(B_v) = \frac{[1+\lambda E(V)]E(S)}{1-\rho} \tag{5}$$

Thereby, the channel utilization is given by

$$\gamma = \frac{E(B_v)}{E(V)+E(B_v)} = \frac{[1+\lambda E(V)]E(S)}{E(V)+E(S)} \tag{6}$$

3. DELAY PERFORMANCE ANALYSIS

Classical SW-ARQ. For classical SW-ARQ protocol, when a packet is retransmitted for n times, the packet equivalent service delay is $l+nl+(n+1)t_T$ with probability $p(1-p)^n$. So we get

$$E(S)_{CSW} = \sum_{n=0}^{\infty}(l+nl+(n+1)t_T)p(1-p)^n = \frac{l+t_T}{p} \tag{7}$$

$$E(S(S-1))_{CSW} = \sum_{n=0}^{\infty} \begin{matrix}(l+nl+(n+1)t_T) \\ (l+nl+(n+1)t_T-1)p(1-p)^n\end{matrix}$$
$$= (l+t_T)[(l+t_T)(2-p)-p]/p^2 \tag{8}$$

By substituting Eq.7, Eq.8 into Eq.3, we get the system average delay of classical SW-ARQ

$$E(T)_{CSW} = \frac{(l+t_T)[2-\lambda-\lambda(l+t_T)]}{2[p-\lambda(l+t_T)]}$$
$$+\frac{2E(V)+\lambda E(V(V-1))}{2[1+\lambda E(V)]} \tag{9}$$

By substituting Eq.7 into Eq.6, we get γ_{CSW} as

$$\gamma_{CSW} = \frac{(1+\lambda l)(l+t_T)}{pl+l+t_T} \tag{10}$$

Multichannel SW-ARQ. Let n_m be the required average number of transmission that successfully transmitted a packet, we have

$$n_m = 1 + \{\textit{the average number of retransmission}\}$$

$$= 1 + p\sum_{i=1}^{\infty} i(1-p)^i = 1/p \tag{11}$$

For multichannel SW-ARQ, in order to reach the best channel utilization, when determining the number of sub channels, the transmission delay should be considered. The t_T is required

$$(N-2)l < t_T \le (N-1)l \tag{12}$$

So, the equivalent service delay of multichannel SW-ARQ is $n_m t_T + (n_m - 1)l \le (Nn_m - 1)l$. We get

$$E(S)_{MSW} = \sum_{n_m=1}^{\infty} (n_m t_T + (n_m - 1)l)\, p(1-p)^{n_m - 1}$$
$$= \frac{l+t_T}{p} - l \tag{13}$$

$$E(S(S-1))_{MSW} = \sum_{n_m=1}^{\infty} \begin{array}{l} (n_m t_T + (n_m-1)l) \\ (n_m t_T + (n_m-1)l - 1) \\ p(1-p)^{n_m-1} \end{array}$$
$$= \frac{(l+t_T)^2(2-p)}{p^2} - \frac{2l(l+t_T)}{p}$$
$$+ l^2 - \frac{(l+t_T)}{p} + l \tag{14}$$

By substituting Eq.13, Eq.14 into Eq.3, we get the system average delay of multichannel SW-ARQ

$$E(T)_{MSW} = \frac{(l+t_T)[2-\lambda+2l\lambda-\lambda(l+t_T)]+pl(\lambda-2-l\lambda)}{2[p-\lambda(l+t_T-pl)]}$$
$$+ \frac{2E(V)+\lambda E(V(V-1))}{2[1+\lambda E(V)]} \tag{15}$$

By substituting Eq.13 into Eq.6, we get γ_{MSW} as

$$\gamma_{MSW} = \frac{(1+\lambda l)(l+t_T-pl)}{l+t_T} \tag{16}$$

GBN-ARQ. For GBN-ARQ protocol, when a packet is retransmitted for n times, the packet equivalent service delay is $l + nNl + (n+1)t_T$ with probability $p(1- \ p)^n$. So we obtain

$$E(S)_{GBN} = \sum_{n=0}^{\infty} (l + nNl + (n+1)t_T)\, p(1-p)^n$$
$$= \frac{Nl+t_T}{p} - (N-1)l \tag{17}$$

$$E(S(S-1))_{GNB} = \sum_{n=0}^{\infty} \begin{array}{l} (l + nNl + (n+1)t_T) \\ (l + nNl + (n+1)t_T - 1)\, p(1-p)^n \end{array}$$
$$= (l+t_T)^2 + \frac{2(l+t_T)(Nl+t_T)(1-p)}{p}$$
$$+ \frac{(Nl+t_T)^2(2-3p+p^2)}{p^2} - (l+t_T)$$
$$- \frac{(Nl+t_T)(1-p)}{p} \tag{18}$$

