

# SIMULATION OF FUEL CONSUMPTION AND ENGINE POLLUTANT IN CELLULAR AUTOMATON

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## ABSTRACT

Among the aspects that require a major interest in the field of traffic modeling is the simulation of the fuel consumption and engine emissions. In this paper, we study the fuel consumption and gas emissions caused by traffic in the Nagel and Schreckenberg (NaSch) model, and some of its variants, with closed boundary conditions. The behavior of the consumption and the emissions in different models are analyzed and compared. The influences of the presence / absence of traffic lights are also investigated.

Keywords: *Cellular automata models, fuel consumption, gas emissions, UML*

## 1. INTRODUCTION

In recent years, the problem of fuel consumption and pollution are more and more serious. On the one hand, the problem caused by a lack of petroleum come forth more frequently, for example, a rising cost of living and international conflict. On the other hand, motor vehicles are the largest source of man-made polluting emissions. Since the late 20th century, many automobile manufacturers and automotive engineers have focused on developing more efficient and powerful vehicles with reduced emissions. Along with those efforts and research, many new technologies have been developed, especially engine improvements such as gasoline direct injection, variable valve timing, variable compression ratio, etc.

In the domain of traffic modeling, some authors have shown their interest in the simulation of the fuel consumption and gas emissions. These simulations are based on cellular automaton, like the NaSch model [1] and some of its variants [2, 3, 4, 5, 6]. The choice of cellular automata models is because they are simpler, and can be easily implemented compared with other dynamical approaches. In cellular automata models, the road, the time and the velocity of vehicles are assumed to take discrete values. When applied to the car traffic, cellular automata uses a set of cells, each of them

has two states to indicate if it is occupied or not by a vehicle.

The paper is organized as follows. Section II is devoted to the survey of the basic model and some of its variants. In section III, the description of our model and the definition of the fuel consumption and gas emissions are detailed. The analysis of the results is given in section IV. Finally, the conclusions are given in section V.

## 2. CELLULAR AUTOMATON MODELS

We cannot mention the cellular automata without addressing the pioneer in this field: the NaSch model [1]. The remainder of this section will therefore be devoted to this model and some of its variants.

The NaSch model [1] is a probabilistic cellular automaton. Space and time (and hence the velocities) are discrete. The road is divided into cells of length 7.5 m. Each cell can either be empty or occupied by just one car. The state of a car  $j$  ( $j = 1 \dots N$ ) is characterized by its momentary velocity  $v_j$  ( $v_j = 0, 1, \dots, v_{\max}$ ). The state of the system at time  $t+1$  can be obtained from the state at time  $t$  by applying the following four rules to all cars at the same time (parallel dynamics):



1. **Acceleration:**  $v(t+1) \leftarrow \min(v(t) + 1, v_{\max})$ .
2. **Deceleration:**  $v(t+1) \leftarrow \min(v(t+1), \text{gap})$
3. **Braking:**  $v(t+1) \leftarrow \max(v(t+1) - 1, 0)$  with probability  $p$ .
4. **Motion:**  $x(t+1) \leftarrow x(t) + v(t+1)$ .

where  $v(t)$  and  $x(t)$  denote the velocity and the position of a car at time  $t$ .  $v_{\max}$  is the maximum velocity.  $\text{gap}$  specifies the number of empty cells in front of the car and it is given by:  $\text{gap} = x_{n+1} - x_n - 1$ .

Using these simple rules, this model is able to reproduce the basic phenomena encountered in real traffic.

Many variants of the NaSch model have been proposed, each with different degree of realism. Some amelioration concerns the rule 3 of the NaSch model. The NaSch model uses a constant probability for this rule, but it seems to be insufficient for modeling metastable state with a very high flow [7]. To include this behavior to simulation in more realistic fashion, new models with “slow-to-start” rule are introduced. For example, we can cite TT[8], BJH [9] and VDR [10] models.

Takayasu and Takayasu model (TT model) [8] were the first to suggest a cellular automata model with a “slow-to-start” rule. According to the authors, a standing vehicle (i. e. a vehicle with the velocity  $v = 0$ ) with exactly one empty cell in front accelerate with probability  $q_t = 1 - p_t$ , while all other vehicles accelerate deterministically. The others rules (2, 3 and 4) of the NaSch model are kept unchanged.

