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ANALYSIS OF RELIABILITY OF THE MATHEMATICAL MODEL OF THE SRM

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ABSTRACT

Switched reluctance motors are of simple construction. Along with salient pole stator systems, they include a rotor without conductor or magnet and are therefore the simplest of all electric motor rotors. Its simplicity makes the SRM inexpensive and reliable, along with its capacity at high speed and high torque in relation to inertia. Mathematical modelling of SRMs still constitutes the only real means of studying them in the fields of scientific research and design. In this article, we propose the combined method, which brings together the field-theoretical method approach used for calculation of the magnetic systems of commutated reluctance motors (SRMs) and analysis of electromagnetic processes in electric drive systems using the electrical circuits theoretical method. We then present a few results of the experiment with a type 8/6 SRM in order to validate the simulation method developed.

Key words: *Switched Reluctance Motor,* Powersys *, Simulink , Combined Method, Experimental Study*

1. INTRODUCTION

Designing methods for calculation and optimisation of the electromechanics characteristics of SRMs is a field of electromechanic that has been rapidly developing over the past few years. The bibliography bearing on the subject includes a large number of articles and scientific works that may be divided up into three major categories:

- Methods based on the concept of electric circuits.
- Methods based on the electromagnetic field theory.
- Combined methods.

2. ANALYSIS OF THE PROBLEM'S POSITIONING :

The first category of methods has been widely used in formulation of theories on induction motors, and has been the subject of considerable examination in a number of manuals and monographs bearing on the question [1-4]. The wide dissemination of such methods may be explained by the limitations of computer technology over recent years and the absence of specialised software and programmes.

The positive qualities of such methods are their simplicity, the accessibility of mathematical tools, and the widespread mastery of their use in calculation of electromagnetic systems, as well as their ability to obtain analytical expressions for approximate calculation of basic parameters. These methods also have their inconveniences, however, including the large volume of work required for design and calculation of components of the equivalent diagram, in particular when account must be taken of the heterogeneity of the distribution of the magnetic field and of the non-linear and anisotropic character of the magnetic circuit. The so-called “field” or “Maxwell” method is one of the most accurate and widely used methods enabling determination of the parameters of electric motors. Accordingly, these methods incorporate difficult mathematical tools, whose application in the initial state of analysis of dynamic processes remains highly complicated.

Combined methods bring together the simplicity of electrical circuit methods and the accuracy and polyvalence of field methods, and are regarded as the most perspective methods. Their distinctive character may be explained by the fact that classic electrical circuit components (electric energy sources, coils, control and commutation components, etc.) figure in the real structure of such motors. Nevertheless, the transitory rates of flow in these components are more or less correct, and may be described with relative ease by electrical circuit theory methods.

This work seeks to illustrate the characteristics of use of the combined method through design of the circuit of the field mathematical model of an SRM, as well as validation of the simulation method developed on the basis of the experimental test bench.

3. THEORETICAL APPROACHES AND SYNTHESES

Combined methods, bringing together the simplicity of electrical circuit methods and the accuracy and polyvalence of field methods, are regarded as the most perspective methods. In the case of switched reluctance motors, the combined method model enables calculation of the electromagnetic field with use of other results obtained in the form of differential equations describing the dynamics of the electrical drive system.

The main problem in the combined approach results from the necessity for simultaneous calculation of the variation of the magnetic field's characteristics and of the characteristics of the corresponding electrical circuits. Resolution of the problem in such a context complicates the simulation process considerably.

Use of the dynamic characteristics method is regarded as one of the possibilities for achievement of the combined method enabling design of the circuit of the field mathematical model of SRM [5]. The essentials of this method, in the case of its application for SRM simulation, consist of establishing a connection between the stator coil's inductance and the rotor's angle of rotation $L(\theta)$, and its introduction into the system of differential equations describing the electromechanical process in switched reluctance motors.

