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MORPHOLOGICAL ADJUNCTIONS, MOORE FAMILY AND MORPHOLOGICAL TRANSFORMS

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ABSTRACT

Mathematical Morphology arose in 1964 by the work of George Matheron and Jean Serra, who developed its main concepts and tools. It uses concepts from algebra and geometry. (Set theory, complete lattice theory, convexity etc,). The notion of adjunction is very fundamental in Mathematical Morphology. Morphological systems is a broad class of nonlinear signal operators that have found many applications in image analysis. Morphological Transforms are a type of non linear signal transform for morphological systems. The Moore family stands for the family of closed objects. When the ETI and DTI systems are related via an adjunction, then there is also a close relationship between their impulse responses. Namely ,let ε be an ETI system, and let Δ be its adjoint dilation. It is easy to show that Δ is a DTI system[11], and therefore $\Delta(f) = f \oplus g$,where g is the lower impulse response. In this paper, we will try to present the inter-relationships between Moore family, adjunctions and Morphological transforms.

Keywords: *Dilation, Moore Family, Adjunction, Slope Transforms, Support Function.*

1. INTRODUCTION

1.1 NOTATION AND IMAGE DEFINITIONS

TYPES OF IMAGES

An image is a mapping denoted as I, from a set, N_P, of pixel coordinates to a set, M, of values such that for every coordinate vector, $\mathbf{p} = (p_1, p_2)$ in N_P, there is a value I(**p**) drawn from M. N_P is also called the image plane.[1]
Under the above defined mapping a real image maps an n-dimensional Euclidean vector space into the real numbers. Pixel coordinates and pixel values are real.
A discrete image maps an n-dimensional grid of points into the set of real numbers. Coordinates are n-tuples of integers, pixel values are real.
A digital image maps an n-dimensional grid into a finite set of integers. Pixel coordinates and pixel values are integers.
A binary image has only 2 values. That is, M= {m_{fg} , m_{bg}}, where m_{fg}, is called the foreground value and m_{bg} is called the background value.
The foreground value is m_{fg} = 0, and the background is $m_{bg} = -\infty$. Other possibilities are {m_{fg}, m_{bg}} = {0,∞}, {0,1}, {1,0}, {0,255}, and {255,0}.

1.2 DEFINITION

The foreground of binary image I is
$FG\{I\} = \{I(\mathbf{p}), \boldsymbol{p} = (p_1, p_2) \epsilon N_p / I(\boldsymbol{p}) = m_{fg}$.
The background is the complement of the foreground and vice-versa

1.3 DILATION AND EROSION

Morphology uses 'Set Theory' as the foundation for many functions [1]. The simplest functions to implement are 'Dilation' and 'Erosion'.

1.3.1 DEFINITION : DILATION

Dilation of the object *A* by the structuring element *B* is given by $A \oplus B = \{x : \hat{B}_x \cap A \neq \emptyset\}$.

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Usually A will be the signal or image being operated on A and B will be the Structuring Element'

1.3.2 DEFINITION EROSION

The opposite of dilation is known as erosion. Erosion of the object *A* by a structuring element *B* is given by

$$A \ominus B = \{x : B_x \subseteq A\}.$$

Erosion of A by B is the set of points x such that B translated by x is contained in A.

1.4 OPENING AND CLOSING

Two very important transformations are *opening* and *closing*. Dilation expands an image object and erosion shrinks it. Opening, generally smoothes a contour in an image, breaking narrow isthmuses and eliminating thin protrusions. Closing tends to narrow smooth sections of contours, fusing narrow breaks and long thin gulfs, eliminating small holes, and filling gaps in contours.

