

SOME METHODS TO TREAT CAPACITY ALLOCATION PROBLEMS

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ABSTRACT

The Constraint Satisfaction Problems (CSP) are proven more and more promising to model and solve a large number of real problems. A lot of approaches using constraint reasoning have been proposed to solve search problems. Our contributions here consist to propose a modeling of Capacity Allocation Problem of an airport (CAP) in the form of a CSP. This modeling produces a model which we call CSPAC. Then, we generalize this model to the fixes of the airport and call the new model GCSPAC. Thereafter, we describe an approach combining the branch-and-bound algorithm and local search to solve both CSPAC and GCSPAC. This method allows assisting the airport managers to regulate the arrival/departure predicted demands and to optimize the utilization of the available capacities of a terminal. Finally, we expose some experimental results showing the utility of our different approaches.

Keywords: *Constraint Optimization, Modeling, Solving, Capacity Allocation Problems.*

1. INTRODUCTION

Capacity allocation problems are the core of many real-world planning problems. The first step to solve a capacity allocation problem consists in formulating and modeling it. Modeling is one of the important themes of Artificial Intelligence (AI). A good model can generally facilitate the problem-solving [12]. For our problem, different modeling techniques can be studied. Among these techniques one can find the constraint satisfaction problems [10].

The Constraint Satisfaction Problem (CSP) is proven more and more promising to model and solve a large number of real problems. A lot of approaches using constraint reasoning have been proposed to solve CSP. Formally, a CSP can be defined by the triplet (X, D, C) , where $X = \{X_1, \dots, X_n\}$, is the set of n variables; $D = \{D_1, \dots, D_n\}$, is the set of n domains; D_i is the domain of values of the variable X_i and $C = \{C_1, \dots, C_m\}$, is the set of m constraints of the problem, specifying compatible values or excluding incompatible values between variables. Solving a CSP consists of assigning a value to each variable such that all constraints are

satisfied. In general, the CSPs are solved by traditional methods combining a mechanism of search and a mechanism of reinforcement of consistencies with each node of search. Various real problems can be represented in the form of a CSP. In this paper, we model the Capacity Allocation Problem of an airport and also of the fixes of this airport in the form of a CSP and apply CSP techniques to solve them.

The present study has several objectives. The prime objective is to model the capacity allocation problem of an airport, on the one hand, and of its fixes, on the other hand, in the form of a CSP. For this purpose, it is enough to identify the variables, the domains of values which these variables can take and the constraints of the problem.

The second objective is to solve those problems by using the resolution algorithms designed specifically for the CSP. These CSP resolution algorithms include the Branch and Bound algorithm and its alternatives, as presented in [7], [9], [11] and [12].

The third objective is to propose optimization approaches in order to assist the airport managers to

regulate the arrival and departure demands and to efficiently use the available capacities of a terminal. These approaches are based on combining the branch and bound algorithm with the local search under time limit constraint to deal with capacity allocation problems of an airport and its fixes.

This paper's organization begins with section 2 which presents the airport capacity allocation problem. In section 3, we present a modeling of this problem in the form of a CSP which we called CSPAC. We generalize this model to several arrival/departure fixes of an airport in section 4. The general model is called GCSPAC. Thereafter, we propose an optimization approach for both CSPAC and GCSPAC and give two algorithms in section 5. In section 6, we expose some experimental results and conclude in section 7.

2. CAPACITY ALLOCATION PROBLEMS

In this paper, we define the airport capacity to be the maximum number of arrivals and departures that can be performed within a fixed time interval under given conditions. We note CT the total capacity of the airport. We define the fix capacity to be the maximum number of flights that can cross a fix in a fixed time interval under given conditions. Finally, the Airport capacity allocation problem consists in determining a balance between arrivals and departures, minimizing the total number of delayed flights for a given time period.

The airport capacity allocation (CAP) is a key problem in aerial traffic. It will be crucial because aerial traffic will greatly increase in the next years and its regulation will become more and more difficult, given the limited capacities both in airports and in aerial sectors. Thus, it is necessary to solve this problem, especially in the case of congestion. The congestion occurs at an airport when the request of the traffic exceeds the available capacity.

The capacity of the airports and the capacity of their fixes become more and more limited compared to the demands, and present real problems for the aerial transportation system in general and more particularly for airports managers. Various solutions can be used to remedy this problem including the construction of new airports, the extension of existing runway systems, the application of new technologies to increase capacity, and optimization of the use of existing capacity. The reader can be referred to [7] for some possible measures to increase airport capacity. Some aspects of the problem are presented in [3-6].

In addition, aerial traffic regulation should be optimized and automated, especially when treating unforeseen events. In [8], we proposed a modeling and a resolution of the aerial conflicts problems by techniques of constraints network (CSP). This modeling and automatic resolution can be possibly introduced into project (FREER) [1] as developed by Eurocontrol following the concept "Free Flight" [2].

3. CSPAC: A CSP MODEL FOR THE AIRPORT CAPACITY ALLOCATION PROBLEM

3.1. Problem Formulation

Figure 1 presents a simplified scheme of an airport comprising the arrival and departure demands during a time interval 'i' ('i' is a period of 15 minutes). The arrival demands are at the point 'A'. The departure demands are at the point 'D'. The point 'T' represents a Terminal.

The problem consists in satisfying the arrival and departure demands, within the limit of the total capacity of the terminal during the time interval 'i'. We will note this limit as CT_i . CT_i is the sum of Pv_i and of Poc_i .

$$CT_i = Pv_i + Poc_i \quad (1)$$

Pv_i is the number of empty places at the Terminal and Poc_i is the number of occupied places at the Terminal. If Xaa_i represents the entries (the actual planes which arrive) and Yad_i , the throughput (actual planes which leave) during a time interval 'i', then the number of empty places for the time interval 'i+1' becomes:

$$Pv_{i+1} = Pv_i + Yad_i - Xaa_i \quad (2)$$

In the same way, the number of occupied places for the time interval 'i+1' becomes:

$$Poc_{i+1} = Poc_i + Xaa_i - Yad_i \quad (3)$$

For the sake of simplification and without loss of generality, let us admit in the sequel that CT_i is a constant independent of time. We will note it as CT such that:

$$CT = Pv_i + Poc_i = Pv_{i+1} + Poc_{i+1} \quad (4)$$

3.2. Problem Modeling

We propose to model this problem of the arrivals/departures at an airport (figure 1) in the form of a CSP $CSPAC_i = (X_i, D_i, C_i)$ during a time interval 'i', where:

- $X_i = \{Xa_i, Xaai, Yd_i, Yadi, Ca_i, Cd_i, Qa_i, Qd_i, Xta_i, Ytd_i, Pv_i, Poc_i\}$ is the set of variables such that:

X_{ai} is the number of predicted arrival demands at the point 'A' (Figure 1); X_{aa_i} is the number of actual arrivals at the point 'A'; Y_{di} is the number of predicted departure demands at the point 'D' (Figure 1); Y_{ad_i} is the number of actual departures at the point 'D'; C_{ai} is the arrival capacity at 'A'; C_{di} is the departure capacity at 'D'; Q_{ai} is the number of arrivals delayed at the point 'A'; Q_{di} is the number of departures delayed at the point 'D'; X_{ta_i} is the total number of arrival demands for the time interval 'i'; Y_{td_i} is the total number of departure demands for the time interval 'i'; P_{vi} and P_{oc_i} are respectively the set of empty and occupied places of the terminal at the point 'T' of Figure 1 during 'i'.