By contrast to the original NaSch model, in the Velocity Dependent Randomization model (VDR model) [10], the randomization parameter depends on the velocity of the vehicle. The randomization in the rule 3 of the NaSch model is now given by  $p = p(v)$  as follows:

$$p(v) = \begin{cases} p_0 & \text{for } v = 0 \\ p & \text{for } v > 0 \end{cases}$$

with  $p_0 > p$ , this means that vehicles which have been standing in the previous time step have a higher probability  $p_0$  of braking than moving vehicles.

Another model was developed by Benjamin, Johnson and Hui (BJH model) [9]. This model is an extension of the NaSch model, in which a “slow-to-start” rule was added. With this rule, the authors attempt to simulate the behavior of drivers which have come to a complete stop in traffic jam. Cars which have velocity 0 either accelerate at their first opportunity (as soon as there is an empty space ahead of them) with the probability  $1 - p_{\text{slow}}$ , or on the time step immediately after that with the probability  $p_{\text{slow}}$ . Otherwise, they follow the NaSch model rules. This scheme is intended to reflect the fact that drivers take longer to accelerate from a complete stop, because they do not immediately notice the car ahead of them moving, or because of the slow pick-up of their car’s engine. The steps of the update rules can be stated as follows:

1. **Acceleration:**  $v(t+1) \leftarrow \min(v(t) + 1, v_{\max})$ .
2. **Slow-to-Start:** if  $\text{flag} = \text{true}$  then  $v(t+1) = 0$  with probability  $p_{\text{slow}}$
3. **Deceleration:**  $v(t+1) \leftarrow \min(v(t+1), \text{gap})$  and, then,  $\text{flag} = \text{true}$  if  $v(t+1) = 0$ , else  $\text{flag} = \text{false}$ .
4. **Braking:**  $v(t+1) \leftarrow \max(v(t+1) - 1, 0)$  with probability  $p$ .
5. **Motion:**  $x(t+1) \leftarrow x(t) + v(t+1)$ .

A modification of the BJH cellular automaton model was proposed by [5]. This new model includes a “slow-to-stop” rule that shows more realistic driver behavior than the BJH model. The “slow-to-stop” rule shows the fact that people who see a stopped car ahead of them would certainly attempt to slow down gradually.

Since this last model is used in our work, we devote the rest of this section to the principles of this model. The model will be noted MBJH for the “Modified BJH”.

The authors of the MBJH have noticed that cars following the BJH rules behave in an unrealistic fashion when approaching a jam. Indeed, if a car B ahead has velocity 0, then a car A may drive up to

B at velocity  $v_{max}$  only to brake down to velocity 0 in one time step in the cell right behind B [5]. This behavior seems to be inaccurate, this leads the authors to add another rule that allows drivers to look farther ahead and slow down gradually earlier when their cars come closer to the jam. In this new version of the BJH model, the car's velocities are adjusted at each time step according to the following rules, where  $v_{next}$  represents the velocity of the next vehicle:

**1. Acceleration:**  $v(t+1) \leftarrow \min(v(t)+1, v_{max})$

**2. Slow-to-Start:** if flag = true then  $v(t+1) = 0$  with probability  $p_{slow}$ .

**3. Deceleration (when the car is near):** if  $gap \leq v(t+1)$  and either  $v(t+1) < v_{next}$  or  $v(t+1) \leq 2$  then  $v(t+1) \leftarrow gap - 1$ . Otherwise, if  $gap \leq v(t+1)$ ,  $v(t+1) \geq v_{next}$  and  $v(t+1) > 2$  then  $v(t+1) \leftarrow \min(gap - 1, v(t+1) - 2)$

**4. Deceleration (when the next car is far):** if  $v(t+1) < gap \leq 2v(t+1)$ , then if  $v(t+1) \geq v_{next} + 4$  then  $v(t+1) \leftarrow v(t+1) - 2$ . Otherwise, if  $v_{next} + 2 \leq v(t+1) \leq v_{next} + 3$  then  $v(t+1) \leftarrow v(t+1) - 1$

**5. Setting of flag:** flag = true if  $v(t+1) = 0$ , else flag = false

**6. Randomization:**  $v(t+1) \leftarrow \max(v(t+1) - 1, 0)$ , with probability  $p_{fault}$

**7. Motion:**  $x(t+1) \leftarrow x(t) + v(t+1)$

Where  $p_{fault}$  is the probability used by the NaSch model.