1.4.1 DEFINITION : OPENING

The opening of *A* by *B*, denoted by $A \circ B$, is given by the erosion by *B*, followed by the dilation by *B*,

that is $A \circ B = (A \ominus B) \ominus B.$

1.4.2 DEFINITION :CLOSING

The opposite of opening is 'Closing' defined by

$$A \bullet B = (A \oplus B) \ominus B.$$

Closing is the dual operation of opening and is denoted by $A \bullet B$. It is produced by the dilation of *A* by *B*, followed by the erosion by *B*:

2 MORPHOLOGICAL OPERATORS DEFINED ON A LATTICE

2.1 DEFINITION: DILATION

Let $(L, \leq)$ be a complete lattice, with infimum and minimum symbolized by $\wedge$and $\vee$, respectively.[1],[2].[11]

A dilation is any operator $\delta : L \rightarrow L$that distributes over the supremum and preserves the least element.

$$\bigvee_i \delta(X_i) = \delta\left(\bigvee_i X_i\right),$$

$$\delta(\emptyset) = \emptyset.$$

2.2 DEFINITION: EROSION

An erosion is any operator $\varepsilon : L \rightarrow L$that distributes over the infimum

$$\bigwedge_i \varepsilon(X_i) = \varepsilon\left(\bigwedge_i X_i\right), \varepsilon(U) = U.$$

2.3 GALOIS CONNECTIONS:

Dilations and erosions form Galois connections. That is, for all dilation δ there is one and only one erosion εthat satisfies

$X \leq \varepsilon(Y) \Leftrightarrow \delta(X) \leq Y$ for all $X, Y \in L$.

Similarly, for all erosion there is one and only one dilation satisfying the above connection.

Furthermore, if two operators satisfy the connection, then δ must be a dilation , and εan erosion.

2.4 DEFINITION : ADJUNCTIONS :

Pairs of erosions and dilations satisfying the above connection are called "adjunctions", and the erosion is said to be the adjoint erosion of the dilation, and vice-versa.

2.5 OPENING AND CLOSING:

For all adjunction (ε, δ), the morphological opening $\gamma : L \rightarrow L$and morphological closing $\phi : L \rightarrow L$are defined as follows:[2]

$\gamma = \delta\varepsilon$, and $\phi = \varepsilon\delta$.

The morphological opening and closing are particular cases of algebraic opening (or simply opening) and algebraic closing(or simply closing). Algebraic openings are operators in *L* that are idempotent, increasing, and anti-extensive. Algebraic closings are operators in *L* that are idempotent, increasing, and extensive.

2.6 PARTICULAR CASES:

Binary morphology is a particular case of lattice morphology, where *L* is the power set of *E* (Euclidean space or grid), that is, *L* is the set of all subsets of *E*, and $\leq$is the set inclusion. In this case, the infimum is set intersection, and the supremum is set union.

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Similarly, grayscale morphology is another particular case, [2] where *L* is the set of functions mapping *E* into $\mathbb{R} \cup \{\infty, -\infty\}$, and $\leq$, $\vee$, and $\wedge$, are the point-wise order, supremum, and infimum, respectively. That is, is *f* and *g* are functions in *L*, then $f \leq g$ if and only if $f(x) \leq g(x), \forall x \in E$; the infimum $f \wedge g$ is given by $(f \wedge g)(x) = f(x) \wedge g(x)$; and the supremum $f \vee g$ is given by $(f \vee g)(x) = f(x) \vee g(x)$.[1]

3 MOORE FAMILY AND MATHEMATICAL MORPHOLOGY

3.1 DEFINITION: MOORE FAMILY:

Let L be a poset.

i) A subset M of L is a Moore family if every element of L has a least upper bound in M. $\forall x \in L, \exists y \in M\ y \geq x \text{ and } \forall z \in M, z \geq x \Rightarrow z \geq y]$

ii) A closure operator on L is an increasing, extensive and idempotent operator from L $\rightarrow$L.

3.2 PROPOSITION:

Let L be a poset. There is a one to one correspondence between Moore families in L and closings on L, given as follows.

i) To a Moore family M, associate the closing φ defined by setting for every x$\in$L; φ(x) is equal to the least y$\in$M such that y$\geq$x.

ii) To a closing φ, one associates the Moore family M which is the invariance domain of φ: M =Inv φ (i.e. M=$\{\varphi(x)/x \in L\}$.

3.3 RESULT:

Let L be a complete lattice. A subset M of L is a Moore family iff M is closed under the infimum operation.