As the simulation subject, we have used an SRM (Figure 1) with the following performances and characteristics: Magnetic circuit material: 3405-quality steel; Magnetic circuit prototype: 8/6 (classic configuration); Stator exterior diameter: 62 mm; Stator interior diameter: 32.5 mm ;Length of active steel: 36 mm; Stator base thickness: 5.8 mm; Stator height: 9 mm; Rotor tooth height: 4.5 mm; Air gap: 0.25 mm; Polar angle of stator: 18˚; Angle of rotor grooves: 19˚; Shaft diameter: 9 mm; Number of spires per phase: 80; Nominal power : 1.5 kW; Nominal rotation frequency: 1500 rpm; Power supply voltage: 480V; Nominal current of stator: 12.4 A; Active coil resistance: 3 ohms.

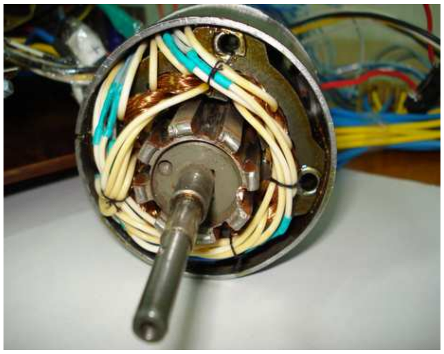


Figure 1. SRM. with 12/8 configuration

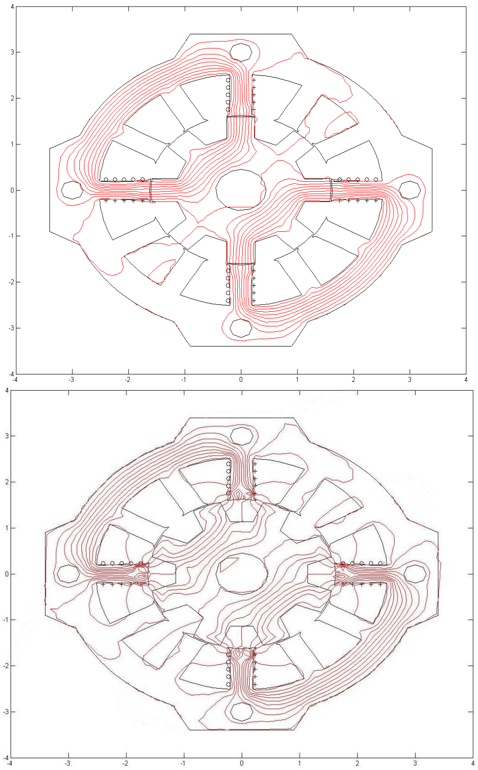


Figure 2. Images of magnetic field lines in an SRM for different rotor positions

Simulation of the SRM's electromagnetic field was carried out using the boundary integral equation method [6], and was implemented with the help of Matlab software. Images of an SRM's magnetic field at different rotor positions for minimum and maximum values are shown in Figure 2. On the basis of results obtained in studying the magnetic circuit, we were able to define the characteristic of the stators' inductance depending on the angle of the rotor $L(\theta)$. The characteristic was determined under the value of the stator coil's nominal current. Values calculated for stator inductance depending on rotor angle are marked in blue in Figure 3. Under real conditions, stator coil inductance also depends on the value of the current $L=L(\theta,i)$ due to the non-linearity of the steel's magnetisation curve.

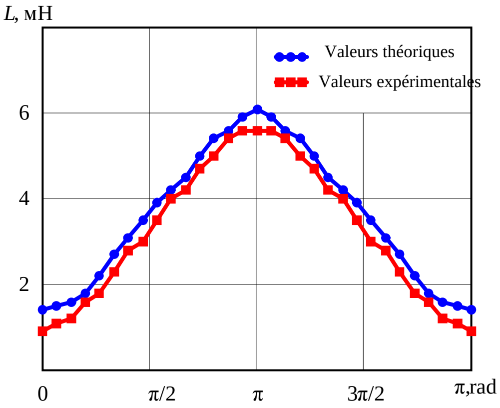


Figure 3. Stator inductance depending on CRM rotor position.

In order to validate the results of theoretical calculation of function $L= f(\theta)$, an experimental test was carried out on a real prototype motor. During the experiment, the rotor was fixed at a specified angle and then a stator phase was connected to an alternative voltage source. For current and phase voltage values measured, and taking account of the coil's active resistance, we calculated the value of the inductance, and went on to deduce the coil's reactance.

