

## ON THE DIVERSITY RECEPTION WITH CORRELATED RAYLEIGH-FADING SIGNALS

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### ABSTRACT

In this paper, the diversity reception with correlated Rayleigh fading signals will be investigated. Results will be evaluated by converting two correlated Rayleigh fading signals into another two independent Rayleigh fading signals using transformation matrix. These results show that Optimum transformation matrix  $F=1$  is better than  $F = \sqrt{2}/2$  which is the elements of the optimum transformation matrix which convert two correlated Rayleigh fading signals into another two independent Rayleigh fading signals. Analysis of the system over Rayleigh fading will be made further, the BER performance will be investigated under BPSK modulation.

**Keywords:** DS-CDMA, The Diversity Technique, Diversity Reception, Fading Signals, Rayleigh Fading

### 1. INTRODUCTION

New techniques are required to improve spectrum utilization to satisfy the increasing demand for many radio services without increasing the used radio frequency spectrum. One of these techniques in a digital cellular system is the use of spread spectrum Code Division Multiple Access (CDMA) technology [1]. Another technique is the diversity system [2] which is a method that is used to combine information from several signals transmitted over independently fading paths in order to reduce the effect of deep fades. There are classifications of diversity system. One view of classification is the transmission and the reception diversity systems. Other classifications are frequency diversity, polarization diversity, spaced diversity, time diversity and angle diversity. All these classifications are presented in detail in [2]. There are many combining techniques to combine uncorrelated faded signals obtained from the diversity branches, namely selective, maximal ratio, and equal gain combining [2]. In this paper, combination between CDMA system and diversity system will be used to improve the BER performance [3, 4]. New results for diversity reception with correlated Rayleigh fading signals will be analysis which is not clarified until now. These results will be evaluated by converting two correlated Rayleigh fading signals into another two independent Rayleigh fading signals using transformation matrix. Optimum transformation matrix will be obtained. The organization of this paper is made as follows:

System evaluation with Rayleigh fading is presented in Section 2. Results and discussion are done in Section 3. Finally, the paper is concluded in Section 4.

### 2. SYSTEM EVALUATION WITH RAYLEIGH FADING

In the Switched Selection Transmission Diversity (SSTD) the system is modeled as two antennas are at BS and one antenna is used in MS Walsh code  $s(t)$  is used. At the receiver, a Matched Filter (MF) is used to depress this code. Sampling is done and a decision is made to obtain the received data.

This technique is installed at the Base Station. It is possible that the same concept can be applied at the Mobile Station with two diversity antennas.

It has been assumed that the diversity antennas are statistically independent. Such assumption is valid only if they are sufficiently separated by a distance generally larger than a half wavelength [6]. Current mobile wireless units are decreasing in size, and using two diversity antennas on them would lead to the case of correlated fading [7].

Ref. [5] introduces a technique that converts two correlated Rayleigh-fading signals into another two independent Rayleigh-fading signals.

The transformation matrix  $T$  that converts two correlated Rayleigh-fading signals into another two independent Rayleigh-fading signals is;

$$T = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 2 & 2 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 2 & 2 \end{bmatrix} \quad (1)$$

Although of this, they did not state some important information such as :-

- 1-How did they get this matrix?
- 2-What did the matrix elements represent?
  - Are they represent attenuation factors, since their values are ( $\sqrt{2}/2$ )?
  - Are they represent phase factor,  $\cos \theta$ , since  $\cos 45 = \sqrt{2}/2$ ?
- 3- What are the basics on which they select the matrix elements?

The proposed transformation matrix T is similar to Hadamard matrix that was used to generate orthogonal sequences of Walsh codes.

Now, the case of correlated fading channels has been studied. Let  $r_1(t)$  and  $r_2(t)$  are two correlated received antenna signals from a dual branch diversity combining system and are given by;

$$r_k(t) = R_k e^{j\alpha_k} e^{j\Psi_m(t)} + n_k(t) \quad k=1, 2 \quad (2)$$

Where

$\Psi_m(t)$  is the transmitted information signal ;

$R_k$  is a Rayleigh – distributed random amplitude

with  $E[R_1^2] = 2\sigma_1^2$  and  $E[R_2^2] = 2\sigma_2^2$ ;

$\alpha_k$  is a uniformly distributed phase factor ;

$n_k(t)$  is zero – mean Additive White Gaussian Noise (AWGN).

