



# FUZZY DECISION MAKING IN TREATMENT FOR ANGINA (2000 MATHEMATICS SUBJECT CLASSIFICATION CODE - 91B06)

<sup>1</sup> Dr. M. THIYAGARAJAN, <sup>2</sup>RT. PANNEER SELVAM

<sup>1</sup> Professor, School Of Computing, Sastra University, Thanjavur, Amilnadu, India

<sup>2</sup> Research Scholar, Sastra University, Thanjavur, Tamilnadu, India

E-mail: [1proftrp@yahoo.com](mailto:1proftrp@yahoo.com)

## ABSTRACT

This paper discusses the application of fuzzy decision making in treatment for angina. The field of decision making is the study both of how decisions are actually made and how they can be made better or more successfully. Bellman[1] has shown that applications of fuzzy sets within the field of decision making have consisted of extensions or fuzzifications of the classical theories of decision making. While decision making under conditions of risk and uncertainty have been modeled by probabilistic decision theories, fuzzy decision making theories attempt to deal with the vagueness or fuzziness inherent in subjective or imprecise determinations of preferences, constraints and goals. Tanaka[4] formulated a fuzzy decision problem and applied successfully to an investment problem. Dubois[2] discussed the use of fuzzy numbers in decision analysis. Multistage decision processes in a fuzzy environment was studied by Kacprzyk [3].

**Keywords:** Bayesian Decision Making, Fuzzy Decision Making, Utility And Optimum Decision, Fuzzy Environments, Fuzzy States Of Nature, Universe Of States, Universe Of Informations And ANGINA

## 1. INTRODUCTION

Decision making is a most important scientific, social and economic endeavour. The problem in making decisions under uncertainty is that the bulk of information we have about the possible outcome, about the value of information, about the way the conditions change with time, about the utility of each outcome-action pair and about our preferences for each action is typically vague, ambiguous and Fuzzy.

## 2. EXTENSION OF BAYESIAN DECISION MAKING TO FUZZY ENVIRONMENTS

The Bayesian decision method can be extended to the possibility that the states of nature are fuzzy and the decision maker's alternatives are fuzzy.

Let  $A_1, A_2, \dots, A_m$  be the different fuzzy alternatives which would be defined on a universe of discomse.

Let  $F_1, F_2, \dots, F_q$  be the different fuzzy  
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states of nature which would be defined on the universe of states.

$$S = \{s_1, s_2, \dots, s_n\}$$

Let  $\phi = \{M_1, M_2, \dots, M_g\}$  be the different fuzzy information which would be defined on the universe of information  $X = \{x_1, x_2, \dots, x_r\}$ . We assume that the fuzzy states of nature are orthogonal.

i.e.

$$\sum_{s=1}^q {}^{\mu} F_s (s_i) = 1, i=1, 2, 3, \dots, n.$$

Also we assume that the fuzzy information is orthogonal.

i.e.

$$\sum_{t=1}^g {}^{\mu} M_t (x_k) = 1, k = 1, 2, \dots, r.$$

Let  $u_{js}$  be the given utility value of fuzzy alternative  $A_j$  corresponding to the fuzzy state  $F_s$



Let  $p(x_k / s_i)$  be the given conditional probability of having information  $x_k$  given that the state is in  $s_i$ .

Let  $p(s_j)$  be the prior probabilities of the singleton state  $s_j$ .

Then the marginal probability of the information  $x_k$  is determined by

$$P(x_k) = \sum_{i=1}^n p(x_k / s_i) p(s_i) \quad \dots (1)$$

The expected utility of the fuzzy alternative  $A_j$  is

$$E(u_j) = \sum_{s=1}^q u_{js} p(F_s) \quad \dots (2)$$

where  $P(F_s) = \sum_{i=1}^n \mu_{F_s}(s_i) p(s_i) \quad \dots (3)$

and the maximum utility is

$$E(u^*) = \max E(u_j) \quad \dots (4)$$

Also we can derive the posterior probability of fuzzy state  $F_s$  given probabilistic information  $x_k$  as follows:

$$P(F_s | x_k) = \frac{\sum_{i=1}^n \mu_{F_s}(s_i) p(x_k / s_i) p(s_i)}{p(x_k)} \quad \dots (5)$$

The posterior probability of fuzzy state  $F_s$  given

fuzzy information  $M_t$  is computed as:

$$P(F_s | M_t) = \frac{\sum_{i=1}^n \sum_{k=1}^r \mu_{F_s}(s_i) \mu_{M_t}(x_k) p(x_k / s_i) p(s_i)}{\sum_{k=1}^r \mu_{M_t}(x_k) p(x_k)} \quad \dots (6)$$