$\forall S \subseteq M, \wedge S \in M$

In particular $\wedge \varphi = 1 \in M$

Given a Moore family M corresponding to a closing $\varphi, (M, \leq)$ is a complete lattice with greatest element 1 and least element $\varphi(o) = \wedge M$, and where the supremum and infimum of a family N$\subseteq$M are given by $\varphi(\vee N)$ and $\wedge$Nrespectively. (φ (1)=1 and $\varphi(\wedge$N) = $\wedge$N)

Also ,$\forall X \subseteq L, \varphi(\vee_{x\in X}\varphi(x)) = \varphi(\vee X)$.

EXAMPLE

Let F be the family of closed sets in a topological space E.

Since F is closed under arbitrary intersections, and contains the empty intersection $\cap \varphi = E$, F corresponds to a closing, which is the topological closure operator cl, where for X$\subseteq$E, cl(X) is the least element of F containing X.□ F is a Moore family of P(E) (ordered by inclusion)

3.4 PROPERTIES OF MOORE FAMILY:

i. $\varphi \in F, cl(\varphi) = \emptyset$where F is the Moore family.

ii. F is closed under binary union for C_1, $C_2 \in F$, $C1 \cup C2 \in F \Longrightarrow cl(C_1 \cup C_2) = C_1 \cup C_2$

F is the set of cl(X) for X$\in$P(E) and C_i=cl(X_i)

$\therefore \forall X_1, X_2 \in P(E), cl(X_1 \cup X_2) = cl(X_1) \cup cl(X_2)$

3.5 RESULT:

In a Poset L, a dual Moore family is a subset M such that every element of L has a greatest lower bound in M.

3.6 DEFINITION: MORPHOGENETIC FIELD

Let X$\neq \varphi$ and W$\subseteq P(X)$ such that i) ϕ, X $\in$ W , ii) If B $\in$ W then its complement $\overline{B}$ $\in$W iii) If $B_i \in$W is a sequence of signals defined in X, then $\bigcup_{n=1}^{\infty} Bi \in$ W.

Let A=$\{\phi : W \rightarrow U / \phi(\cup A) = \vee\phi(A) \& \phi(\wedge A) = \wedge\phi(A)\}$

Then W_U is called Morphogenetic field [7]where the family W_u is the set of all image signals defined on the continuous or discrete image Plane X and taking values in a set U .The pair ($W_{u,}$ A) is called an operator space where A is the collection of operators defined on X.

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3.7 DEFINITION : MORPHOLOGICAL SPACE

The triplet (X, W_u, A) consisting of a set X, a morphogenetic field W_u and an operator A(or collection of operators) defined on X is called a Morphological space.

Note: If X = Z^2 then it is called Discrete Morphological space

3.8 DEFINITION: ADJUNCTION

Let $(X, W_u, A) \& (Y, W_u, \bar{A})$ be a morphological spaces. The pair $(A, \bar{A})$ is called an adjunction iff $A(X) \le Y \Leftrightarrow X \le \bar{A}(Y)$ where $\bar{A}$ is an inverse operator of A.

3.9 PROPOSITION:

Let $(X, W_u, \delta) \& (Y, W_u, \varepsilon)$ be a morphological spaces with operators dilation and erosion on A. Then $\delta(X) \le Y \Leftrightarrow X \le \varepsilon(Y)$.

3.10 PROPOSITION (FOR LATTICE):

Let $(X, W_u, A) \& (Y, W_u, \bar{A})$ be a morphological spaces. The pair $(A, \bar{A})$ is called an adjunction iff $\forall u, v \in X, \exists$ an adjunction $(l_{u,v}, m_{v,u})$ on U such that $\bar{A}(x(u)) = \vee_{v \in X} m_{v,u}(x(v))$ and $A(y(v)) = \wedge_{u \in X} l_{u,v}(y(u))$, $\forall\ u, v \in X, x, y \in W_U$.

3.11 DEFINITION:

The operator $\phi = \varepsilon \circ \delta$ defines a closure called morphological closure and $\phi^* = \delta \circ \varepsilon$ defines a kernel, called morphological kernel.