The two received signals can be modeled by :

$$r_k(t) = [X_k + jY_k] e^{j\Psi_m(t)} + n_k(t) \quad k=1,2 \quad (3)$$

Where  $X_1, X_2, Y_1,$  and  $Y_2$  are all Gaussian random variables with zero mean and variance  $\sigma^2$ . Also  $X_1$  and  $X_2$  are correlated with each other with correlation coefficient  $\rho$  [8]. Similarly  $Y_1$  and  $Y_2$ .

In order to transform the two correlated received signals  $r_1(t)$  and  $r_2(t)$  into two new uncorrelated signals  $r_3(t)$  and  $r_4(t)$  we define the transformation matrix T as :

$$T = \begin{bmatrix} F & F \\ -F & F \end{bmatrix} \quad (4)$$

Where F could be an attenuation factor, gain factor, or phase factor,  $\cos \theta$ . Therefore ;

$$\begin{bmatrix} r_3(t) \\ r_4(t) \end{bmatrix} = T \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} \quad (5)$$

Which are expressed in the form

$$r_k(t) = [X_k + jY_k] e^{j\Psi_m(t)} + n_k(t) \quad k=3,4 \quad (6)$$

Where

$$X_3 = F.X_1 + F.X_2 \quad (7)$$

$$X_4 = -F.X_1 + F.X_2 \quad (8)$$

$$Y_3 = F.Y_1 + F.Y_2 \quad (9)$$

$$Y_4 = -F.Y_1 + F.Y_2 \quad (10)$$

$$n_3(t) = F.n_1(t) + F.n_2(t) \quad (11)$$

$$n_4(t) = -F.n_1(t) + F.n_2(t) \quad (12)$$

Using (7) and (8), the covariance of  $X_3$  and  $X_4$  can be calculated as ;

$$\begin{aligned} \text{Cov}(X_3, X_4) &= E[X_3 X_4] = F^2 E[X_2^2 - X_1^2] \\ &= F^2 E[X_2^2] - F^2 E[X_1^2] = F^2 [\sigma_2^2 - \sigma_1^2] = 0 \end{aligned} \quad (13)$$

Since it is assumed that  $\sigma_1^2 = \sigma_2^2 = \sigma^2$

This indicates that  $X_3$  and  $X_4$  are uncorrelated. Similarly, with (9) and (10), it can be concluded that  $Y_3$  and  $Y_4$  are uncorrelated.

Now,  $X_3, X_4, Y_3$  and  $Y_4$  are functions of Gaussian random variables, so they are also Gaussian random variables; in addition they mutually independent.

Also the power of  $X_3$  and  $X_4$  can be calculated by :

$$\begin{aligned} E[X_3^2] &= F^2 E[X_1^2 + 2X_1 X_2 + X_2^2] \\ &= F^2 [2\rho \sigma^2 + 2\sigma^2] = 2\sigma^2 F^2 [1 + \rho] \end{aligned} \quad (14)$$

Since  $E[X_1 X_2] = \rho \sigma^2$

$$\begin{aligned} E[X_4^2] &= F^2 E[X_1^2 - 2X_1 X_2 + X_2^2] \\ &= F^2 [2\sigma^2 - 2\rho \sigma^2] = 2\sigma^2 F^2 [1 - \rho] \end{aligned} \quad (15)$$

Since it is assumed that the noise power at each receiver for the original correlated signals is the same, then an SNR can be defined as ;

$$\Gamma = \sigma^2 / N$$

Where  $N = N_1 = N_2 = E[n^2(t)]$

The average SNRs of  $r_3(t)$  and  $r_4(t)$  are ;

$$\Gamma_3 = 2F^2(1 + \rho)\Gamma \quad (16)$$

$$\Gamma_4 = 2F^2(1 - \rho)\Gamma \quad (17)$$

Here, the same BER expression for a two – branch diversity system with Selection Combining introduced in [5] can be used .

$$BER = \frac{1}{2} \left[ 1 - \frac{a\Gamma_3}{a\Gamma_3 + 1} - \frac{a\Gamma_4}{a\Gamma_4 + 1} + \frac{a\Gamma_3\Gamma_4}{a\Gamma_3\Gamma_4 + \Gamma_3 + \Gamma_4} \right] \quad (18)$$

where

a = 1 for BPSK

a = 0.5 for QPSK

### 3. RESULTS AND DISCUSSIONS

The object now is to look for the optimum value of F that gives BER better than that is reported in [5]. The expression (18) is plotted in Figure 1 with  $\rho = 0.8$  and F varies between 0.4 and 2.