### 3. PROBLEM FORMULATION

#### A. Estimate Function for the fuel consumption and gas emissions

In the last few years, some authors were interested by the fuel consumption and gas emissions, so, models were presented [1, 2, 3, 9]. The model adopted in this paper used the following function  $\Delta F$ , given in equation 3.1. This function is used to estimate the value of the fuel consumption (mL) or emission produced (g), during a simulation interval ( $\Delta T$  seconds):

Equation 3.1: function used for estimating fuel consumption and gas emission

$$\Delta F \begin{cases} \alpha \Delta t, \text{ for } R_T \leq 0, \\ [\alpha + \beta_1 R_T v + (\beta_2 M_v a^2 v / 1000)_{a>0}] \Delta t, \text{ for } R_T > 0, \end{cases}$$

where

$R_T$  = total traction force (KN) required to drive the vehicle, which is the sum of rolling resistance, air drag force, cornering resistance, inertia force and grade force. For simplicity, we have not considered the vehicles turning and the road gradient. The total traction force can therefore be formulated as follows:  $R_T = F_f + F_w + F_j = M_v g C_r + \frac{1}{2} C_D A \rho v^2 + \delta M_v a$ ,

$M_v$  = vehicle mass (kg) including occupants and any other load,

$v$  = instantaneous speed (m/s) =  $v$  (km/h)/3.6,

$a$  = instantaneous acceleration rate ( $m/s^2$ ), negative for deceleration,

$\alpha$  = constant idle fuel rate (mL/s) or emission rate (g/s), which applies during all modes of driving (as an estimate of fuel used to maintain engine operation),

$\beta_1$  = the efficiency parameter which relates fuel consumed and pollutant emitted to the energy provided by the engine, i.e. fuel consumption or emission per unit of energy (mL/kJ or g/kJ),

$\beta_2$  = the efficiency parameter which relates fuel consumed and pollutant emitted during positive acceleration to the product of inertia energy and acceleration, i.e. fuel consumption or emission per unit of energy-acceleration (mL/(kJ.m/s<sup>2</sup>) or g/(kJ.m/s<sup>2</sup>)).

Table 1: Parameters for fuel consumption used for light vehicles. These values are the same used in [3]

Parameter	Description	Value of fuel
A	Idle fuel consumption or emission rate	1350 mL/h
$\beta_1$	Efficiency parameter	$900 * 10^{-4}$ mL/kJ
$\beta_2$	Energy-acceleration efficiency parameter	$300 * 10^{-4}$ mL/(kJ.m/s <sup>2</sup> )
$M_v$	Average vehicle mass for light vehicle	1400 kg
g	Gravitation force	9.8 m/s <sup>2</sup>
$C_r$	Rolling friction coefficient	0.01
$C_D$	Air resistance coefficient	0.3
A	Vehicle frontal area	2 m <sup>2</sup>
$\rho$	Air density	1.2 kg/m <sup>3</sup>
$\delta$	Rotating mass inertia factor	1
$u_r$	Relative speed of the vehicle and air	$u_r \approx v$
$\Delta t$	Simulation interval	7 s

For the simulation interval ( $\Delta t$ ), and the formulas to calculate the values of the mean velocity and the acceleration in every interval, we adopt the approach used in [9]. If we denote  $x_i(t)$ ,  $v_i(t)$  and  $a_i(t)$  the position, the velocity and the acceleration of the  $i$ th car at time  $t$ , then the values of the mean velocity and the mean acceleration are given as follows :

$$\langle v_i(t) \rangle = \frac{x_i(t) - x_i(t-1)}{\Delta t}$$

$$\langle a_i(t) \rangle = \frac{2(x_i(t) - x_i(t-1))}{\Delta t^2} = \frac{2(v_i(t) - v_i(t-1))}{\Delta t}$$

**B. Object Oriented Modeling**

Before tackling the algorithmic part, it seems important to deal with the traffic network structure. For this, another requirement is important: the Object Oriented Modeling (OOM). The Object Oriented character of the designed model is very important, because we want to have the possibility of easy and half-automatic creation of the simulation program. The OOM also means that the

simulation program should be divided into parts, representing urban traffic entities (road, vehicle, light ...), that are connected together and the whole simulation is performed by communication among them. This Object Oriented Modeling (OOM) is realized with the Unified Modeling Language (UML).