Measurements were made for various frequencies between 200-1000 Hz in order to decrease the uncertainty of the experimental data. After making the required measurements, the rotor was changed to the next position. In this way, experimental inductance values were obtained discretely throughout the period of its variation, i.e. 45°. Experimental values for stator inductance depending on angle of rotor are marked in red in Figure 3.

Comparison of theoretical and experimental results of function $L= f(\theta)$ show relative inaccuracy not exceeding 5%, going to show the accuracy of results of simulation of the motor's magnetic field with the help of the proposed model.

The next step in simulating the SRM's dynamic processes was to establish the system of differential equations, on a basis taking the previously determined function $L(\theta)$ into full consideration.

Taking account of the complexity of electromagnetic phenomena in a SRM with a passive rotor, it is desirable to adopt the following hypotheses:

- Absence of mutual inductance between phase bobbins;
- Inverter switches are perfect – commutation is produced immediately without loss of energy;
- Infinite power supply with energy recuperation;
- No losses due to hysteresis or Foucault currents.

The above hypotheses enable simplification of the system of differential equations by rejecting insignificant factors and processes.

The basic equations required for description of electromagnetic and electromechanical processes are well known [3].

As with all types of electric motors, the control system for commutated reluctance motors is made up of equations of electrical balance for each of the motor's stator phases, and equations for rotor movement.

The electrical relationship of the balance position of the CRM's stator coil is defined as follows:

$$U = iR + \frac{\partial \Psi}{\partial t} \qquad (1)$$

$$\frac{d\omega}{dt} = \frac{1}{J} \cdot \left(M_e - M_c \right), \qquad (2)$$

Where U is the supply voltage; R - the coil's active resistance; M_c – static torque opposed by charge; M_e – electromagnetic torque developed by the motor; J – moment of inertia; ω – speed of the rotor's angular rotation

In general, magnetic flow across coil k in an electric motor with several coils is equal to:

$$\Psi_k = L_k i_k + \sum_{j=1, j\neq k}^{N} M_{jk}\, i_j \tag{3}$$

With:

L_k: Own inductance of phase k;

M_{jk}: Mutual inductance between phases j and k.

As, with symmetrical control in full step, currents overlap slightly depending on time, the second term in equation (2) may be ignored for a certain relation between motor charge and number of stator coils. In this case, a coil's magnetic flow may be entirely determined by its own inductance.

As the inductance of a SRM coil depends on rotor angle compared to the latter $L_s=L_s(\theta,i)$, equation (1) may be written as follows:

$$\begin{aligned} u &= Ri + \frac{\partial \Psi}{\partial i}\frac{di}{dt} + \frac{\partial \Psi}{\partial \theta}\frac{d\theta}{dt} \\ &= Ri + L_\partial(\theta,i)\frac{di}{dt} + \omega\frac{\partial L_s(\theta,i)}{\partial \theta}i, \end{aligned} \tag{4}$$

Or

$$u = Ri + \frac{\partial L_s(\theta,i)}{\partial\theta}\omega i + \left(\frac{\partial L_s(\theta,i)}{\partial i}i + L_s(\theta,i)\right)\frac{di}{dt} \tag{5}$$

Where ,

$$L_\partial(\theta,i) = \frac{\partial \Psi(\theta,i)}{\partial i},$$

$$L_s(\theta,i) = \frac{\Psi(\theta,i)}{i},$$

$$\omega = \frac{d\theta}{dt}$$

Designating dynamic inductance, static inductance and angular rotor speed respectively.

In certain cases [3 and 4], equation (4) is used in the form of:

$$u = Ri + L(\theta)\frac{di}{dt} + \omega\frac{\partial L(\theta)}{\partial\theta}i \tag{6}$$

Ignoring the difference between static and dynamic inductance, it may be noted that this hypothesis is valid in the absence of magnetic saturation of the system. Consequently, in order to better describe the electrical processes, we can use equation (6).

The electromagnetic torque that develops under the influence of phase i current

$$M_e = \frac{\partial L(\theta)}{\partial\theta}\cdot\frac{i^2}{2}\cdot \tag{7}$$

Therefore, the generalised system of differential equations for the CRM is :

$$\left.\begin{aligned} u_k &= Ri_k + L(\theta_k)\frac{di_k}{dt} + \omega\frac{\partial L(\theta_k)}{\partial\theta}i_k \\ M_e &= \frac{\partial L(\theta_k)}{\partial\theta_k}\cdot\frac{i_k^{\,2}}{2} \\ \frac{d\omega}{dt} &= \frac{1}{J}\cdot(M_e - M_c) \\ \frac{d\theta_k}{dt} &= \omega \end{aligned}\right\} \tag{8}$$

Where k = 1... m – phase number; u_k, i_k – designate voltage and current of phase k respectively.