$D_i = \{DX_{ai}, DX_{aa_i}, DY_{di}, DY_{ad_i}, DC_{ai}, DC_{di}, DQ_{ai}, DQ_{di}, DX_{ta_i}, DY_{td_i}, DP_{vi}, DP_{oc_i}\}$ is the set of variable domains. We consider that all the domains of these variables are a set of natural integers such as $\{0, 1, 2, \dots, CT\}$, except $D(C_{ai}, C_{di})$ which is the set of pairs of natural integers. We assume that all domains are independent of i, i.e., $D1=D2=D3\dots$

Note that the values of C_{ai} and C_{di} are dictated by the airport managers according to the predicted arrival and departure demands (see Table 1) to minimize the total delay.

The values of C_{ai} and C_{di} are interdependent and generally should be chosen from a set of pairs such as $D(C_{ai}, C_{di}) = \{(18, 29), (24, 24), (26, 19), (28, 15), (17, 30), (20, 27)\}$ (see Table 2).

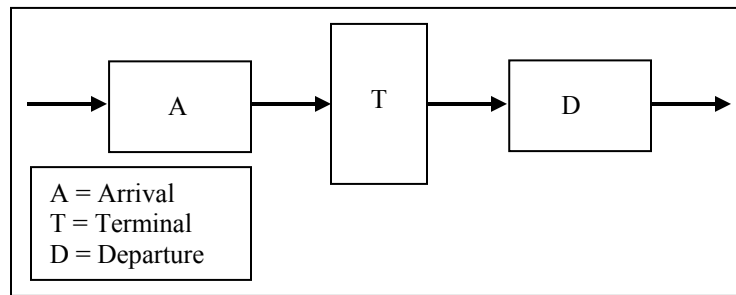


Figure 1. Example of a simplified scheme of an airport

Table 1. Example of predicted number of arriving flights (X_{ai}) and predicted number of departing flights (Y_{di}) per each $i=15$ minute time interval of a 3 hour period (extracted from [3-6]).

i	1	2	3	4	5	6	7	8	9	10	11	12	TT
X_{ai}	26	38	42	29	06	13	14	20	40	25	13	12	278
Y_{di}	36	32	09	15	07	10	17	33	34	22	13	01	229

Table 2. Example of pairs of capacities dictated by airport managers to use per each $i=15$ minute time interval of a 3 hour period. C_{ai} is the arrival capacity, C_{di} is the departure capacity (extracted from [3-6]).

C_{ai}	18	24	26	28	17	20
C_{di}	29	24	19	15	30	27

Note that for each value of 'i', we add the delay $Q_{a_{i-1}}$ recorded in the previous interval 'i-1' to the predicted arrival demands X_{ai} . We have $X_{ta_i} = X_{ai} + Q_{a_{i-1}}$ and $X_{ta1} = X_{a1}$ since $Q_{a0} = 0$. In the same way, for each value of 'i', we add the delay $Q_{d_{i-1}}$ recorded in the previous interval 'i-1' to the predicted departure demands Y_{di} . We have $Y_{td_i} = Y_{di} + Q_{d_{i-1}}$ with $Q_{d0} = 0$.

The first pair in the set means that if $C_{ai} = 18$ then $C_{di} = 29$ and vice versa. The meaning of the other pairs is similar. The purpose of our approach is to assist the managers to efficiently determine the optimal values or accepted values of C_{ai} and C_{di} .

- C_i is the set of constraints of the problem.

We can formulate them as follows:

$$0 \leq Pv_i \leq CT \quad (5)$$

$$0 \leq Poc_i \leq CT \quad (6)$$

$$0 \leq Pv_i + Poc_i \leq CT \quad (7)$$

$$0 \leq Pv_{i+1} + Poc_{i+1} \leq CT \quad (8)$$

$$0 \leq Xaa_i \leq Pv_i \quad (9)$$

$$0 \leq Yad_i \leq Poc_i \quad (10)$$

$$0 \leq Xaa_i \leq Xa_i \quad (11)$$

$$0 \leq Yad_i \leq Yd_i \quad (12)$$

$$0 \leq Xaa_i + Yad_i \leq CT \quad (13)$$

$$Pv_{i+1} = Pv_i + Yad_i - Xaa_i \quad (14)$$

$$Poc_{i+1} = Poc_i + Xaa_i - Yad_i \quad (15)$$

$$Qa_i = Xa_i - Xaa_i + Qa_{i-1} \quad (16)$$

$$Qd_i = Yd_i - Yad_i + Qd_{i-1} \quad (17)$$

$$[Xaa_i + Yad_i] \leq [Xa_i + Yd_i] \quad (18)$$

Qa_{i-1} (in Relation (16)) and Qd_{i-1} (in Relation (17)) are respectively the number of arrival and departure planes delayed during the time interval 'i-1'. These two numbers should be added respectively to the arrival and departure predicted demands during the time interval 'i'. So the total number of predicted arrival demands during 'i' is the sum of demands actually expressed Xa_i during 'i' increased by the number of planes Qa_{i-1} which were waiting in the queue due to the fact that they were not been served during 'i-1'. Similarly, the total number of predicted departure demand during 'i' is the sum of demands really expressed Yd_i during 'i' increased by the number of planes Qd_{i-1} which were waiting in the queue due to the fact that they were not been served during the time interval 'i-1'. The planes which were waiting in the queue during the time interval 'i-1' are served in priority during the time interval 'i'.

Relations (5)-(8) are evident and easy to understand. Relations (9) and (10) are explained above. Relations (14) and (15) are, respectively, the same as relations (2) and (3). Relations (11) and (12) mean that at each arrival/departure point, the number of flights actually served is less than or equal the number predicted to arrive/to leave at the same point.

Relation (13) means that the total number of flights actually served over the arrival and departure points must not exceed the total capacity of the terminal. Relations (16) and (17) allow computing the delay at each arrival/departure point. Relation (18) means that only a part of the predicted demands is served. Other flights are delayed.

The problem consists in assigning a value to each variable by respecting the constraints described above. Thus, our constraint network (CSP) is now well identified. In particular, the variables, the domains of value which these variables can take and the constraints of the problem are explicitly defined. Now it remains to apply one of the CSP resolution approaches to solve the capacity allocation problem. We think in particular to use the Branch and Bound algorithm and its alternatives, as presented in [7], [11] and [12].

4. GCSPAC: A GENERALIZED CSPAC FOR THE FIXES OF AN AIRPORT

4.1. Problem Presentation

In the previous section, we modeled the capacity allocation problem of an airport in form of a CSP called CSPAC_i. We considered that there was only one arrival way and only one departure way, this, because, the airport managers deliver the capacities only in form of pairs (arrival-capacity, departure-capacity). However, there are really several arrival ways (called arrival "fixes") at the point 'A' which will be noted now 'AF' (see Figure 2). In the same way, at the point 'D' which will be noted 'DF', there are several departure ways (called departure "fixes"). Let us note naf_i the number of arrival 'fixes' during 'i' and ndf_i the number of departure 'fixes' during 'i'.

