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SIMULATION OF CODE TRACKING ERROR VARIANCE WITH EARLY LATE DLL FOR GALILEO/GPS BANDLIMITED RECEIVERS

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ABSTRACT

Both, the modernized GPS and Galileo, the new European Global Navigation Satellite System (GNSS), plan to incorporate in their signal schemes the Binary Offset Carrier (BOC) modulation. This modulation provides several advantages with respect to the Binary Phase Shift Keying (BPSK) modulation. Among others, it provides a significant reduction of the code tracking error. Code tracking accuracy is an important aspect of the GPS/Galileo navigation services. Errors in the code tracking affect the pseudorange computation and thus the GNSS based position performance. To support the design of future GPS/Galileo receivers, this paper provides analytical and numerical analysis of the code tracking accuracy for GPS/Galileo signals utilizing BOC modulation as a function of key parameters like carrier-to noise density (C/N0), receiver bandwidth, and Early-Late correlator spacing. It will be showed that a remarkable performance improvement is obtained when compared with signals utilizing BPSK modulation.

Keywords: *Code Tracking Error, Galileo, GPS, GNSS, Band Limited Recievers*

1. INTRODUCTION

Increasing demand for high quality navigation services for aviation is having a direct impact on the required performance of GNSS based navigation solutions. With the continuous reduction of satellite clock and ephemeris errors and the future removal of ionospheric induced errors due to the availability of dual frequency measurements (L1 and L5 bands), impact of code tracking errors in the navigation solution becomes more relevant.

In order to reduce the code tracking error both, modernized GPS and Galileo, plan to transmit signals (L1C for GPS and E1 and E5 for Galileo) that use BOC modulation, or some variation like MBOC or altBOC, instead of BPSK. Several studies [1]-[6] have shown that BOC clearly outperforms BPSK with respect to code tracking accuracy. Legacy GPS receivers utilized delay locked loops (DLLs) with early-late spacing equal to one. This was due to the easy analog implementation and the desire to minimize the hardware complexity of the receiver [7]-[8]. With the improvement of digital signal processing rates and the advent of narrow correlator technology multipath and code tracking errors were greatly reduced. The current code tracking accuracy achieved using narrow correlators is very close to the theoretical limit so in order to further improve the performance a new type of modulation is required. This is one of the motivations for the utilization of the BOC modulation in the news signals to be transmitted by GPS and Galileo. Other reasons are spectrum separation and improved multipath rejection. It should be noted that spectrum characteristics of the BOC modulation scheme compared to the traditional BPSK shows remarkable distinctions and conversely, they affect the DLL tracking accuracy differently.

Previous work in code-tracking accuracy of BPSK includes [9] and references therein. Dierendonck et al. in [7] provides results widely used in GPS, under assumptions of white noise and infinite receiver precorrelation bandwidth. However, the infinite receiver precorrelation bandwidth does not correspond to the real scenario in most cases. This work extended to variable receiver bandwith operating also in the presence of non-white spurious interference components has been extended by Betz et al. in [10]-[11].

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The aim of this paper is three fold. First, based on the analytical results reported in [10]-[11] for generalized case of baseband modulation, the code tracking error variance of the DLL for BOC and BPSK as a function of the limited RF filter bandwidth and discriminator chip spacing is analyzed. Then, numerical results for the code tracking error variance in the presence of Additive White Gaussian Noise (AWGN) are provided. It will be shown that these numerical results match perfectly the analytical expectations and therefore support the previously published studies. Finally, the detailed explanation of the oscillatory behavior of the code tracking error characteristics over the range of the investigated front-end bandwidths and discriminator spacings is provided. It will be shown that the slope of the detector characteristic in the stable lock point does not vary proportionally with the input signal bandwidth which explains the presence of ripples in the code tracking error characteristics.

In Section 2, the basic system model used throughout the paper is introduced. Then, in Section 3 the analytical derivation of the code tracking error variance for particular coherent Early-Late DLL is presented. Section 4 introduces the simulation model considered in the numerical results. Section 5 is devoted to the results of the numerical simulations. Conclusions and follow-up research are drawn in Section 6.