Let us now consider in detail the operational principle governing the SRM's conversion system. U phase's power-supply voltage during a commutation cycle can take on three values:

(U_s): When the bobbin is connected to a power supply source and transistors T1 and T2 are open (Figure 4a);

(0): When the bobbin is short-circuited on itself with transistor T1 closed and T2 open (Figure 4b);

($-U_s$): When both transistors are closed and the bobbin's polarities are inverted in relation to the power-supply source (Figure 4c). This configuration is used for forced extinction of current within the stator, in order to eliminate the electromagnetic torque's negative values, and possible implementation in the event of power supply from an accumulator battery or one with a reversible rectifier.

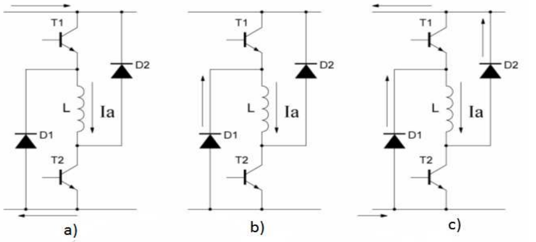


Figure 4. Ways of connecting a stator coil to a power source.

Thus, the voltage at a phase's terminals varies according to the following division: :

$$u = \begin{cases} U_s, \alpha \le \theta < \beta; \\ -U_s, \theta \ge \beta, i \ne 0; \\ 0, \beta < \theta < 2\pi \cup \theta < \alpha, i = 0. \end{cases} \tag{9}$$

Where α, β – designate the angles of commutation.

4. MODELLING OF AN SRM

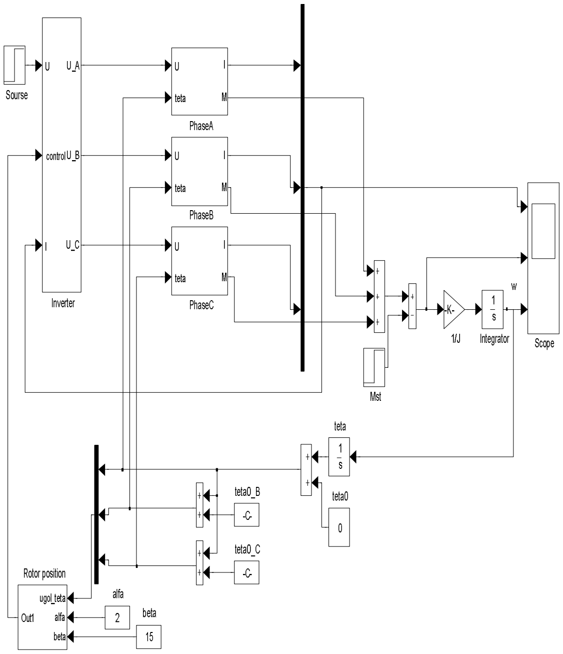


Figure 5. Functional diagram of the model for SRM simulation

The model for simulation is developed from the above SRM equations. Figure 5, describing the dynamics of a four-phase SRM with independent control. A structure diagram of the electrical-drive model based on the SRM, created with Matlab Simulink software [7-8], is presented in Figure 5.

The power source is presented in the *source* block with voltage U supplying the transistorised *inverter* simulation block. The order of commutation of phases is defined depending on the rotor's angle of rotation by the *rotor position* block, which, by imitating the operation of the position captor, generates control pulses by power switches within the limits of specified phase commutation angles. These pulses are generated by blocks *alpha* and *beta* respectively. From the *inverter* block, voltage is supplied at entrance U of one of the *Phase A-D* subsystems, so simulating phase operation. With release of phase switches, the supply block is polarised under inverse voltage, and current circulates through the back-off diodes. When the current in the phase cancels out, the diodes are blocked and the current in the phase is extinguished. For specified torque values of the various phases, the *sum* block generates the resulting electromagnetic torque. The *Mst* block simulates the static torque opposed by the motor charge. Block *1/J* represents the moment of the motor's inertia.