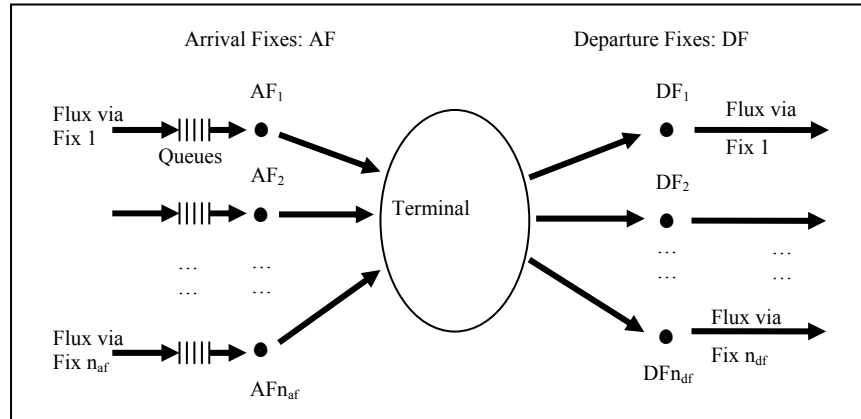


Figure 2. Example of an airport system (inspired from [3-6]).

Thus, if the airport admits n_{af} ($n_{af} > 1$) arrival fixes and n_{df} ($n_{df} > 1$) departure fixes, then it is necessary to distribute these pairs of capacities over all the arrival/departure fixes.

In the sequel, we will consider that there is 'j' arrival fixes AF_{aj} with $j = 1, \dots, n_{af}$ and 'k' departure fixes DF_{dk} with $k = 1, \dots, n_{df}$. Each fix 'j' or 'k' admits its own capacity noted CF_{ji} or CF_{ki} during the same time interval 'i'.

The arrival and departure fixes have limited capacities which show the maximum number of flights that can cross a fix in a 15-minute interval (or other interval) under given conditions. These capacities are generally variable and interdependent.

The traffic demands for an airport and for the fixes of this airport are given by the predicted number of arriving and departing flights per each 15 minute time interval of a considered period (see Table 1).

4.2. Problem Modeling

We model the capacity allocation problem of the fixes of an airport (Figure 2) in form of a CSP $GCSPAC_i = (X_i, D_i, C_i)$ during a time interval 'i', where:

- $X_i = \{X_{a_i}, X_{aa_i}, Y_{d_i}, Y_{ad_i}, Q_{a_i}, Q_{d_i}, C_{a_i}, C_{d_i}, X_{1af_i}, \dots, X_{naf_i}, X_{1aaf_i}, \dots, X_{naaf_i}, Y_{1df_i}, \dots, Y_{ndf_i}, Y_{1adf_i}, \dots, Y_{nadf_i}, C_{1af_i}, \dots, C_{naf_i}, C_{1df_i}, \dots, C_{ndf_i}, Q_{1af_i}, \dots, Q_{naf_i}, Q_{1df_i}, \dots, Q_{ndf_i}, X_{ta_i}, Y_{td_i}, P_{v_i}, P_{oc_i}\}$ is the set of variables.

$X_{a_i}, X_{aa_i}, Y_{d_i}, Y_{ad_i}, Q_{a_i}, Q_{d_i}, P_{v_i}, P_{oc_i}$ are the same variables than those described above in the $CSPAC_i$.

X_{jaf_i} is the number of arrival demands at fix 'j' with $j = 1, \dots, n_{af}$. X_{jaaf_i} is the number of actual

arrivals at fix 'j'. Y_{kdf_i} is the number of departure demands at fix 'k' with $k = 1, \dots, n_{df}$. Y_{kadf_i} is the number of actual departures at fix 'k'. Q_{jaf_i} is the number of arrivals delayed at fix 'j'. Q_{kdf_i} is the number of departures delayed at fix 'k'. C_{jaf_i} is the arrival capacity at fix 'j'. C_{kdf_i} is the departure capacity at fix 'k'.

- $D_i = \{DX_{a_i}, DX_{aa_i}, DY_{d_i}, DY_{ad_i}, D(C_{a_i}, C_{d_i}), DQ_{a_i}, DQ_{d_i}, DX_{1af_i}, \dots, DX_{naf_i}, DX_{1aaf_i}, \dots, DX_{naaf_i}, DY_{1df_i}, \dots, DY_{ndf_i}, DY_{1adf_i}, \dots, DY_{nadf_i}, DC_{1af_i}, \dots, DC_{naf_i}, DC_{1df_i}, \dots, DC_{ndf_i}, DQ_{1af_i}, \dots, DQ_{naf_i}, DQ_{1df_i}, \dots, DQ_{ndf_i}, DX_{td_i}, DY_{td_i}\}$ is the set of variable domains.

We consider that all the domains of these variables are a set of natural integers such as $\{0, 1, 2, \dots, CT\}$, except $D(C_{a_i}, C_{d_i})$ which is the set of pairs of natural integers.

We assume that all domains are independent of i, i.e., $D_1=D_2=D_3\dots$

- C_i is the set of constraints of the problem. We can formulate them as follows:

$$0 \leq X_{jaaf_i} \leq X_{jaf_i}, \forall j \in \{1, \dots, n_{af}\} \quad (19)$$

$$0 \leq Y_{kadf_i} \leq Y_{kdf_i}, \forall k \in \{1, \dots, n_{df}\} \quad (20)$$

$$0 \leq \sum_{j=1}^{n_{af}} X_{jaaf_i} + \sum_{k=1}^{n_{df}} Y_{kadf_i} \leq CT \quad (21)$$

$$X_{a_i} = \sum_{j=1}^{n_{af}} X_{jaf_i} \text{ and } Y_{d_i} = \sum_{k=1}^{n_{df}} Y_{kdf_i} \quad (22)$$

$$X_{ta_i} = \sum_{j=1}^{n_{af}} X_{tjaf_i} \text{ and } Y_{td_i} = \sum_{k=1}^{n_{df}} Y_{tkdf_i} \quad (23)$$



$$Xaa_i = \sum_{j=1}^{naf} Xj_{aafi} \text{ and } Yadi = \sum_{k=1}^{ndf} Yk_{adfi} \quad (24)$$

$$Ca_i = \sum_{j=1}^{naf} Cj_{aafi} \text{ and } Cdi = \sum_{k=1}^{ndf} Ck_{adfi} \quad (25)$$

$$Qa_i = \sum_{j=1}^{naf} Qj_{aafi} \text{ and } Qdi = \sum_{k=1}^{ndf} Qk_{adfi} \quad (26)$$

$$Xtj_{afi} = Xj_{aafi} + Qj_{aafi}, \forall j \in \{1, \dots, naf\} \quad (27)$$

$$Ytk_{dfi} = Yk_{adfi} + Qk_{adfi}, \forall k \in \{1, \dots, ndf\} \quad (28)$$

$$Xaa_i \leq Ca_i \quad (29)$$

$$Yad_i \leq Cdi \quad (30)$$

$$Xtj_{afi} \leq Cj_{aafi}, \forall j \in \{1, \dots, naf\} \quad (31)$$

$$Ytk_{dfi} \leq Ck_{adfi}, \forall k \in \{1, \dots, ndf\} \quad (32)$$

$$Qj_{aafi} = Xtj_{afi} - Xj_{aafi}, \forall j \in \{1, \dots, naf\} \quad (33)$$

$$Qk_{adfi} = Ytk_{dfi} - Yk_{adfi}, \forall k \in \{1, \dots, ndf\} \quad (34)$$

$$Xaa_i + Yadi \leq Xai + Ydi \quad (35)$$

Relations (19) and (20) mean that at each arrival/departure fix (j or k), the number of flights actually served is less than or equal to the number predicted to arrive/to leave at the same fix. Relation (21) means that the total number of flights actually served over all the arrival and departure fixes must not exceed the total capacity of the terminal.