2. SYSTEM MODEL

Let us model the complex envelope of the received GNSS signal as

$$x(t)=\alpha e^{i\phi}s(t-\tau)+w(t) \qquad (1)$$

where $\alpha\in\Re$, $\alpha>0$ is the channel multiplicative factor, ϕ represents the signal phase, *s(t)* is the navigation signal being transmitted from the GNSS satellite, and finally *t* and *w(t)* denotes unknown navigation delay and AWGN contribution, respectively. Furthermore, it is assumed that the received signal *x(t)* is coherently received, i.e. the signal phase ϕ is perfectly known over the observation interval *T* in the receiver.

In the further analysis, *w(t)* stands for the noise equivalent complex envelope for systems with finite bandwidth. The corresponding power spectral density equals to *GW(f) = 2N0*. Moreover, τ and α are considered to be constant over *T* and thus, no dynamics are considered explicitly. However, this is reasonable assumption, clarified in e.g. [8]. To further simplify the forthcoming analysis, neither multipath distortion, nor intended interference occurs in the system.

Navigation signal *s(t)* can be further expressed as

$$s(t)=\sum_n d_n h(t-nT_P) \qquad (2)$$

where d_n represents the data symbols of the navigation message and $h(t)\in<0,T_P)$ is the period of the continuous signal formed as a product of the digitally modulated pseudorandom sequence. Furthermore, $T_P = T_C N_C$, where T_C and N_C represent the chip period and number of chips occuring in the pseudo-ranging code period, respectively. For our next analysis, we assume $d_n = 1$, in order to avoid the integration issues over the observation interval T, $T > T_P$.

Coherent DLL estimator can be derived from the likelihood function related to the considered channel model. In the AWGN channel model, the likelihood function can be adjusted to the form [7]

$$\rho(\tau)=\Re\left\{\frac{1}{T}\int_{kT}^{(k+1)T}x(t)s^*(t-\tau)e^{j\phi}dt\right\}. \qquad (3)$$

To estimate the signal delay τ, maximum likelihood strategy might be used with advantage. Then, it follows from the probability theory that estimated $\hat{\tau}=\arg\max_{\breve{\tau}}\rho(\breve{\tau})$, where $\hat{\tau}$, $\breve{\tau}$ represents delay estimate and trial parameter, respectively.

$$\left.\frac{\partial\rho(\tau)}{\partial\tau}\right|_{\tau=\hat{\tau}}=0. \qquad (4)$$

The function *h(t)* consists of rectangular pulses which presence consequently dictates that the first derivative of $\phi(t)$ contains Dirac deltas. To avoid implementation issues, the Early-Late approximation is used and thus, the derivative is replaced by the simple difference according to the equation

$$\left.\frac{\partial\rho(\tau)}{\partial\tau}\right|_{\tau=\hat{\tau}}=\frac{1}{T}\int_{kT}^{(k+1)T}\Re\left\{x(t)\frac{\partial s^*(t-\tau)}{\partial\tau}\right\}\Bigg|_{\tau=\hat{\tau}}$$

$$\approx\frac{1}{T}\int_{kT}^{(k+1)T}\Re\left\{x(t)\frac{s^*\left(t-\left(\tau+\Delta/2-s^*\left(t-\left(\tau-\Delta/2\right)\right)\right)\right)}{\Delta}e^{-i\phi}\right\}dt\Bigg|_{\tau=\hat{\tau}}$$

$$=\mu[k]=0. \qquad (5)$$

To solve the eq. (5) iteratively, the Feed-Back System (FBS) is employed. According to the FBS theory, the iterative solver utilizes loop error signal

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to correct the current value of the estimate which yields to the new parameter estimate [13]. It is assumed that the solution converges, hence

$$\hat{\tau}[k+1]=\hat{\tau}[k]+K_V G\{\mu[k]\}, \quad (6)$$

where $K_V \in \Re$ and $G\{.\}$ is general operator including memory (in practical realization $G\{.\}$ is implemented as filter).

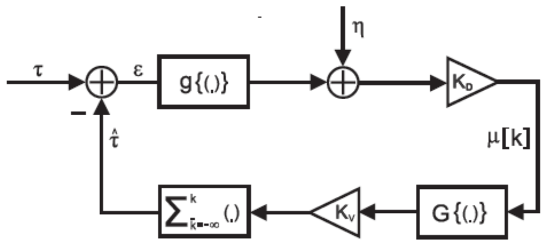

Fig.1 Early-Late DLL equivalent model

Corresponding equivalent model is depicted in the Fig. 1. Here, ε denotes the tracking error ($\varepsilon = \tau-\hat{\tau}$), $g(\varepsilon)$ represents nonlinear memoryless deterministic function normalized such that the first derivate of the function in the stable lock point equals to one (also known as S-curve [8]), η represents equivalent loop noise given by presence of AWGN in the real FBS input signal, and finally KD is the discriminator gain. For the purposes of our next analysis, we write