Further the expected utility given probabilistic information  $x_k$  and that given fuzzy information  $M_t$  are computed as:

$$E(u_j | x_k) = \sum_{s=1}^q u_{js} p(F_s | x_k) \quad \dots (7)$$

$$E(u_j | M_t) = \sum_{s=1}^q u_{js} p(F_s | M_t) \quad \dots (8)$$

The maximum conditional expected utility for probabilistic information  $x_k$  and that for fuzzy information  $M_t$  are computed as:

$$E(u^*_{x_k}) = \max_j E(u_j | x_k)$$

$$E(u^*_{M_t}) = \max_j E(u_j | M_t) \quad \dots (9)$$

$$\dots (10)$$

Finally the unconditional expected utility for fuzzy states and probabilistic information and that for fuzzy states and fuzzy information are determined by:

$$E(u^*_x) = \sum_{k=1}^r E(u^*_{x_k}) p(x_k) \quad \dots (11)$$

$$E(u^*_\phi) = \sum_{t=1}^g E(u^*_{M_t}) p(M_t) \quad \dots (12)$$



$$\text{where } p(M_t) = \sum_{k=1}^r \mu_{Mt}(x_k) p(x_k) \dots (13)$$

Further the value of probabilistic information is determined by:

$$V(x) = E(u^*_x) - E(u^*) \dots (14)$$

and the value of fuzzy information is determined by:

$$V(\phi) = E(u^*_\phi) - E(u^*)$$

The decision in each case is taken bas ... (15) values computed so far.

### 3. FUZZY DECISION MAKING IN THE TREATMENT FOR ANGINA

Suppose a physician wants to decide the nature of treatment to be administered to a patient suffering from angina.

The different fuzzy alternatives may be taken as below[5,7]:

A1: Routine outpatient medical treatment

~

A2: Intensive inpatient medical treatment which

~ include treatment which include treatments like percutaneous Translumious coronary Angio plasty.

A3: Very Intensive inpatient medical treatment

~ which include treatments like coronary Artery Byepass Grafting.

The universe of states may taken as

S = {s1, s2, s3, s4, s5 }, where

s1 - Secondary angina

s2 - Stable angina

s3 - Unstable angina

s4 - infarction

s5 - post infarction angina

he different fuzzy states of nature may taken as

F1 - mild angina

~

F2 - moderate angina

~

F3 - severe angina

~

The universe of information[9] may be taken as

X = { x1, x2, x3, x4, x5 }

Where

x<sub>1</sub> = abnormal Hb%

x<sub>2</sub> = abnormal ECG chart finding

x<sub>3</sub> = abnormal Echo cardiogram finding

x<sub>4</sub> = abnormal Tread Mill Test finding

x<sub>5</sub> = abnormal angiogram finding

The different fuzzy information is

φ = { M<sub>1</sub>, M<sub>2</sub>, M<sub>3</sub> } where

~ ~ ~

M<sub>1</sub> = Poor information

~

M<sub>2</sub> = Moderate information

~

M<sub>3</sub> = Good information

~

Out of past experience and information[6,8], the prior probabilities of the various states are

p ( s<sub>1</sub> ) = .2, p ( s<sub>2</sub> ) = .3, p ( s<sub>3</sub> ) = .3, p ( s<sub>4</sub> ) = .1, p ( s<sub>5</sub> ) = .1

The utilities for fuzzy states and alternatives are computed as:



Table-1 Utilities for fuzzy and alternatives

	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
	~	~	~
A <sub>1</sub>	10	3	0
~			
A <sub>2</sub>	4	9	6
~			
A <sub>3</sub>	1	7	10
~			

The orthogonal fuzzy sets for fuzzy states of nature given as:

Table-2 orthogonal fuzzy sets for states

	s <sub>1</sub>	s <sub>2</sub>	s <sub>3</sub>	s <sub>4</sub>	s <sub>5</sub>
F <sub>1</sub>	1	5	0	0	0
~					
F <sub>2</sub>	2	5	1	25	0
~					
F <sub>3</sub>	0	0	0	75	1
~					

The orthogonal fuzzy sets for fuzzy information are given as:

Table-3 Orthogonal fuzzy sets for fuzzy information

	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>
M <sub>1</sub>	1	4	0	0	0
~					
M <sub>2</sub>	0	6	1	6	0
~					
M <sub>3</sub>	0	0	0	4	1
~					

The conditional probabilities p( x<sub>k</sub> / s<sub>j</sub> ) for uncertain information are given as:

Table-4 Conditional probabilities ( x<sub>k</sub>/ s<sub>j</sub> )

	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>
p ( x <sub>k</sub> / s <sub>1</sub> )	.44	.35	.17	.04	0
p ( x <sub>k</sub> / s <sub>2</sub> )	.06	.32	.16	.33	.13
p ( x <sub>k</sub> / s <sub>3</sub> )	.02	.13	.20	.33	.32
p ( x <sub>k</sub> / s <sub>4</sub> )	.03	.30	.10	.21	.36
p ( x <sub>k</sub> / s <sub>5</sub> )	0	.04	.17	.35	.44

Table -5 Utility values for crisp states

	s <sub>1</sub>	s <sub>2</sub>	s <sub>3</sub>	s <sub>4</sub>	s <sub>5</sub>
A <sub>1</sub>	10	8	6	2	0
~					
A <sub>2</sub>	4	6	9	6	4
~					
A <sub>3</sub>	1	2	6	8	10
~					

4. CASE STUDY

4.1 UTILITY AND OPTIMUM DECISION GIVEN NO INFORMATION

Here, we compute the prior probabilities for fuzzy states using equation (3).