Relations (22) and (23) mean that the sum of planes predicted to arrive/leave at all the arrival/departure fixes is equal to the total number predicted to arrive/leave at the airport. Relation (23) takes in account the number of flights delayed at the previous time interval.

Relation (24) means that the sum of planes actually arrived (respectively left) at all arrival (respectively departure) fixes is equal to the total number actually arrived (respectively left) at the airport. Relation (25) means that the sum of capacities at all the arrival/departure fixes is equal to the total arrival/departure capacity of the airport.

Relation (26) means that the sum of arrival/departure planes delayed at all the arrival/departure fixes is equal to the total number of planes delayed at the airport. Relations (27) and (28) are described above. Relations (29), (30), (31) and (32) mean that the numbers of served

arrival/departure flights at each fix must be not larger than the capacity allocated to these fixes.

Relations (33) and (34) allow computing the delay at each arrival/departure fix. Relation (35) means that only a part of the predicted demands is served. Other flights are delayed.

Now our constraint network GCSPAC for Fixes is well identified. In order to optimize and automate the aerial traffic regulation, we propose in the next section an optimization model for the best use of airport and fixes capacities.

5. OPTIMIZATION OF THE UTILIZATION OF EXISTING CAPACITY

We seek to optimize the utilization of existing capacity of an airport and of its fixes, during the congestion periods. For that, we must find the best allocation of capacities between arrivals and departures which can satisfy the predicted traffic demand for a time period and can also minimize the delay.

5.1. Optimization of the use of existing airport capacity

Let us note that in the case of congestion during a time interval 'i' the sum of numbers of actual arrivals (Xaa_i) and of actual departures (Yad_i) is lower than the sum of numbers of arrival demands (Xa_i) and of departure demands (Yd_i). In other words, only a part of the demands is served (equation (18)). When solving a CSP, a number of questions should be asked. For example:

- Is there any solution for this problem? In other words, is it the problem satisfiable?

- How can we find a solution?

- How can we find all solutions?

- ...

For our CSPAC, we are interested in the following questions:

- Is it possible to serve all the requests without any delay?

$$\sum_{i=1}^N [Qa_i + Qd_i] = 0?$$

Where Qa_i and Qd_i are respectively the number of arrival and departure planes delayed during the time interval 'i' and N is the number of intervals constituting the given time period.

- Is it possible to serve all the requests with the given delay 'r' (r = 1, 2, ...):

$$\sum_{i=1}^N [Q_{ai} + Q_{di}] = r ?$$

- Is it possible to serve all the requests without exceeding a certain given delay 'r_{max}'?

$$\sum_{i=1}^N [Q_{ai} + Q_{di}] \leq r_{max} ?$$

Our objective is to minimize the sum of the numbers of arrival planes delayed $\sum_{i=1}^N [Q_{ai}]$ and to minimize the sum of the numbers of departure planes delayed $\sum_{i=1}^N [Q_{di}]$ during a given time period (15*N minutes). In other words, we seek to minimize:

$$\sum_{i=1}^N [Q_{ai} + Q_{di}] \tag{36}$$

subject to constraints (5)-(18).

We recall that at a time interval i:

$$Q_{ai} = X_{ai} - X_{aai}$$

$$Q_{di} = Y_{di} - Y_{adi}$$

5.2. Optimization of the use of existing capacity of the fixes of an airport

Given values of (X_{ai}, Y_{di}) (arrival and departure demands for a time interval 'i' and a number of possible values of (C_{ai}, C_{di}), the problem consists in assigning a value to each variable by respecting the constraints described above. The problem GCSPAC is a problem of decision. It is also an optimization problem. The question is to determine the real repartition of the arrival and departure demands among the fixes, and the real repartition of arrival and departure capacities among the fixes by respecting the constraints described above. Thereafter, it suffices to apply one of the CSP reasoning procedures (Branch and Bound, Local Search, Backtracking, etc.) to solve the capacity allocation problem.

Unfortunately, in many cases, only a part of predicted demands, arrivals or departures, can be served. In order to deal with this problem, we propose below an optimization method able to give a best allocation of capacities of an airport and of its fixes between arrivals and departures. This

optimization method under constraints also takes in account the time limit constraint, i.e., it is able to give an acceptable solution within a given time limit.

In other words, for a period formed by N intervals we seek to minimize

$$\varphi = \sum_{i=1}^N \left(\sum_{j=1}^{naf} Q_{jafi} + \sum_{k=1}^{ndf} Q_{kdfi} \right) \tag{37}$$

subject to constraints (19)-(35) where

$$\begin{aligned} Q_{jafi} &= X_{jafi} - X_{jaafi} + Q_{jafi-1}, \\ \forall j \in \{1, \dots, n_{afi}\} & \quad \text{and} \\ Q_{kdfi} &= Y_{kdfi} - Y_{kadfi} + Q_{kdfi-1}, \\ \forall k \in \{1, \dots, n_{dfi}\} & \end{aligned}$$

These two relations numbered (38) and (39) are, respectively, the number of planes arriving and leaving which are delayed during a time interval 'i=15 min'. Note that φ is the sum of flights delayed in all intervals. We equivalently have:

$$\varphi = \sum_{i=1}^N \left(\sum_{j=1}^{naf} (X_{jafi} - X_{jaafi}) + \sum_{k=1}^{ndf} (Y_{kdfi} - Y_{kadfi}) \right) \tag{40}$$

Our goal is to minimize φ. For this goal, we propose an algorithm in the next section.

5.3. BLSTL: An algorithm for capacity allocation problems

We first describe an algorithm to distribute a given pair of arrival and departure capacities (C_{ai}, C_{di}) among all fixes for time interval i, in order to serve the flight demands in interval i.

Algorithm 1:

1. We first distribute the predicted arrival demands (i.e. the values of X_{ai}) of an airport for its arrival fixes and the predicted departure demands (i.e. the values of Y_{di}) for its departure fixes. This distribution takes into account the flights delayed at each fix in interval i-1, so that all fixes have almost the same number of flight demands.
2. Secondly, we affect (C_{ai}, C_{di}) to predicted arrival and departure demands (X_{ai}, Y_{di}) to obtain the actual arrival and departure demands (X_{aai}, Y_{adi}) of the airport in interval i.



3. Thirdly, we distribute (X_{aa_i}, Y_{ad_i}) given above respectively to arrival fixes $X_{1aaf_i}, \dots, X_{naaf_i}$ and to the departure fixes $Y_{1adf_i}, \dots, Y_{nadf_i}$. This distribution takes into account the number arrival/departure demands at each fix, so that there are almost the same numbers of delayed flights at each fix, and all constraints are satisfied.