By substituting Eq.17, Eq.18 into Eq.3, we get the system average delay of GBN-ARQ

$$E(T)_{GBN} = \{\lambda(Nl+t_T)p(1-p)$$
$$[(Nl+t_T) - 2(l+t_T)] - \lambda p^2 (l+t_T)^2$$
$$+ (2-\lambda)[p^2(l+t_T) + (Nl+t_T)$$
$$p(1-p)]\}/2p\{p - \lambda[p(l+t_T)$$
$$+ (Nl+t_T)(1-p)]\} + [2E(V) +$$
$$\lambda E(V(V-1))]/2[1+\lambda E(V)]$$

(19)

By substituting Eq.20 into Eq.7, we get γ_{GBN} as

$$\gamma_{GBN} = \frac{(1+\lambda l)[p(l+t_T)+(Nl+t_T)(1-p)]}{p(2l+t_T)+(Nl+t_T)(1-p)} \tag{20}$$

From Eq.7, Eq.13 and Eq.17, we find $E(S)_{MSW} = E(S)_{CSW} - l$, $E(S)_{GBN} = E(S)_{CSW}$ (when $N = 1$). The first moment of equivalent service delay for multichannel SW-ARQ is less than the classical SW-ARQ by one packet length. Besides, choosing $N = 1$ in Eq.19, we obtain $E(T)_{GBN} = E(T)_{CSW}$. When the length of the sliding window equals one, the GBN-ARQ protocol is naturally a classical SW-ARQ.

4. NUMERICAL RESULTS AND ANALYSIS

In this section, we present two numerical simulations to study the effect of the varying parameters on the system average delay of the three ARQ protocols. In all plots, we choose $N = 3$, $t_T = 1.2l$.

The system average delay performances are presented in Figure 1 and Figure 2. Figure 1 describes the effect of l and λ ($\lambda = 0.001, 0.002$) on the system average delay. Assuming that $p = 0.25$, in Figure1, it is observed that the system average delay decrease with the decreasing values of λ. In addition, when $l < 2$, the multichannel SW-ARQ protocol has the smallest average delay. While the GBN-ARQ protocol has the smallest average delay when $l > 2$.

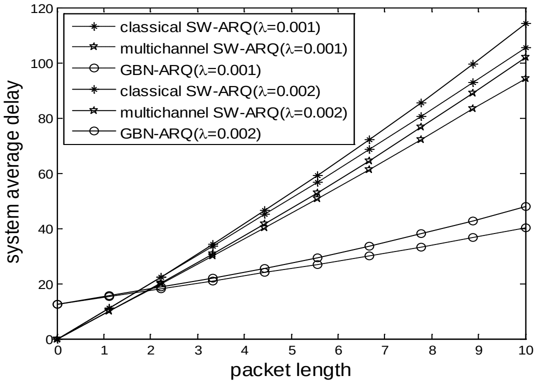

Figure 1 System average delay versus packet length and arrival rate

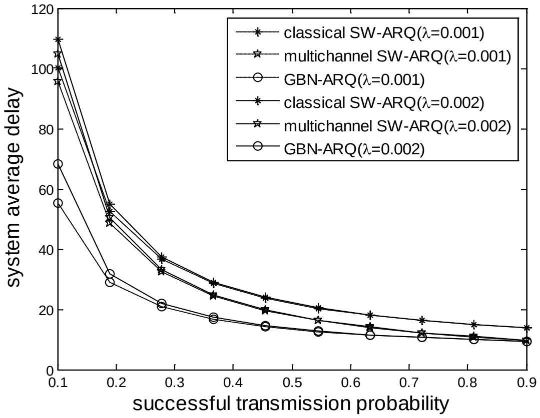

Figure 2 System average delay versus successful transmission probability and arrival rat

In Figure 2, we depict the behaviour of the system average delay against the parameters p and λ ($\lambda = 0.001, 0.002$). As to be expected, the system average delay decrease with the decreasing values of λ. Whereas, the values of system average delay decrease with increasing values of p . However, if the successful transmission probability is larger ($p > 0.6$), the arrival rate bring less effect on the system average delay.

5. SUMMARY

In this paper, the delay performance of the three ARQ protocols has been analyzed. The discrete-time Geom./G/1 queuing model with setup has been established sententiously, from which the expressions of the system average delay and channel utilization have been obtained. The discrete-time queuing model could be applied to more elaborate ARQ protocols, and possibly to HARQ.

6. ACKNOWLEDGMENT

This research was supported by grant 61167005 from the Natural Science Foundation of China, grant Z2006-1-2006 from the "Chun-hui Program" from the Ministry of Education of China and grant 0809RJZA019 from the Natural Science Foundation of Gansu Province.

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