They are:

P ( F<sub>1</sub> ) = .35, P ( F<sub>2</sub> ) = .475, P ( F<sub>3</sub> ) = .175

~ ~ ~

Also the expected utilities for the various alternatives are determined by

Using equation (2).

They are:

E ( u<sub>1</sub> ) = 4.925, E ( u<sub>2</sub> ) = 6.725, E ( u<sub>3</sub> ) = 5.425

The optimum expected utility of the fuzzy alternatives[10] for the cases of no information is

E ( u\* ) = max {4.925, 6.725, 5.425 } = 6.725

So the alternative A2 is the optimum choice.



Thus, in the case of no information, the physician prefers to administer the intensive inpatient treatment to the patient.

**4.2 UTILITY AND OPTIMUM DECISION GIVEN UNCERTAIN PROBABILISTIC INFORMATIONS**

In this case, the marginal probabilities of the probabilistic information  $x_k$  are computed by using equation(1). They are:

$$P ( X_1 ) = .115, P ( X_2 ) = .239, P ( X_3 ) = .169, P ( X_4 ) = .262, \text{ and } P ( X_5 ) = .215$$

The posterior probabilities of fuzzy sets  $F_s$  given

probabilistic information  $x_k$  are computed by using equation (5) and they are presented in Table 6.

Table-6 Posterior probabilities  $P ( F_s / X_k )$

	$F_1$	$F_2$	$F_3$
$X_1$	.843	.137	.020
$X_2$	.493	.395	.112
$X_3$	.343	.417	.240
$X_4$	.219	.587	.194
$X_5$	.091	.579	.330

Further the expected utilities for fuzzy alternatives given probabilistic information  $x_k$  are computed by using equation(7) and these expected utilities are presented in Table 7.

Table-7 Expected utilities for fuzzy alternatives with probabilistic information

	$A_1$	$A_2$	$A_3$
$X_1$	8.841	4.725	2.002
$X_2$	6.115	5.014	4.378
$X_3$	4.681	6.565	5.662
$X_4$	3.951	7.323	6.268
$X_5$	2.647	7.555	7.444

The optimum conditional expected utilities for probabilistic information  $x_k$  are determined by using equation(9). They are found to be

$$E( u^*x_k ) = \max_j E( u_j / x_k ) = \{ 8.841, 6.115, 6.565, 7.323, 7.555 \}$$

Hence the optimum choice is  $A_1$ ,

Thus, in the case of given probabilistic information, the physician prefers to routine inpatient medical treatment to the patient.

Further the unconditional expected utility for fuzzy states and probabilistic information is determined by using equation(11), as

$$E( u^*_x ) = \sum_{k=1}^r E( u^*x_k ) p( x_k ) = 7.130$$

And the value probabilistic information for fuzzy states is computed as

$$V(x) = E( u^*_x ) - E( u^* ) = 7.130 - 6.725 = .405$$

**4.3 UTILITY AND OPTIMUM DECISION GIVEN FUZZY INFORMATION**

The posterior probabilities for fuzzy information with fuzzy states are computed by using equation(6) and their values are presented in Table 8.

Table-8 Posterior probabilities  $P ( F_s / M_i )$

	$M_1$	$M_2$	$M_3$
$F_1$	.685	.354	.133
$F_2$	.254	.501	.582
$F_3$	.061	.151	.285



The expected utilities given fuzzy information  $M_t$  are obtained by Equation (8), ~ as Table 9.

Table-9 Expected utilities  $E(U_j/M_t)$

	$M_1$	$M_2$	$M_3$
	~	~	~
$A_1$	7.612	5.043	3.076
~			
$A_2$	5.392	6.831	7.480
~			
$A_3$	3.073	5.371	7.057
~			

The optimum conditional expected utilities for fuzzy information  $M_k$  are computed by

Equation (10), as:

$$E(u^* M_t) = \max_j E(u_j | M_t) = \max \{7.612, 6.831, 7.480\} = 7.612$$

Hence the optimum choice associated with value in this case is alternative  $A_1$ .

Thus, in the case of given fuzzy information, the physician prepares to give routine outpatient medical treatment.

Further the unconditional expected utility for fuzzy states and fuzzy information is obtained by using equation(12) and (13), as

$$E(u^*_\phi) = \sum_{t=1}^g E(u^* M_t) P(M_t) = 7.202$$

Finally the value of the fuzzy information is computed as

$$V(\phi) = E(u^*_\phi) - E(u^*) = 7.202 - 6.725 = .477$$

5. CONCLUSION

This approach is reliable and provides wide spread of information to help the physician in reaching a more logical conclusion for a more accurate diagnosis. The system has been successfully tested for disease diagnostics for number of combinations of healthy and diseased categories. With the help of the system, it is possible to know the degree of severity as well as the type of disease.

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