4. Finally, we compute the number of delayed flights $(\sum_{j=1}^{naf} Q_{jaf_i} + \sum_{k=1}^{nafi} Q_{kafi})$ for interval i and then the sum S of number of delayed flights for intervals $1, 2, \dots, i$.

problems use a branch and bound procedure. The capacity allocation is done in the order of time intervals. In other words, we allocate capacities for interval i , only after we have allocated capacities for all intervals $j < i$.

In addition, the resolution and the optimization of a CSPAC problem is a particular case of GCSPAC. The difference is that in the CSPAC there is only one arrival fix and only one departure fix. So, for the CSPAC there is no need to use Algorithm 1, i.e. Algorithm 2 is applied directly.

The branch and bound algorithm consists of recursively testing all possible pairs of arrival departure capacities in $D(C_{a_i}, C_{d_i})$ in order to find the minimum number of delayed planes.

Obviously, after the execution, I is the solution for the capacity allocation problem, giving the capacity allocation for each interval. We put a time cutoff T_{fixed} for the branch and bound algorithm. If the execution of the algorithm reaches T_{fixed} , we stop it and run a local search algorithm to find an acceptable solution satisfying all constraints.

Algorithm 1: Distribution Algorithm

```

Procedure BLSTL( $I, (X_{a_i}, Y_{d_i})_i, a$ )
 $I = I \cup \{(X_{a_i}, Y_{d_i})_i, a\}$ 
If ( $I$  checks all the constraints) AND
    ( $\varphi = \min$ ) AND ( $T_{execution} < T_{fixed}$ ) then
    If ( $(X_{a_i}, Y_{d_i})_i = (X_{a_i}, Y_{d_i})_n$ ) then
         $I$  is an optimal solution
    else
        For all  $b \in (D_{X_{a_i}}, D_{Y_{d_i}})$  do
            BLSTL( $I, (X_{a_i}, Y_{d_i})_{i+1}, b$ )
        end If
    else
        If ( $T_{execution} \geq T_{fixed}$ ) then
            Perform Local Search Algorithm
        end If
    end If
End
    
```

6.1. Experimental Results for Airport

Table 3 presents an example of a CSPAC problem. We report in this table the predicted arrival and departure demands and available capacities of an airport as presented in Tables 1 and 2.

The first column of Table 3 presents the time intervals of 15 minutes. Columns 2 and 3 represent, respectively, the arrival and departure demands on these same time intervals. Columns 4 and 5 present, respectively, the arrival and departure flows that have been actually served. Columns 6 and 7 present the demands which were not served (these demands which were not served are queues which must be taken into account during the next time interval). Columns 8 and 9 are, respectively, the available arrival and departure capacities at the terminal. For our example, we notice that the period is 3 hours and that the number of intervals N forming this period is equal to 12. The predicted arrival/departure demands are those given in Table 1 and the available arrival/departure capacity pairs are values given in Table 2.

Algorithm 2: Algorithm BLSTL

We called this algorithm BLSTL for (Branch and Bound + Local Search + Time Limit constraint). Note that I is the instantiation for intervals $1, 2, \dots, i-1$; a and b are values in $(D_{X_{a_i}}, D_{Y_{d_i}})$; $(X_{a_i}, Y_{d_i})_i$ is the i^{th} variable; $(X_{a_i}, Y_{d_i})_n$ is the last variable and \min is the best number of delayed flights found so far.

The BLSTL algorithm consists in recursively testing all possible pairs of arrival and departure capacities in $(D_{X_{a_i}}, D_{Y_{d_i}})$ dictated by the airport manager to find the minimum delay.

6. IMPLEMENTATION AND EXPERIMENTAL RESULTS

The Arrival/Departure predicted demands are those given in Table 1 and the available capacities values (domain) are given in Table 2. Note that the instances are extracted from [3-6]. The resolution and the optimization of a CSPAC and of GCSPAC

The resolution of the CSPAC consists in selecting and assigning values C_{a_i} (column 8) and C_{d_i} (column 9) respectively to variables X_{a_i} (column 4) and Y_{d_i} (column 5) so that constraints



(5)-(18) are all satisfied and minimized.

$$\sum_{i=1}^N [Q_{ai} + Q_{di}]$$

is

As mentioned previously, our method gives optimal solutions, but it takes enough time before to find it, especially when the number of time intervals is greater than or equal to 17.

Table 3 shows a solution to the problem. The number of arrival delays is 143 and the number of departure delays is 77. The total number delayed is 220 (see Table 4). It is an optimal value.

Thus, local search can be used for example when the number of time slots becomes greater than or equal to 17. It can also be used in case of unexpected events or during an emergency. For $i = 16$, the execution time is approximately 2 minutes. For $i = 12$, it is about 60 seconds and for $i = 28$, it is about an hour.

We present another example in Tables 4, 5 and 6. In this example, the period is 7 hours and the number N of intervals (of 15 minutes) forming this period is equal to 28. The description of this table is similar to that given for Table 3. The available arrival/departure capacity pairs delivered by airport managers for this period are given in Table 4. They are values belonging in domain $D(C_{ai}, C_{di})$ where $D(C_{ai}, C_{di}) = \{(07, 14), (10, 12), (13, 10), (14, 08)\}$.

We compared our approach to other methods proposed in the literature (especially [3-6]). The results for various instances of the problem show that our approach provides solutions at least as good as those found by other techniques. In our knowledge, the authors in [3-6] do not show how they proceed and do not describe any algorithm and do not give any information on the execution time.

The arrival/departure predicted demands are given in Table 5 and a solution of this problem is given in Table 6.

We implemented this method with Java (JDK1.5) on an Athlon 1.67GHz with 512MB of RAM. We performed preliminary experiments of the proposed approach and the results obtained for the various instances of the problem are optimal.

Table 1. Example of predicted number of arriving flights (X_{ai}) and predicted number of departing flights (Y_{di}) per each $i=15$ minute time interval of a 3 hour period.

i	1	2	3	4	5	6	7	8	9	10	11	12	TT
X_{ai}	26	38	42	29	06	13	14	20	40	25	13	12	278
Y_{di}	36	32	09	15	07	10	17	33	34	22	13	01	229

Table 2. Example of pairs of capacities dictated by airport managers to use per each $i=15$ minute time interval of a 3 hour period. C_{ai} is the arrival capacity, C_{di} is the departure capacity.

C_{ai}	18	24	26	28	17	20
C_{di}	29	24	19	15	30	27

Table 3: A Solution of the CSPAC Problem (Airport capacity allocation problem).

1	2	3	4	5	6	7	8	9
i	Xa_i	Yd_i	$Xaai$	$Yadi$	Qa_i	Qd_i	Ca_i	Cd_i
1	26	36	24	24	2	12	24	24
2	38	32	24	24	16	20	24	24
3	42	09	24	24	34	5	24	24
4	29	15	26	19	37	1	26	19
5	06	07	28	15	15	0	28	15
6	13	10	28	15	0	0	28	15
7	14	17	17	30	0	0	17	30
8	20	33	20	27	0	6	20	27
9	40	34	24	24	16	16	24	24
10	25	22	24	24	17	14	24	24
11	13	13	24	24	6	3	24	24
12	12	01	18	29	0	0	18	29
ToT	278	229			143	77		

Table 4. Capacities delivered by airport managers.