A class diagram in the UML language is a type of static structure diagram that describes the structure of a system by showing the system's classes, their attributes, and the relationships between the classes.

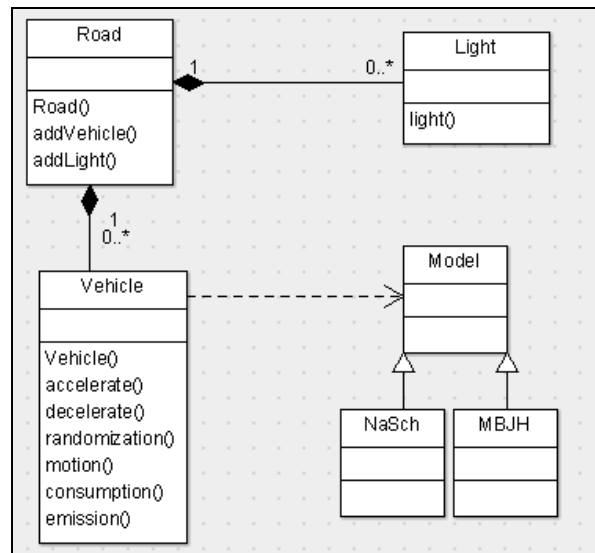


Figure 1: The Object Oriented Modeling of our simulation program.

In the above figure, a road is made up of a set of traffic lights. A road is composed of one or more vehicles. This structure is better represented with a composition relationship, since the relationship between the 'Road' class and the 'Vehicle' class on one hand, and the 'Light' class on the other hand represents a 'part-whole' or 'part-of' relationship.

**4. SIMULATION AND ANALYSIS RESULTS**

In the simulation, two models NaSch and MBJH are used for the fuel consumed and gas emitted. The results are then compared and analyzed. A closed periodic road is adopted, in which N cars are initially located randomly on the circuit of 1000 cells. The velocity of each car is designated by an integer chosen from zero to the maximum velocity  $v_{max}$ . The table 2 shows the parameters for the two models.

Table 2: Parameters selected in the simulation

	NaSch	MBJH
	model	model
Length of each cell (m)	7.5	7.5
Length of each car (cell)	1	1
Total number of cells	1000	1000
Maximum velocity (cell)	5	5
Randomization probability for the NaSch (p <sub>fault</sub> )	0.3	0.3
Randomization probability for slow-to-start (p <sub>slow</sub> )		0.75

Our discussion has two distinct parts. The first will analyze and compare the fuel consumption for both models: the Nasch model and the MBJH model. In the second part, we will analyze the effect of the presence of a traffic light on consumption in the NaSch model.

Before going on in our discussion, note that under the condition of uniform motion, the optimal speed (v<sub>optimal</sub>) whose fuel-efficiency is the highest is equal to v<sub>max</sub> [6].

Let's begin with the fuel consumption for the two models NaSch and MBJH. Our discussion will focus on three areas of the diagram shown in Figure 2; the free flow region, the seriously jammed region and the area of medium densities.

In the free flow region (when the density is below than 0.1), all the vehicles almost have the maximum speed v<sub>max</sub>(that is a constant value, see figure 5 below), that's why the curves of the two models are almost horizontal lines in this region.

When the density becomes more serious, the accelerations and velocities of most vehicles are zero in every time step. The fuel consumed by a vehicle i for ΔT is then ΔF<sub>i</sub> ≈ α \* ΔT. The average consumption of all vehicles at each interval ΔT is then given as follows:

$$\Delta F(\Delta T) = (1/N)\sum \Delta F_i = (1/N) \sum \alpha * \Delta T \approx \alpha * \Delta T$$

To travel 100 km, the vehicles need to consume ΔF = ∑ α \* ΔT ≈ α \* T, where T is the time required for 100 km, and its value is given as d = 100 km = <v> \* T. By replacing T by its value, the overall consumption becomes:

$$\Delta F = \alpha * T = \frac{\alpha * d}{\langle v \rangle}$$

When the density becomes more serious, the average velocity tends to zero, ΔF tends to infinity.