$$K_T = K_D K_V. \quad (7)$$

For the sake of brevity, no loop filter is considered in the numerical analysis presented in this paper ($G(i\omega)$), and hence closed loop frequency response $H(i\omega)$ has the following form

$$H(i\omega)=\frac{K_T}{i\omega+K_T}. \quad (8)$$

The one-sided DLL loop bandwidth is given as

$$B_L=\int_0^{\infty}\left|H(f)\right|^2 df. \quad (9)$$

The previous equations directly dictate that B_L finally equals to

$$B_L=\frac{K_T}{4}. \quad (10)$$

It can be shown that for the linearized equivalent model in the stable lock point (in the close vicinity of $\varepsilon = 0$) the error variance σ_ε^2 equals to [10]

$$\sigma_\varepsilon^2=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|1-H(i\omega)\right|^2 G_T(\omega)\,d\omega+2B_L G_\mu(0). \quad (11)$$

where $G_T(\omega)$, $G_\eta(\omega)$ represent the power spectral density (PSD) of the input signal delay and equivalent loop noise, respectively.

From the general equation above, it can be easily seen that σ_ε^2 is always trade-off between dynamic of the tracking parameter (represented by $G_T(\omega)$) and the noise in the loop (represented by $G_\eta(\omega)$).

Specifically, when no tracking parameter dynamic is assumed, eq. (11) is reduced to

$$\sigma_\varepsilon^2=2B_L G_\eta(0). \quad (12)$$

3. ANALYTICAL ANALYSIS

The previous works delivers the performance of the Cramer-Rao Lower Bound (CRLB) achievable by an unbiased Time of Arrival (TOA) estimator, for the given integration time T and two-sided precorrelation bandwidth βr, when the reference signal is phase coherent with the received signal. According to [10], [11], CRLB for coherent DLL can be formulated as follows

$$\sigma_{LB}^2=\frac{B_L(1-0.5B_L T)}{(2\pi)^2\frac{C_S}{2N_0}\int_{-\beta/2}^{\beta_r/2} f^2 G_S(f)\,df}. \quad (13)$$

where $C_S/|N_0|$ is the carrier power to noise density ratio and $G_S(f)$ is the power spectral density of the navigation signal normalized to unit power over the infinite bandwidth.

The frequency squared term in eq. (13) indicates that modulations with higher frequency components (i.e. higher offset from the center frequency) will demonstrate better code tracking accuracy. Spreading modulations, such as BOC, provide way to increase the desired signal high frequency power within a limited bandwidth and to decrease the variance of the code tracking error [10].

The analytical derivation of the code tracking error variance for particular coherent Early-Late DLL estimator is introduced in [11]. The final equation expressing the error variance of the Coherent Early-Late estimator (CELP) is given as

$$\sigma_{CELP}^2=\frac{B_L(1-0.5B_L T)\int_{-\beta/2}^{\beta_r/2} G_S(f)\sin^2(\pi f\Delta)df}{(2\pi)^2\frac{C_S}{2N_0}\left(\int_{-\beta/2}^{\beta_r/2} f G_S(f)\sin(\pi f\Delta)df\right)^2}. \quad (14)$$

It is important to note that the term

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$$K_D = C_S \int_{-\beta/2}^{\beta_r/2} 4\pi f G_S(f) \sin(\pi f \Delta) df \qquad (15)$$

included in the denominator of eq. (14) represents the discriminator gain. It can be seen that K_D varies with both, the code chip spacing Δ and the input signal bandwidth βr.

4. SIMULATION MODEL

The GNSS simulator model of the code tracking error variance is established according to the general recommendations introduced in [12]. The fundamental blocks of the error variance simulator are given in the Fig 2. Moreover, the simulation setup being used in the following Section is showed in Table 1.