8	9
Ca_i	Cd_i
07	14
10	12
13	10
14	08



Table 5. Arrival and Departure demands. Table 6. A solution of the CSPAC presented in table 5.

1	2	3
<i>i</i>	<i>Xa_i</i>	<i>Yd_i</i>
1	17	09
2	09	10
3	09	03
4	14	14
5	15	11
6	10	15
7	09	10
8	20	12
9	10	09
10	12	08
11	06	14
12	10	14
13	09	08
14	11	13
15	09	16
16	15	08
17	06	09
18	15	09
19	08	12
20	04	10
21	13	05
22	07	11
23	10	13
24	16	12
25	05	12
26	06	09
27	05	11
28	11	06
ToT	291	293

4	5	6	7	8	9
<i>Xa_i</i>	<i>Yd_i</i>	<i>Qa_i</i>	<i>Qd_i</i>	<i>Ca_i</i>	<i>Cd_i</i>
13	10	04	00	13	10
13	10	00	00	13	10
14	08	00	00	14	08
13	10	01	04	13	10
13	10	03	05	13	10
13	10	00	10	13	10
07	14	02	06	07	14
13	10	09	08	13	10
13	10	06	07	13	10
13	10	05	05	13	10
10	12	01	07	10	12
10	12	01	09	10	12
10	12	00	05	10	12
10	12	01	06	10	12
10	12	00	10	10	12
13	10	02	08	13	10
07	14	01	03	07	14
13	10	03	02	13	10
10	12	01	02	10	12
07	14	00	00	07	14
14	08	00	00	14	08
07	14	00	00	07	14
10	12	00	01	10	12
13	10	03	03	13	10
07	14	01	01	07	14
07	14	00	00	07	14
07	14	00	00	07	14
14	08	00	00	14	08
		44	102		

6.2. Experimental Results for Fixes of Airport

Given a time period composed of a number of time intervals, the predicted arrival and departure demands in each of these intervals, and a set of compatible distributions of airport capacity between arrival and departure flights, our goal is to determine the real distribution of the demands among fixes, and the real distribution of arrival and departure capacities among fixes, according to the actual demands of each fix.

We give, below, an example to illustrate our approach, in which there are 4 arrival fixes noted af1, af2, af3 and af4 and 4 departure fixes noted df1, df2, df3 and df4 at the airport (Figure 2). The period is 3 hours and the number of intervals N forming this period is equal to 12. The predicted arrival/departure demands and the available arrival/departure capacity pairs are the same as those given in the preceding sub-section. We give a solution found using our approach in Table 7.

Column 0 shows the time intervals of 15 minutes. For each interval, column 1 shows the number of predicted arrival/departure demands X_a/Y_d of the airport. Columns 2, 3, 4 and 5 present respectively arrival/departure flows actually served (or number of planes actually arrived/left) X_1/Y_1 , X_2/Y_2 , X_3/Y_3 and X_4/Y_4 at the fixes af1/df1, af2/df2, af3/df3 and af4/df4. Columns 6, 7, 8 and 9 present respectively the arrival/departure demands which were not served (queues or delays) Q_{1a}/Q_{1d} , Q_{2a}/Q_{2d} , Q_{3a}/Q_{3d} and Q_{4a}/Q_{4d} at the arrival/departure fixes af1/df1, af2/df2, af3/df3 and af4/df4. The arrival/departure demands which were not served are queues which must be taken into account in the next time interval.

The resolution of the problem proceeds as follows. We first distribute the values 26/36 (column 1) of the predicted arrival/departure demands of the first interval to the arrival/departure fixes (we do not show explicitly this distribution because it is easy to understand). The optimal capacity allocation which we found for interval $i=1$ (for (Ca_1, Cd_1)), using our approach, is 24/24 (column 10). So the number of actual arrivals/departure is 24/24 in interval $i=1$. This pair (24/24) is distributed among all fixes (X_1/Y_1 , X_2/Y_2 , X_3/Y_3 and X_4/Y_4) (columns 2, 3, 4 and 5). The total number of delayed flights is 14 (i.e. 00/00 + 00/00 + 00/03 + 02/09 for, respectively, Q_{1a}/Q_{1d} , Q_{2a}/Q_{2d} , Q_{3a}/Q_{3d} and Q_{4a}/Q_{4d}) (columns 6, 7, 8 and 9) in interval $i=1$. We verify that all constraints are satisfied. We then distribute the values 38/32 of the predicted arrival/departure demands (column 1)

of the second interval $i=2$ to the arrival/departure fixes. This distribution takes into account the number of delayed flights in fixes Q_{1a}/Q_{1d} , Q_{2a}/Q_{2d} , Q_{3a}/Q_{3d} and Q_{4a}/Q_{4d} (columns 6, 7, 8 and 9) in interval $i=1$.

The optimal capacity allocation Ca_2/Cd_2 for interval $i=2$ that we found after executing branch and bound is also 24/24, so the number of actual arrivals/departures is 24/24 in interval 2, which is distributed among all fixes. The number of delayed flights is 36 in interval 2. The total number of delayed flights until interval 2 is then $14+36=50$ (see Table 7).

We proceed in the same manner for the other intervals. The total number of planes delayed is 220. It is an optimal value. Note that the distribution of predicted demands and capacity allocation Ca/Cd among the fixes is not deterministic. We always find optimal solution if the distribution respect the balance between the fixes. In other words, in any interval we should not have a case where a fix has available capacity not used and another fix has demands not served.

Table 8 shows a different distribution, for which we also find the same optimal solution.

The resolution methods presented here allow finding the optimal solution, but spend much time before delivering it, especially when the number of time intervals is greater or equaling than 17.

However, the optimization approaches presented give an optimal solution in about 2 minutes if the number of time intervals is 16, in about 60 seconds if the number of intervals is 12. Note that the local search can be used only if we introduce in advance the time limit T_{fixed} .

6.3. Other GCSPAC problem

We consider that the predicted arrival/departure demands and the available arrival/departure capacity pairs are the same as those given in the preceding sub-section. We always consider that there are 4 arrival fixes and 4 departure fixes at the airport (Figure 2) and the period considered is also 3 hours.

In Table 9, column 0 shows the time intervals of 15 minutes. For each interval, columns 1 and 2, respectively, show the number of predicted arrival demands X_{a_i} and departure demands Y_{d_i} of the airport. These predicted demands will be distributed among the different fixes. The predicted arrival demands of the airport will be distributed among the arrival demands of its fixes X_{1afi} , X_{2afi} , X_{3afi} and



X4af_i as indicated respectively in columns 3, 4, 5 and 6 for arrival fixes af1, af2, af3 and af4 (see Table 9). In the same way, the predicted departure demands of the airport will be distributed among the departure demands of its fixes Y1df_i, Y2df_i, Y3df_i and Y4df_i as indicated, respectively, in columns 7, 8, 9 and 10 for departure fixes df1, df2, df3 and df4 (Table 9).

In Table 10, columns 13, 14, 15 and 16 present, respectively, arrival flows actually served X1aaf_i, X2aaf_i, X3aaf_i and X4aaf_i at the fixes af1, af2, af3 and af4. Columns 17, 18, 19 and 20 of the same table present respectively departure flows actually served Y1adf_i, Y2adf_i, Y3adf_i and Y4adf_i at the fixes df1, df2, df3 and df4.