Since the MBJH model uses a "slow-to-start" rule, the cars that have come to a complete stop need time to move (accelerate), by contrast to the NaSch model where stopped cars moved immediately whenever the headway allows. That is why the curve of the MBJH model increase rapidly compared to the NaSch one in this region.

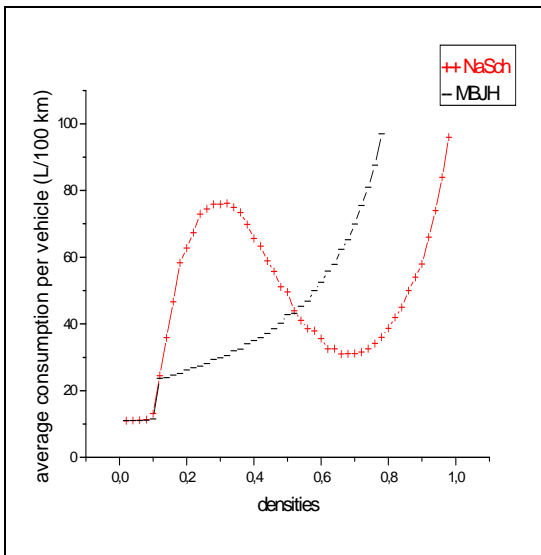


Figure 2: Average fuel consumption per vehicle (L/100 km) vs. traffic density

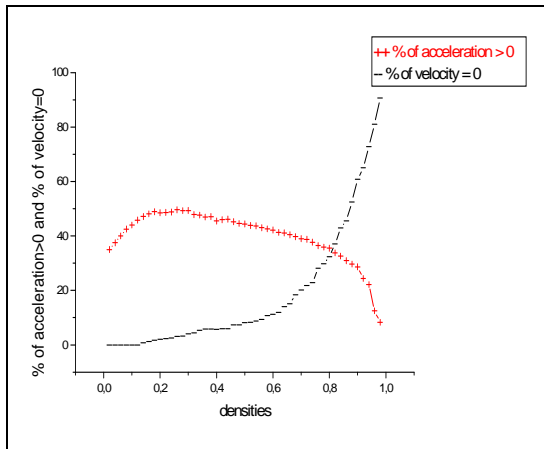


Figure 3: The percentage of vehicles speeding and those who are idle for the Nasch model

In the medium density (i.e. the region between the free flow region and the seriously jammed one), the acceleration first increases and then decreases, while the velocity is still decreasing (figure 3); this justifies the shape of the curve representing the fuel consumption for the model NaSch. For the MBJH model, we can notice that the curve of the consumption is just a translation of the NaSch one. This translation is due to the fact that bottling starts earlier (around 0.2) because of the effect of the “slow-to-start” rule, while in the NaSch model, it is from 0.7 that bottling gets really serious.

Now we add a traffic light to the NaSch model. The figure 4 represents the fuel consumed in the NaSch model without and with traffic lights. In this figure we are interested to discover the impact on fuel economy that traffic lights would have on the NaSch model. In the rest of the discussion, we note NSL and NS the NaSch model with and without traffic lights respectively. We can see clearly that for very low or very high densities, the two curves have fairly similar fuel consumption characteristics. It seems obvious, in fact, for lower densities, the speed is constant, and the curves are, then, almost horizontal lines. For higher densities, the velocities and accelerations of most vehicles are zero in every time step regardless of the presence of traffic lights. These results can be interpreted as follows:

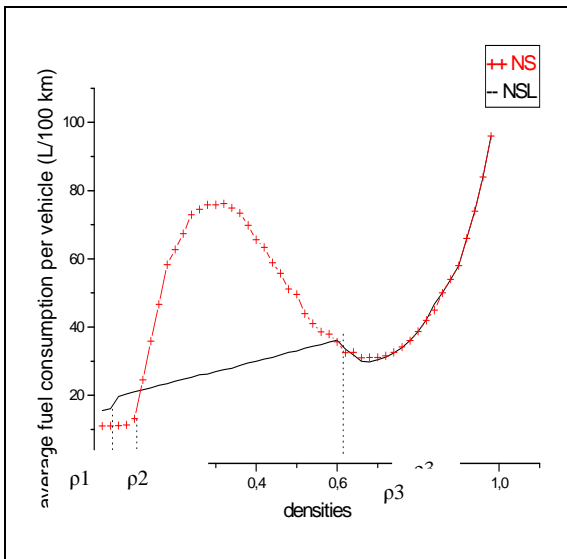


Figure 4 : Average fuel consumption per vehicle (L/100 km) vs. traffic density. The curves represent the fuel consumed for NaSch model without (red curve) and with (black curve) traffic lights

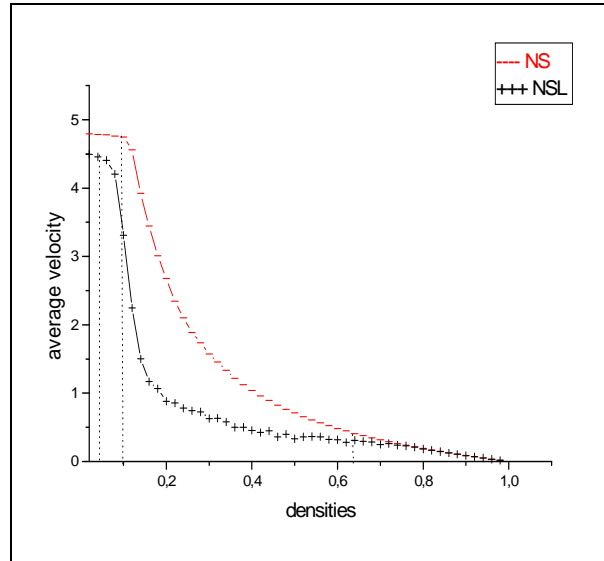


Figure 5 : Average velocity vs density for the NaSch model without and with traffic lights

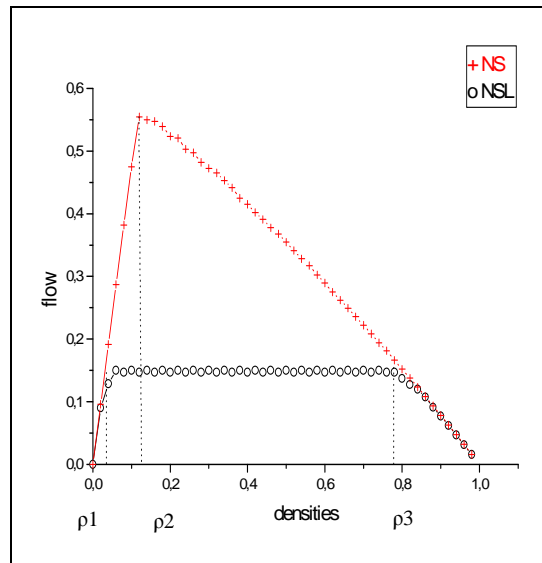


Figure 6 : fundamental diagrams of the NaSch model without and with traffic light

For densities  $\rho < \rho_1$ , the means velocity of vehicles is  $\langle v \rangle \approx v_{max}$  (that is a constant value) (figure 5), that’s why the two curves are almost horizontal lines in this region. Since the values of velocities for the NSL are lesser than the ones of NS in this region, cars in the NSL need more time to travel 100 km than in NS, the consumption of NSL is, then, greater than in NS.

When the densities  $\rho$  is between  $\rho_1$  and  $\rho_2$  ( $\rho_1 < \rho < \rho_2$ ), the velocities of NSL begin to decrease

rapidly, while in NS, the velocities of most vehicles have always  $v_{max}$  value (see figure 5). The consumption in the NS is still a horizontal line, but this consumption begins to increase rapidly in NSL.

Between  $\rho_2$  and  $\rho_3$ , the average velocity of the NSL begins to stabilize (figure 5) and the traffic flow is then represented by a horizontal line (figure 6). The consumption  $\Delta F$  has a linear form.