3.12 PROPERTIES:

Let (X, W_u, A) be a morphological space and let ε and δ in A. Then

i. ε and δ are increasing, $\varepsilon = \varepsilon\delta\varepsilon$ and $\delta = \delta\varepsilon\delta$.[14]

ii. $\varepsilon\delta$ is a closing on A, $\delta\varepsilon$ is an opening on B.

iii. Inv($\varepsilon\delta$) = ε(B) and Inv ($\delta\varepsilon$) = δ(A).

iv. ε(B) defines a Moore family.

v. δ(A) defines a dual Moore family

vi. The restriction of δ to ε(B) is an isomorphism from $\varepsilon(B) \to \delta(A)$ whose inverse $\delta(A) \to \varepsilon(B)$ is the restriction of ε to $\delta(A)$.

vii. ε:B→ A is an erosion if it commutes with the infimum operation .That is $\forall$ (x_i, i∈I) ⊆ B, $\varepsilon(\wedge_{i\in I} x_i) = \wedge_{i \in I} \varepsilon(x_i)$.

viii. δ: A → B is a dilation if it commutes with the supremum operation. That is $\forall$ (x_i, iεI) ⊆ B, $\delta(\vee_{i\in I} x_i) = \vee_{i\in I} \delta(x_i)$.

3.13 PROPOSITION:

Let (X, W_u, A) be a morphological space and ε and δ in A. Let V and W be two sets in X. Let P be a relation between elements of V and of W. Define δ_ρ: P(V) → P(W), the dilation by ρ and ε_ρ: P(W) →P(V), the erosion by ρ as: ∀X∈P(V), δ_ρ(X) = $\{w \epsilon W / \exists v \in X, v\rho w\}$

$\forall y \in P(w), \varepsilon_\rho(Y) = \{v \in V / \forall w \in W, v\rho w \Rightarrow w \in Y\}$

3.14 DEFINITION:

Let (X, W_u, A) be a morphological space and ε and δ in A. Let V and W be two sets in X. Let P be a relation between elements of V and of W. Let N: V→P(W) and $\check{N}$: W →P(V)

$\forall v \in V, \forall w \in W, \{w \epsilon N(v) \Leftrightarrow v \in \check{N}(w) \Leftrightarrow v\rho w\}$

i.e., N(v) = $\{w \in W / v\rho w\}$ and

$\check{N}$ (W)= $\{v \in V / v\rho w\}$

When V=W, the set N(v) is called a neighbourhood function [14]or a window function.

3.15 PROPOSITION:

Let (X, W_u, A) be a morphological space and ε and δ in A. Let V and W be two sets in X. Let P be a relation between elements of V and of W. Let N: V→P(W) and $\check{N}$: W →P(V)

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For every $x \in P(V)$,

$\delta_N(x) = \bigcup_{v \in X} N(v) = \{w \in W / \tilde{N}(W) \cap X \neq \varphi\}$

For every $y \in P(W)$,

$\varepsilon_N(Y) = \{v \in V / N(v) \subseteq Y\}$

Also $(\varepsilon_N, \delta_N)$ is an adjunction.

3.16 PROPOSITION:

Let (X, W_u, A) be a morphological space .Consider a relation ρ on a set E [14]in X and the corresponding maps, N, $\tilde{N}$: E→P(E).

Then

a) ρ is reflexive.

b) δ_N is extensive

c) ε_N is anti-extensive

d) $\delta_{\tilde{N}}$ is extensive

e) $\varepsilon_{\tilde{N}}$ is anti-extensive are equivalent statements.

Proof: Let (X, W_u, A) be a morphological space .Consider a relation ρ on a set E in X. Let E be in X. Let P be a relation between elements of E and E. Let N: E→P(E) and $\tilde{N}$: E →P(E)

$\forall v \varepsilon E, \forall w \in W, \{w \in N(v) \Leftrightarrow v \in \tilde{N}(w) \Leftrightarrow v \rho w\}$

i.e., $N(v) = \{w \in W / v \rho w\}$ and

$\tilde{N}(W) = \{v \in V / v \rho w\}$

By using the definitions of Dilation, Erosion and Neighbourhood function we can prove the above.