In Table 11, columns 21, 22, 23 and 24 present, respectively, the arrival demands which were not served (queues or delays) Q1af_i, Q2af_i, Q3af_i and Q4af_i at the arrival fixes af1, af2, af3 and af4. Columns 25, 26, 27 and 28 of the same Table 11 present, respectively, the departure demands which were not served Q1df_i, Q2df_i, Q3df_i and Q4df_i at the departure fixes df1, df2, df3 and df4. The arrival/departure demands which were not served are queues which must be taken into account during the next time interval.

6.3. GCSPAC problem resolution

The problem consists in distributing the values of column 1 over columns 3, 4, 5 and 6 (Table 9) of arrival fixes af1, af2, af3 and af4 and in distributing the values of column 2 on columns 7, 8, 9 and 10 of departure fixes df1, df2, df3 and df4. This distribution is arbitrary and is more or less equitable. Then, it acts of the distribution of the arrival capacities of the terminal (column 11 of table 10) on the capacities of arrival fixes which are in columns 13, 14, 15 and 16 (table 10) and also, the distribution of the departure capacities of the terminal (column 12 of table 10) on the capacities of departure fixes which are in columns 17, 18, 19 and 20. But before that, let us notice, for each line, the choice of the capacity (value) among the various capacities to affect to arrival and departure demands (variables) of the different fixes. This choice is selected according to the heuristic which allows to respect all the constraints and to minimize the number of delayed planes. The resolution is done in an incremental way and follows the law of line per line.

In columns 21, 22, 23 and 24 of Table 11, we add, at each time interval i, the number of the planes which had demanded to arrive at one of the arrival fixes af1, af2, af3 and af4 but which has not been served. These demands which have not been served will constitute, respectively, arrival queues for these same fixes for the next time interval i+1.

In the same way, in columns 25, 26, 27 and 28 (Table 11), we add, at each time interval i, the number of the planes which have required to leave at each departure fixes df1, df2, df3 and df4 but which has not been served. These demands which have not been served constitute, respectively, departure queues for these same fixes for the next time interval i+1.

The objective of this study is to minimize the sum of planes delayed which are in columns 21, 22, 23 and 24 for arrival fixes and in columns 25, 26, 27 and 28 for departure fixes. In a general way, we seek to reduce the total sum of all delayed planes. For the example of the problem presented above, the objective function which is the sum of the columns 21, 22, 23 and 24 for arrival fixes and of the columns 25, 26, 27 and 28 for the departure fixes (table 11) will be written:

$$\varphi = \sum_{i=1}^{12} \left(\sum_{j=1}^4 Qj_{afi} + \sum_{k=1}^4 Qk_{dfi} \right) \tag{40}$$

That is to say:

$$\varphi = \sum_{i=1}^{12} \left(\sum_{j=1}^4 (Xj_{afi} - Xj_{aafi}) + \sum_{k=1}^4 (Yk_{dfi} - Yk_{adfi}) \right) \tag{41}$$

The number of the arrival planes delayed is:

$$\sum_{i=1}^{12} \left(\sum_{j=1}^4 (Xj_{afi} - Xj_{aafi}) \right) = 37 + 35 + 30 + 41 = 143$$

(columns 21, 22, 23 and 24), and the number of the departure planes delayed is:

$$\sum_{i=1}^{12} \left(\sum_{k=1}^4 (Yk_{dfi} - Yk_{adfi}) \right) = 18 + 21 + 17 + 21 = 77$$

(columns 25, 26, 27 and 28).

The total number of planes delayed is 143+77 = 220 (see Table 11).

Table 7: Real flows and delays recorded on the different fixes of the airport.

0	1	2	3	4	5	6	7	8	9	10
	X_a/Y_d	X_1/Y_1	X_2/Y_2	X_3/Y_3	X_4/Y_4	Q_{1a}/Q_{1d}	Q_{2a}/Q_{2d}	Q_{3a}/Q_{3d}	Q_{4a}/Q_{4d}	C_a/C_d
1	26/36	08/09	06/09	06/06	04/00	00/00	00/00	00/03	02/09	24/24
2	38/32	09/08	04/00	00/00	11/16	00/00	05/08	11/11	00/01	24/24
3	42/09	00/03	14/10	00/08	10/03	12/00	01/00	21/05	00/00	24/24
4	29/15	20/06	00/00	08/08	00/01	00/00	08/03	20/00	07/02	28/15
5	06/07	01/01	09/04	10/04	08/03	00/00	00/00	13/00	00/00	28/15
6	13/10	03/02	04/04	16/02	03/02	00/00	00/00	00/00	00/00	28/15
7	14/17	03/04	03/04	03/04	05/05	00/00	00/00	00/00	00/00	26/19
8	20/33	05/08	02/06	05/08	05/08	00/00	03/03	00/00	00/00	17/30
9	40/34	10/08	13/11	00/00	01/05	00/00	00/00	10/08	09/05	24/24
10	25/22	03/05	06/05	00/04	15/10	03/00	00/00	17/11	00/00	24/24
11	13/13	06/03	00/04	15/14	03/03	00/00	04/00	05/00	00/00	24/24
12	12/01	03/01	07/00	08/00	03/00	00/00	00/00	00/00	00/00	28/15
ToT	278/229	71/58	68/57	71/58	68/56	15/00	21/14	97/38	18/17	

Table 8: Other example of Real flows and delays recorded on the different fixes of the airport.

0	1	2	3	4	5	6	7	8	9	10
	X_a/Y_d	X_1/Y_1	X_2/Y_2	X_3/Y_3	X_4/Y_4	Q_{1a}/Q_{1d}	Q_{2a}/Q_{2d}	Q_{3a}/Q_{3d}	Q_{4a}/Q_{4d}	C_a/C_d
1	26/36	06/09	06/00	06/09	06/06	00/00	02/09	00/00	00/03	24/24
2	38/32	00/00	11/17	04/00	09/07	09/08	00/00	07/08	00/04	24/24
3	42/09	12/10	12/03	00/05	00/06	07/00	00/00	17/05	10/00	24/24
4	29/15	11/03	00/01	00/08	17/03	03/00	08/05	24/00	00/00	28/15
5	06/07	04/01	09/06	14/04	01/01	00/00	00/00	13/00	00/00	28/15
6	13/10	03/02	03/02	16/02	04/04	00/00	00/00	00/00	00/00	28/15
7	14/17	03/04	05/05	03/04	03/04	00/00	00/00	00/00	00/00	26/19
8	20/33	02/06	05/08	05/08	05/08	03/03	00/00	00/00	00/00	17/30
9	40/34	00/00	10/08	10/10	04/06	13/11	00/00	00/00	06/02	24/24
10	25/22	19/16	00/01	00/00	05/07	00/00	06/04	07/07	07/00	24/24
11	13/13	03/03	09/07	10/10	02/04	00/00	00/00	00/00	09/00	24/24
12	12/01	03/01	03/00	03/00	12/00	00/00	00/00	00/00	00/00	28/15
ToT	278/229	66/55	73/58	71/60	68/56	35/22	16/18	68/20	32/09	