Once the density is above a certain value  $\rho_3$  ( $\rho > \rho_3$ ), the velocities and accelerations of most vehicles are zero in every time step regardless of the presence of traffic lights. The consumption in NS and NSL, then, increases rapidly.

The same reasoning can be used in interpreting the results of Figures 7 and 8, representing the emission of HC and NOX, respectively, as function of traffic densities for NaSch and MJBH models. The only difference between consumption and emission rates is that the gas emitted by vehicles is calculated as a function of time ( $\Delta T = 7s$ , in our case), as opposed to consumption which is calculated according to the distance ( $L = 100$  km). It is for this reason that for higher densities, consumption is increasing exponentially, while the gas emitted rate for the two models tends to  $\alpha$  ( $\alpha=0.005$  for HC and  $\alpha = 0.0002$  for NOX [3]). It is obvious that the MJBH model emits less gas than the NaSch model. This is due to the slow-to-start rule which slow the starting of standing vehicles.

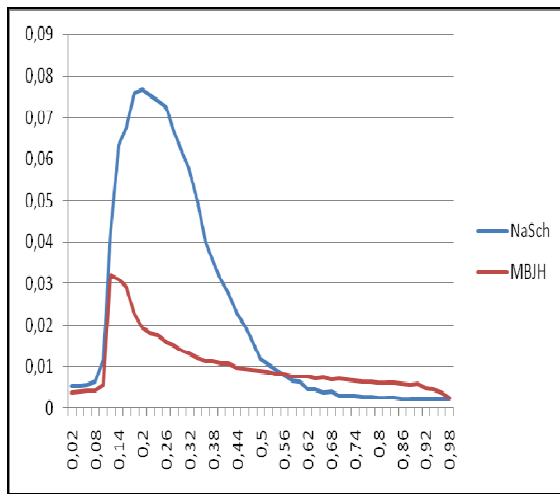


Figure 7 : HC emission rates vs. traffic density

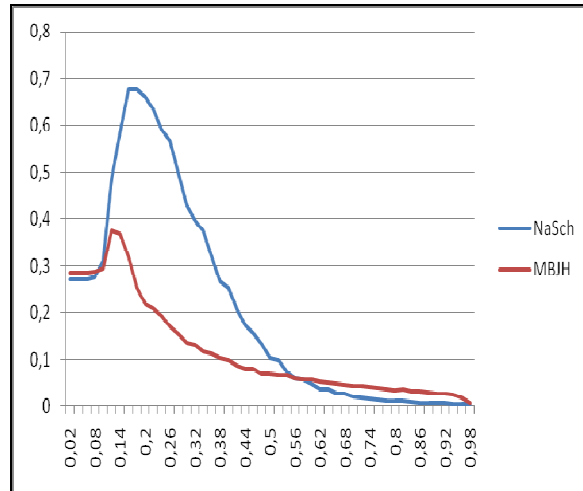


Figure 8 : NOX emission rates vs. traffic density.

## 5. CONCLUSION

In this paper we have studied the fuel consumption and the engine emissions in the NaSch model and its variant, the MJBH model (the BJH model with the “slow-to-stop” rule). We are interested, then, to the impact on fuel economy that traffic lights would have on the NaSch model. The simulation program we have constructed is Object Oriented based. The program is designed with UML language and then implemented with Java language. It is, then, simple to understand and to modify to adapt it to others more complicated situations.

The results of the analysis show that, firstly, the fuel consumption in the MJBH model is effective for lower and medium densities, while the Nasch model is more efficient for lower and higher densities. On the other hand, the presence of a traffic light has a big effect on fuel consumption. Indeed, the addition of a traffic light optimizes consumption for medium and seriously jammed densities. But for the lower densities, the Nasch model remains the best.

By comparing the fuel consumption in the MJBH model and that of the Nasch one with traffic lights, we can notice that in medium densities the shapes of the corresponding curves are very similar. This is mainly due to the fact that traffic lights add, in some way, a “slow-to-start” rule. Indeed, red lights slow the starting of vehicles from a complete stop, like in the case of a “slow-to-start” rule.



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