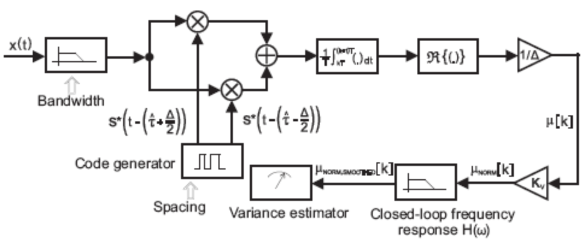

Fig.2 Simulator model

Tab.1 Simulation setup

Parameters	GPS	Galileo
Modulation	BPSK	BOC(1,1)
Pseudorandom Code	L1, Satellite1	E1b,Satellite1
Number of samples per chip	50	20
C/N0	45	45
KT	1	1
DLL loop bandwidth [Hz]	1/4	1/4
Number of iterations	10000	10000

As can be seen from eq. (11), the loop bandwidth B_L has important impact on the overall tracking error variance. During the numerical simulation presented in the following section, the multiplicative constant K_V is chosen to keep loop bandwidth B_L constant over the range of the investigated front-end bandwidths βr and discriminator spacings Δ. Conversely, K_V compensates for the detector gain K_D to keep both K_T and B_L, constant. Only in that particular case, the results obtained for the different values of βr and Δ correspond to the same physical conditions in the FBS realization.

5. NUMERICAL RESULTS

In this section, we provide the analytical and numerical performance evaluation of both BPSK (representing GPS) and BOC(1,1) (representing Galileo) modulations accuracy capabilities. In fact, for the numerical evaluation, the simulator configuration introduced in Section 4 has been considered, whereas the analytical results are solely provided by the analytical framework given in Section 3.

In order to better visualize the positioning accuracy given by both modulation schemes, the code ranging error $\sigma_{CELP}*c$ is used in the simulation results. Here, c denotes the light speed and thus, positioning accuracy in terms of distance will be showed.

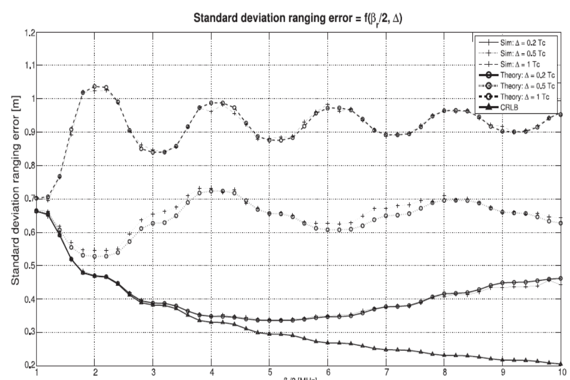

Fig.3 BPSK Standard Deviation Ranging Error

In Fig. 3, BPSK the code ranging error vs. Input filter bandwidth $\beta r/2$ for the set of chip spacings $\{\Delta=0.2T_C, 0.5T_C, 1T_C\}$ is illustrated. Moreover, theoretical lower bound represented by CLRB is also present, indicating inherent limits of accuracy under given conditions. As it can be appreciated, the code ranging error is substantially decreased with reduced Δ, however, for the larger filter input bandwidths, the position accuracy is significantly impaired by the larger amount of the noise presented in the system. One should also notice almost ideal agreement between the analytical and simulation results, which support our assumptions used throughout this paper. Furthermore, the reader

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might notice oscillatory behavior of the code ranging error curves, which will be explained in detail in the end of this section.

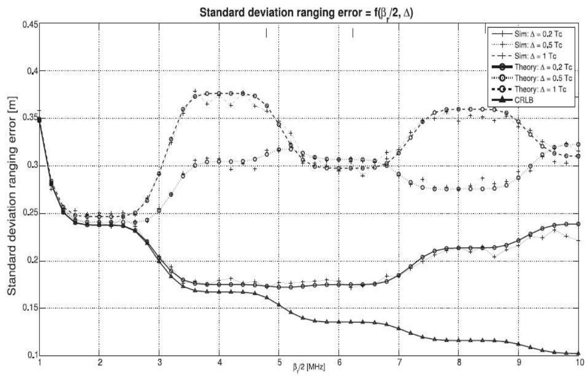

Fig.4 (BOC,1,1) Standard Deviation Ranging Error

Fig. 4 represents the analogous scenario, when BOC(1,1) modulation is being employed in the system. Here, again, the code ranging error is significantly reduced towards lower chip spacing Δ being considered in the system. The correctness of the numerical results is again comfirmed by the analytical results, where one can observe almost perfect match. As the simulation results provided in this section suggest, the BOC(1,1) modulation is capable of providing huge accuracy improvement over the large set of input filter bandwidths $\beta_r/2$ and chip spacings Δ compared to BPSK and thus, is of more interest in recent positioning improvements.