Initial value of the rotor's angle of rotation is determined from the *teta0* block. Monitoring of variation of the different parameters is carried out by the bloc representing the scope.

Figure 6 presents the structure diagram of the model of a phase (bloc phase A). For certain laws of instantaneous variance of supply voltage and the rotation angle of rotor θ, this block calculates the value of the current and the electromagnetic moment. It is composed of blocks *L(teta)* and *dL_dteta*, forming the liaison between *L*(θ) and *dL*(θ)/*d*θ respectively and their proposed approximate functions.

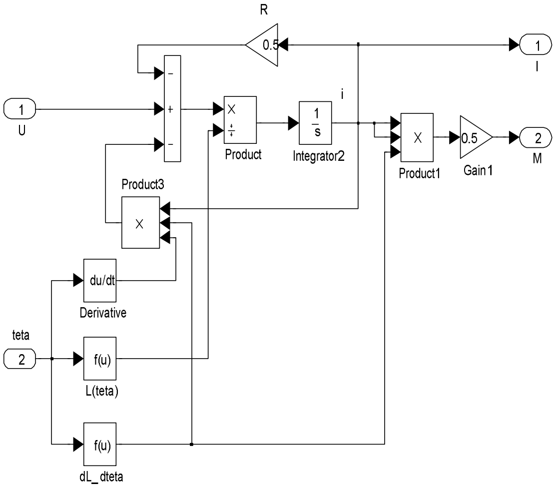
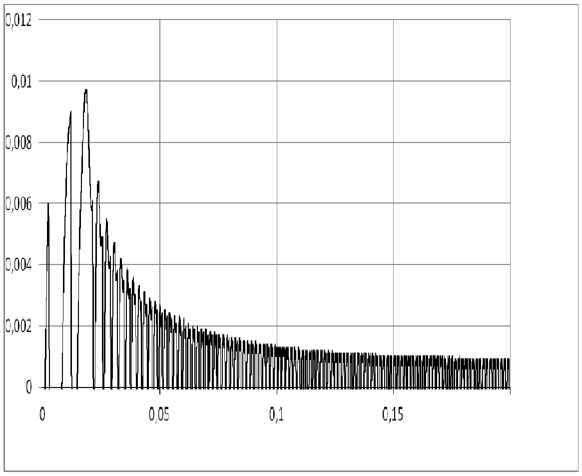


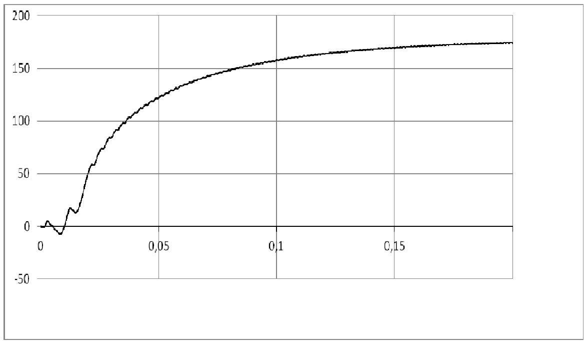
Figure 6. Model of simulation of an SRM phase

5. SRM SIMULATION RESULTS

The results of simulating the start-up process for the electrical drive based on the SRM model with a resistive charge are presented in Figure 7 in the form of chronograms of phase currents, the resulting electromagnetic torque and the rotor's angular rotation speed. Forms of phase currents during stationary motor operation are represented in Figure 8.