Table 9: Distribution of predicted Arrival and Departure flows over Different Arrival and Departure

0	1	2	3	4	5	6	7	8	9	10
i	Xa_i	Yd_i	X1af_i	X2af_i	X3af_i	X4af_i	Y1df_i	Y2df_i	Y3df_i	Y4df_i
i = 1	26	36	6	7	6	7	9	9	9	9
i = 2	38	32	10	9	10	9	8	8	8	8
i = 3	42	9	10	11	10	11	2	2	2	3
i = 4	29	15	8	7	7	7	4	4	4	3
i = 5	6	7	1	2	1	2	1	2	2	2
i = 6	13	10	4	3	3	3	3	2	3	2
i = 7	14	17	3	4	3	4	4	4	4	5
i = 8	20	33	5	5	5	5	9	8	8	8
i = 9	40	34	10	10	10	10	9	8	9	8
I = 10	25	22	6	6	6	7	5	6	5	6
I = 11	13	13	3	4	3	3	4	3	3	3
I = 12	12	1	3	3	3	3	0	1	0	0
TOT	278	229	69	71	67	71	58	57	57	57

Table 10: Arrival and Departure flows actually served

0	11	12	13	14	15	16	17	18	19	20
i	Ca_i	Cd_i	X1aaf_i	X2aaf_i	X3aaf_i	X4aaf_i	Y1adf_i	Y2adf_i	Y3adf_i	Y4adf_i
i = 1	24	24	9	10	1	4	6	6	6	6
i = 2	24	24	9	10	1	4	6	6	6	6
i = 3	24	24	8	10	2	4	6	6	6	6
i = 4	26	19	8	10	3	5	5	5	5	4
i = 5	28	15	10	10	4	4	2	2	2	2
i = 6	28	15	5	5	9	9	1	3	3	3
i = 7	17	30	4	0	4	6	4	4	4	5
i = 8	20	27	2	3	7	8	7	7	7	6
i = 9	24	24	3	1	10	10	6	5	7	6
I = 10	24	24	3	1	10	10	6	6	6	6
I = 11	24	24	2	5	7	10	6	6	6	6
I = 12	18	29	4	9	1	4	1	2	0	1
TOT			68	74	61	75	56	58	58	57

Table 11: Delayed Flows Recorded

0	11	12	21	22	23	24	25	26	27	28
i	Ca_i	Cd_i	Q1af_i	X2af_i	Q3af_i	Q4af_i	Q1df_i	Q2df_i	Q3df_i	Q4df_i
i = 1	24	24	1	1	0	0	3	3	3	3
i = 2	24	24	4	5	2	5	5	5	5	5
i = 3	24	24	11	7	7	9	1	1	1	2
i = 4	26	19	12	12	6	7	0	0	0	1
i = 5	28	15	4	4	4	3	0	0	0	0
i = 6	28	15	0	0	0	0	0	0	0	0
i = 7	17	30	0	0	0	0	0	0	0	0
i = 8	20	27	0	0	0	0	2	1	1	2
i = 9	24	24	2	1	4	9	4	4	4	4
I = 10	24	24	1	2	6	8	3	5	3	3
I = 11	24	24	2	3	1	0	0	2	0	1
I = 12	18	29	0	0	0	0	0	0	0	0
TOT			37	35	30	41	18	21	17	21

7. HOW TO CONTROL AND REGULATE THE CAPACITIES

We saw in Relation (7) that:

$$0 \leq Pv_i + Poc_i \leq CT_i$$

Let us suppose that one wishes to reduce or increase the capacities of an airport for an unspecified reason or, more precisely, if one wishes to control and regulate these capacities. Then, in this case, it is necessary to introduce some control parameters. The formula, above, becomes:

$$\alpha_i Poc_i + \beta_i Pv_i = \lambda_i CT_i \tag{41}$$

The parameters α_i , β_i and λ_i are parameters which control the capacities. They depend on the objectives expressed by the airport managers according to certain conditions on weather, priorities, authorities, rush hours, etc. They have values between 0 and 1. We are, thus, brought to introduce these parameters into all the formulas which we clarified, above. More precisely, we will have the following formulas:

$$\alpha_i Xaa_i \leq \lambda_i Ca_i \leq \beta_i Pv_i \tag{42}$$

$$\alpha_i Yad_i \leq \lambda_i Cd_i \leq \beta_i Poc_i \tag{43}$$

Such that Ca_i and Cd_i are the total capacities, respectively, of arrival and departure fixes (Figure 2) during the time interval ‘i’ and α_i , λ_i and β_i are parameters of control. Over one period of (15*N) minutes, we will have:

$$[\alpha_i Xaa_i + \alpha_i Yad_i] \leq [\lambda_i Ca_i + \lambda_i Cd_i] \leq [\beta_i Pv_i + \beta_i Poc_i] \tag{44}$$

Concretely, if the capacity is rather large so that no congestion takes place, the problem would not arise. But, if there are more arrival demands than departure demands during one time period, the problem of congestion appears. In this case, it is necessary to minimize the number of delayed planes or to maximize departure flows as much as possible.

The following formula allows us to answer partly this problem of congestion, while ensuring a certain balance between arrivals and departures.

$$\sum_{i=1}^N [\alpha_i Poc_i - \gamma_i Yad_i + \beta_i Pv_i - \delta_i Xaa_i]$$

N represents the number of intervals over one period of (15*N) minutes; Poc_i is the number of occupied places during the time interval ‘i’; Pv_i is the

number of empty places during the time interval 'i'; Yad_i is the number of actual departures during 'i'; Xaa_i is the number of actual arrivals during 'i'; CTd_i is the departure capacity at the terminal during 'i'; CTa_i is the arrival capacity at the terminal during 'i'; α_i , β_i , γ_i , δ_i , λ_i and μ_i are parameters of regulation.

The response to this load balancing problem between arrivals and departures consists in minimizing the following function which we call the objective function.

$$\min \sum_{i=1}^N \{[\alpha_i P o c_i - \gamma_i Y a d_i] + [\beta_i P v_i - \delta_i X a a_i]\} \quad (46)$$

8. CONCLUSION

In this paper, we presented a modeling of the capacity allocation problem (CAP) of an airport and of its fixes in the form of a CSP which we called, respectively, CSPAC and GCSPAC. Thereafter, we proposed an optimization method under constraints for those problems. The method presented combines a branch and bound algorithm (BnB) with Local Search (LS). LS being called if BnB fails to find the optimal solution within a certain Time Limit constraint (TL) fixed by airport managers. Thus, we always have at least a solution which may not be optimal, but which is acceptable.

We presented some examples illustrating our approach which can assist airport managers to monitor and regulate arrival/departure capacities to minimize the delay.

We performed preliminary experiments of the proposed approaches and compared them to other methods proposed in the literature (especially [3-6]). The results for various instances of the problem show that our approach provides solutions at least as good as those found by other techniques.

Let us note that our approaches of modeling, resolution and optimization presented in this paper is valid not only for air traffic, but also for road traffic, railway and maritime. Another possible solution to reduce these problems of congestion of the airports (or any other system) is to carry out certain cooperation and complementarities between the various systems, such as air, maritime, road and rail transport. Each mode of transportation has its own advantages, e.g. potential capacity, high levels of safety, flexibility, low energy consumption, low environmental impact.

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