3.17 PROPOSITION:

a) ρ is symmetrical b) $\varepsilon_{\tilde{N}} \delta_N$ is extensive

c) $\delta_N \varepsilon_{\tilde{N}}$ is anti-extensive d) $\varepsilon_N \delta_{\tilde{N}}$ is extensive

e) $\delta_{\tilde{N}} \varepsilon_N$ is anti-extensive

3.18 PROPOSITION:

a) ρ is transitive b) $\delta_N^2 \leq \delta_N$

c) $\varepsilon_N^2 \geq \varepsilon_N$ d) $\delta_{\tilde{N}}^2 \leq \delta_{\tilde{N}}$

e) $\varepsilon_{\tilde{N}}^2 \geq \varepsilon_{\tilde{N}}$.

4. MORPHOLOGICAL TRANSFORMS:

4.1 LINEAR TIME – INVARIANT SYSTEMS: DEFINITION (LTI SYSTEMS):

An LTI system is defined as a signal operator L, mapping on input signal x(t) to an output L[(x(t)] which obeys the linear super position principle L $[\sum_i a_i x_i(t)] = \sum_i a_i L[x_i(t)]$ and is time-invariant.

i.e. $L[x(t-t_0)] = [L(x)](t-t_0)$ where $\{x_i\}$ is a finite collection of signals, t_0 is an arbitrary time shift, and α_i are real or complex weights.

4.2 DEFINITION: DILATION TRANSLATION INVARIANT (DTI) SYSTEM

A signal operator $D: x \longrightarrow y = D(x)$ is called a dilation translation invariant (DTI) system if it is translation invariant.

i.e. D[x(t-to)+c]=C+[D(x)(t-to)] for any real constants to and c.

Equivalently, a system is DTI if it is time - invariant and obeys the Morphological Supremum of sums superposition principle $D[\bigvee_i c_i + x_i(t)] = \bigvee_i c_i + D(x_i(t))]$

4.3 PROPOSITION:

Any morphological dilation is a DTI system. A system is DTI if and only if its output signal is the morphological dilation of the input by its impulse response.

D is DTI $\Leftrightarrow D(x) = x \oplus g$, $g \triangleq D(\mu)$

The lines, i.e. affine signals $x(t) = \alpha(t) + b$ are Eigen functions of any DTI system D because

$D(\alpha t + b) = \bigvee_\tau \alpha(t-\tau) + b + g(\tau)$

$= \alpha t + b + G(\alpha)$ where the corresponding Eigen value is $G(\alpha) = \bigvee_t g(t) - \alpha t$ which is the slope response of the DTI system. It measures the amount of shift in the intercept of the input lines.

4.4 DEFINITION: EROSION – TRANSLATION INVARIANT (ETI) SYSTEM

The Morphological erosion is a dual operation of the dilation with respect to signal negation.

A signal operator $\varepsilon: x \rightarrow y = \{(x)\}$ is called an Erosion translation Invariant (ETI) system if it is a

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lattice erosion i.e. distributes over any infimum of input signals and is translation invariant. Equivalently ε is an ETI system if it is time - invariant and obeys the morphological infimum of sums super position principle $\varepsilon[\wedge_i c_i + x_i(t)] = \wedge_i c_i + \varepsilon[x_i(t)]$

A system is ETI iff its output is the infimal convolution of the input with the impulse response.

The affine signals x(t) = αt+b are Eigen functions of any ETI system ε because

ε (αt+b)= αt+b+F(α) with corresponding Eigen value F(α) = $\wedge_t f(t)$-α(t). F(α) is the slope response of the ETI system.

4.5 DEFINITION: CONVEX AND CONCAVE SIGNALS

Given a function f: →R, f is concave iff

f(t)>, $\frac{pf(t-q)+qf(t+p)}{p+q}$ ∀p ,q,>0 and ∀t.

A concave function is called proper if f(t)>– ∞ for atleast one t and f(t) < +∞ ∀t.

A function f is convex if – f is concave.