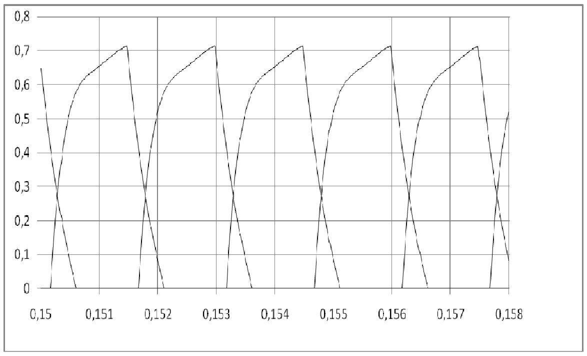


a) Electromagnetic torque

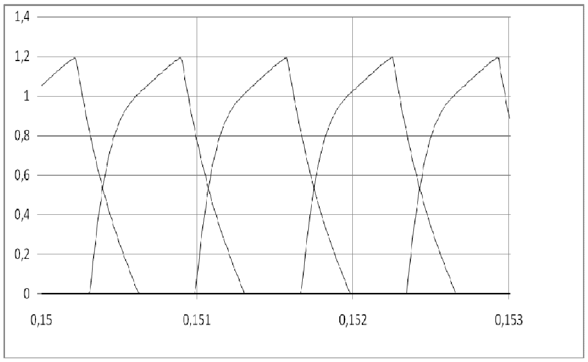


b) Angular rotation speed

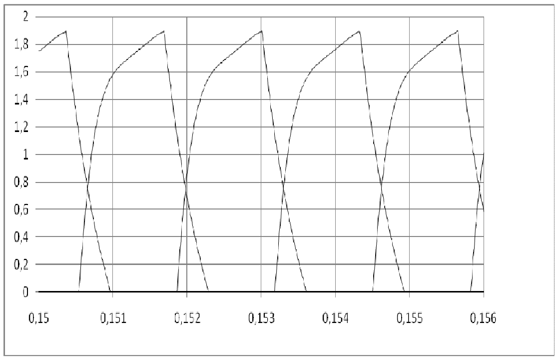
Figure 7 . Transition process of SRM at start-up



a) U = 5 V, M = 0,007 Nм



b) U = 12 V, M = 0,007 Nм



c) U = 12 V, M = 0,018 Nм

Figure 8. Forms of phase currents

6. DESCRIPTION OF THE EXPERIMENTAL PLATFORM

With a view to carrying out the experimental study, a test bench for the SRM's electrical drive was created (Figure 9). The three-phase SRM prototype described above formed the basis for the test bench. The distinctive feature of this 12/8-configuration three-phase motor is that it has four bobbins in each phase, set at an angle of π/2; the bobbin-connection diagram is mixed, i.e. series – parallel.

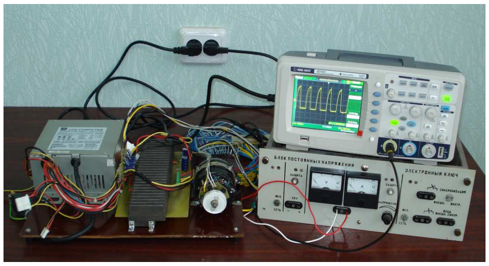


Figure 9: Experimental platform for the SRM electrical drive

A block based on photoelectric elements located around the circumference of the disc with machined segments was used as rotor position captor. The advantages of this type of captor include its simple construction, low cost, reliability, high performance and ease of signal-captor manipulation.

The power commutator was created in the form of a classic three-phase bridge inverter with two IGBT transistors and two ultra-fast inverse diodes in each phase (Figure 10). Universal drivers with upper- and lower-arm pilots were used in order to ensure the conditions required for commutation of transistors.

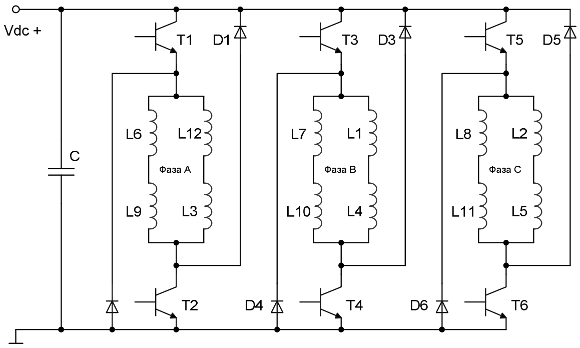


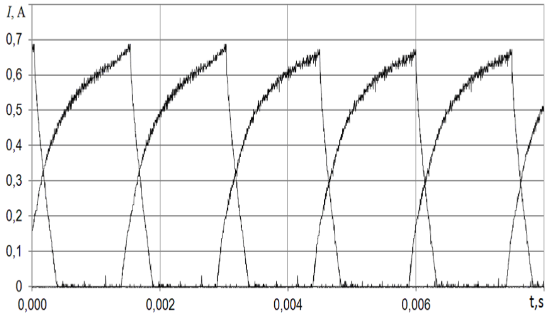
Figure 10. Three-phase bridge inverter

A digital oscilloscope was used to record forms of phase-current curve waves, with the possibility of capturing data in vector form and with a 4-μs discretisation rate.