As seen in this Figure, the BER decreases as F increases. At this moment, we can answer the first part of our second question. If F has values of 0.4 or 0.707 ( $\sqrt{2}/2$ ), it means that F represents an attenuation factor that decreases SNR in each branch. On the second hand, if F has values greater than one, this means that F represents a gain factor. In this case, more sophisticated power amplifiers are needed to ramp up the signal power, which brings up cost issues. From this discussion it can be concluded that for F =1, is the optimum value that does not need any attenuation or amplification. Also, means low complexity for hardware. At the same time it gives BER better than that at F=0.707 ( $\sqrt{2}/2$ ).

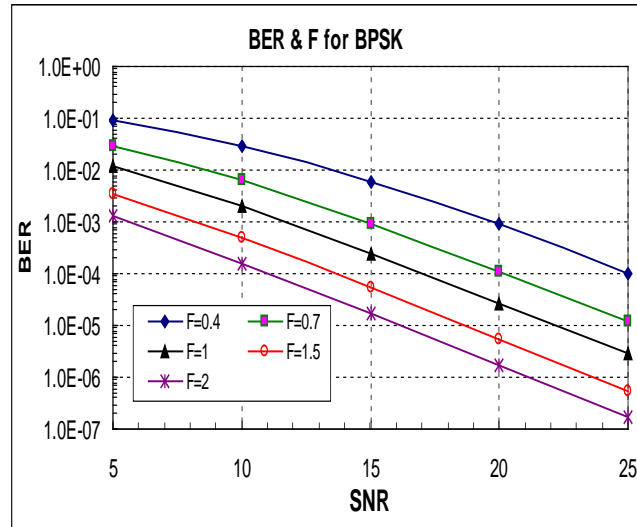


Figure1 BER vs. SNR with different values of F.

On the other hand, if F is considered as a phase factor,  $\cos \theta$ .

i.e.  $F = \cos \theta$

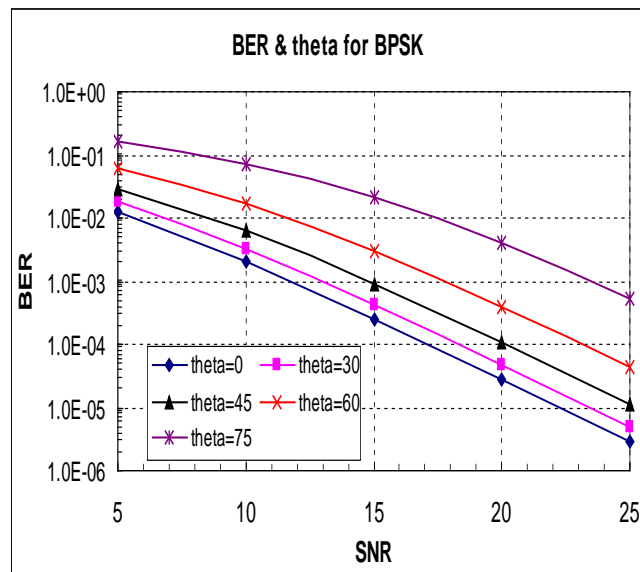


Figure 2 BER vs. SNR with different values of theta.

In this case the optimum value of  $\theta$  that gives better BER must take into consideration. As shown in Figure 2, increasing  $\theta$  will increase the error rate. BER at  $\theta=0$  is better than that at  $\theta=45$ , which means that  $F=1$  is better than  $F=\sqrt{2}/2$ . This matches up well with the previously reported result that  $F=1$  is the best value. From this discussion, it can be concluded that the optimum transformation matrix T is:

$$T = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

#### 4. CONCLUSIONS

In this paper, the diversity reception with correlated Rayleigh fading signals was studied. Results have been evaluated and show that  $F=1$  is better than  $F=\sqrt{2}/2$  which is the elements of the optimum transformation matrix which convert two correlated Rayleigh fading signals into another two independent Rayleigh fading signals. Analysis of the system over Rayleigh fading was made. BER performance was investigated under BPSK modulation.

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