4.6 LEGENDRE TRANSFORM:

Let the signal x(t) be concave and have an invertible derivative x$'$ = $\frac{dx}{dt}$. The Legendre transform [13]of x is based on the concept of imagining the graph of x, not as a set of points (t, x(t)) but as the linear envelope of all its tangent lines.

4.7 PROPOSITION:

Let Fun (R^d) be the function mapping R^d into $\overline{R}$=R$\cup\{-\infty, \infty\}$, which defines a complete lattice under the partial order given by point wise inequality $f_1 \leq f_2$,$f_1(x) \leq f_2(x)$ for every x∈R^d, f∈C⟹f(x) = C∀x ∈R^d

4.8 FUN R^D DEFINES A MOORE FAMILY:

Since **Fun R^d** defines a complete lattice, ∃ a least upper bound for every subset M of **Fun R^d**. Therefore we can prove that **Fun R^d** defines a Moore Family.

4.9 PROPOSITION:

f∈M⊆Fun R^d is upper semi continuous iff M is a Moore family.

Proof:

Since the function f∈M⊆Fun R^d is upper semi continuous if, for every t ∈ $\overline{R}$ and x∈R^d ,

f(x)$< t \Rightarrow f(y) < t$ for every y in some neighbuorhood of x. ∴M is a Moore family. Similarly M ⊆Fun R^d defines a Moore family implies that f is upper semi continuous. [M=Fun$_u$R^d].

Similarly l.s.c functions defines a dual Moore familiy.

4.10 DEFINITION: UPPER SLOPE TRANSFORM

Given a signal f, its upper slope transform[13] is defined as $A_\vee$(f)(v) = $\vee_{x\in R^d} f(x)$ – <x,v>, v∈R^d.

Upper and lower slope transforms provide information about the slope content of signals. It also give a description of morphological systems in a slop domain.

4.11 DEFINITION: ADJOINT UPPER SLOPE TRANSFORM

The adjoint upper slope transform $\overleftarrow{A_\vee}$ is defined as $\overleftarrow{A_\vee}(g)(x) = \wedge_{v\in R^d} g(v) + < x, v >$for a function $g: R^d \to \overline{R}$. The upper slope transform maps the affine function $x \mapsto < x, v_0 > +b$ onto an upper impulse which equals b for $v = v_0$ and +∞ elsewhere. By applying $\overleftarrow{A_\vee}$ to this upper impulse, the original input function $x \mapsto < x, v_0 > +b$ is obtained.

4.12 PROPOSITION:

$(\overleftarrow{A_\vee}, A_\vee)$ is an adjunction on Fun (R_d) .

i.e. $A_\vee(f) \leq$g$\Leftrightarrow f \leq \overleftarrow{A_\vee}(g)$

Proof:

If part: $A_\vee(f) \leq$g $\Rightarrow f(x)$ – <x,v>$\leq g(v), x \in R^d$, v∈R^d.

$f(x) \leq g(v) + < x, v >$for $x \in R^d$, v∈R^d

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$\Rightarrow f(x) \leq \wedge_{v\epsilon R^d} g(v) + < x, v > \ for\ x \in R^d$

.

Ie. $f \leq \overleftarrow{A_\vee}(g)$.

Similarly, only if part.

4.13 PROPERTIES OF AV AND $\overleftarrow{A_\vee}$:

$(\overleftarrow{A_\vee}, A_\vee)$be an adjunction implies

i. $\overleftarrow{A_\vee}$ and $A_\vee$ are increasing operators.

ii. $A_v = A_v \overleftarrow{A_\vee} A_v$ and $\overleftarrow{A_\vee} = \overleftarrow{A_\vee} A_v \overleftarrow{A_\vee}$

iii. $\overleftarrow{A_\vee} A_\vee$ is a closing on Fun (R^d).

iv. $A_\vee \overleftarrow{A_\vee}$ is an opening on Fun (R^d).

v. $A_\wedge$ defines a Moore family where $A_\wedge$is lower slope transtor.

vi. Av is a Dual Moore family.