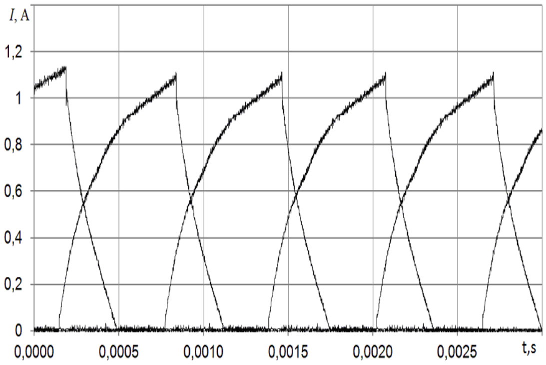
7. EXPERIMENTAL RESULTS

During the experimental test, after start-up of the motor, phase supply voltage of a fixed value was established, and a specified motor torque was brought to bear on the motor shaft. The required measurements were taken upon reaching almost stationary rate of flow.

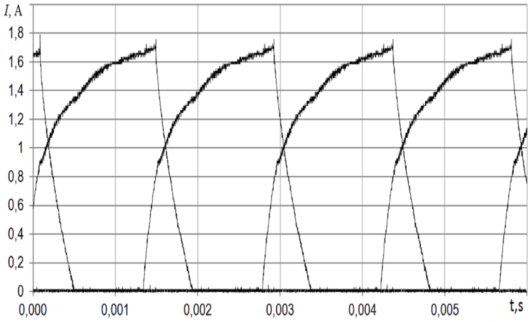
Figure 10 shows the experimental curves of the motor's phase currents for various voltage and resistant torque values.



a) U = 5 V, M = 0,007 Nм



b) U = 12 V, M = 0,007 Nм



c) U = 12 V, M = 0,018 Nм

Comparing the experimental curves of phase currents for different operational rates of flow with the simulated curves of the proposed model, we may observe that amplitude, form and period values coincide with sufficient accuracy. We may therefore conclude that the simulation model proposed describes a commutated reluctance motor's electrodynamics process with a high degree of precision.

8. CONCLUSION

In this article, a mathematical model of the SRM has been developed using the combined approach, which brings together the field method enabling calculation of the magnetic system and the electrical circuits method. A chain-field model of a switched reluctance motor has been calculated using the dynamic characteristics method. Comparison of the results of calculation of the parameters of the motor's magnetic field and the characteristics of electro-dynamic processes with the experimental results reaffirms the chosen method's validity and viability. The model developed may be used for study of the SRM's electromechanical and energy properties. The universal nature of the mathematical methods and software tools used for development of the combined method creates the right conditions required for future improvements.

REFRENCES:

[1]. T.J.E. Miller, Switched Reluctance Motor and Their Control, Magna Physics and Clarendon Press, Oxford, 1993.

[2]. John R. Suriano. Design methodology for low speed variable reluctance motors. Purdue University, School of Electrical Engineering, 1992. – 90 p.

[3]. Krishnan Ramu. Switched reluctance motor drives: modeling, simulation, analysis, design, and applications. – Boca Raton London New York Washington, D.C., 2001. – 416 p.

[4]. Gollandcev Ю.A. Switched Reluctance Motor. – Saint-Pétersbourg.: State scientific center of the RUSSIAN FEDERATION – Central Research Institute « Electrical Drive», 2003. – p.148 .

[5]. Vaskovsky J. M. *Field analysis of electric machines*: T. Handbook, KIEV: NTU "KPI", 2007. p.192.

[6]. O. Tozoni, I. Mayergoyz. Computation of 3-Dimensional Electromagnetic Fields. Technika, Kiev, USSR, 1974. – 352 p.[7] Math Works. SIMULINK Version 1 User's Guide (1996)

[8] Elliot, C. R., Stephenson, J. M., and McClelland, M. L.(1995)Advances in switched reluctance drive system dynamic simulation. In *Proceedings of EPE '95*, **3** (1995), 622—626.