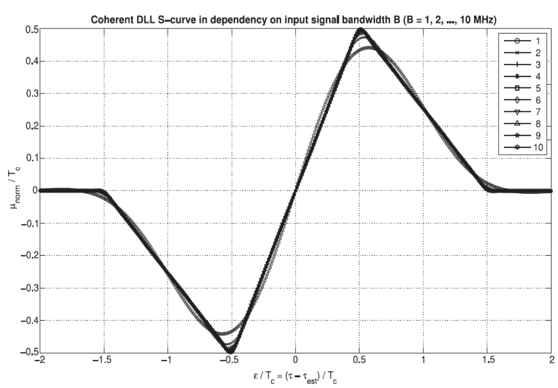

Fig.5 Coherent DLL S-curve vs. Input Signal Bandwidth

Now, our last goal is to explain the oscillatory behaviour of the code ranging error curves that is presented in both investigated cases. To make this explanation more tractable, we illustrate in Fig. 5 the coherent DLL S-Curve for the given signal bandwidth. The rigorous explanation of presence of the ripples in the ranging error characteristics comes directly from eq. (15) describing discriminator gain K_D on the input signal bandwidth β_r for the given chip spacing Δ. Since the discriminator gain is defined as a slope of the detector characteristic g(ε) in the origin, the following illustrations of the detector characteristics in the dependency on the input signal bandwidth describe this phenomenon comprehensibly and more intuitively. It can be easily seen that around the origin of the characteristic ([Detail A] , Fig. 6), the slopes do not change proportionally with the input signal bandwidth. Nevertheless, Fig. 7 and Detail B illustrated therein show that such a behaviour is not observed around the peak of the

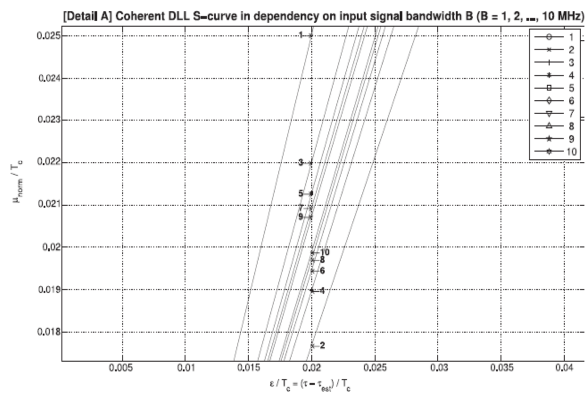

Fig.6 Coherent DLL S-curve vs. Input Signal Bandwidth [Detail A]

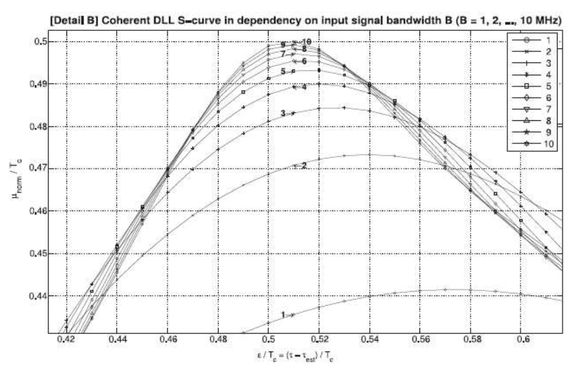

Fig.7 Coherent DLL S-curve vs. Input Signal Bandwidth [Detail B]

characteristic. This is exactly the reason why the ripples in Fig. 3-4 occur.

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6. CONCLUSIONS

This paper presents analytical and numerical evaluation of the code-tracking error performance of both, GPS and Galileo navigation systems in the presence of the input receiver filter and AWGN. It verifies that the analytical results precisely correspond to the results of Monte Carlo simulations. Galileo using BOC modulation scheme may provide a substantial better performance mainly due to the specific signal spectra of BOC modulation. The impact of proper discriminator gain compensation on the error performance is also discussed in this paper. The results presented in this paper might be used in the design process when looking for an optimal combination of receiver bandwidth and Early-Late correlator spacing. It should be noted, though, that no interference, which impact is closely related to the receiver bandwidth, has been assumed in this paper. Therefore, in our follow-up research, we will focus on the more complex environment, where both, the narrow bandwidth interference and more complex channels, will be considered.

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