4.14 DEFINITION:SUPPORT FUNCTION

For a set X$\sqsubseteq R^d$, its support function σ(x) is defined by[13] σ(x) (v)=$\vee_{x\epsilon X} < x, v > , v\epsilon R^d$. Support function is the point wise supremum of the affine functions v$\longmapsto$<x,v>,x$\in$X

$\overleftarrow{\sigma}$ (f) is defined as $\overleftarrow{\sigma}$ (f) $= \cap_{v\epsilon R^d} \overline{H}(v, f(v))$.

$\overleftarrow{\sigma}$ is a closed convex set for every function f.

$\overline{H}$(v, f(v))= {xϵR^d/<v, x> $\leq$ f(v)}

4.15 PROPOSITION:

$(\sigma, \overleftarrow{\sigma})$is an adjunction between Fun (R^d) and P(R^d).

i.e σ(x) $\leq$ f$\Leftrightarrow X \sqsubseteq \overleftarrow{\sigma}$ (f)

Proof:

Let σ(x) $\leq$ f and x$\in$X.

If part: $< x, v > \leq \sigma(x)(v) \leq f(v)$

$\Rightarrow x \in \overline{H}(v, f(v)) \ \forall\ v\epsilon R^d$.

Only if part:

Let $X \sqsubseteq \overleftarrow{\sigma}$ (f)= $\cap_{v\epsilon R^d} \overline{H}(v, f(v))$.

.$\Rightarrow X \sqsubseteq H(v, f(v)) \ \forall\ v\epsilon R^d$.

$\therefore \sigma(x)(v) \leq f(v) \Rightarrow \sigma(x) \leq f$

$\because X \sqsubseteq \overline{H}(v,b)$ iff $\sigma(x)(v) \leq b$.

4.16 PROPOSITION:

σ defines a Dual Moore family.

Since X $\sqsubseteq \overline{H}$ (v ,b), σ (x) (v) $\leq b \Rightarrow \sigma$ defines a Dual Moore family.

Also $\sigma_\wedge(X)(v) = \wedge_{x\epsilon X} < x, v >$ defines a Moore family.

4.17 PROPOSITION:

Let N: P (R^d) $\rightarrow$Fun (R^d) and$\tilde{N}$: Fun (R^d) $\rightarrow$P(R^d).

Define a relation ρ as $\forall$ v$\in$R^d, w$\in$Fun (R^d)

w$\in$Fun (R^d) iff (w) iff v$\in \tilde{N}(w)$ Iff v ρ w where

v ρ w $\Rightarrow$Av (f)(v) = W = $\vee_{x\epsilon R^d} f(x)$ – <x,v>

v$\in$R^{d}and also N(v) = { w$\in$Fun (R^d) / v ρ w},

$\tilde{N}(w)$ = {v$\in$R^d / v ρ w}.

4.18 PROPOSITION:

For every X$\in$P(R^d),

$\sigma_N(X) = \cup_{v\epsilon X = R^d} N(v) = \{w\epsilon Fun R^d / \tilde{N}(w) \cap X \neq \phi\}$

For every Y$\in$Fun(R^d) $\overleftarrow{\sigma_{\tilde{N}}}(y) = \{v\epsilon R^d / N(v) \sqsubseteq Y\}$.

4.19 PROPOSITION:

$A_\vee(f)$=$\vee_{v\epsilon R^d} N(v)and\overleftarrow{A_\vee}(f) = \{v\ \epsilon R^d / N(v) \sqsubseteq Y\}$

4.20 PROPOSITION:

Let V be a non empty set. Define the binary operation $\oplus$ as Dilation on V such that (V,$\oplus$) is an abelian Monoid. Let '$\leq$' be a partial order relation on V such that (v,$\leq$) is a poset. Define an equivalence relation '~'on V, then (V, $\oplus$, $\leq$, N) is said to be MOPE if the following conditions are satisfied.

i. If a ~b, c ~d then a$\oplus$c~b$\oplus$d, a,b,c,d $\in$V

ii. If e$\leq$a, e$\leq$b then e$\leq a\oplus$b

MOPE is an algebraic structure – Monoid , Poset, Equivalence

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5. CONCLUSIONS:

In this paper we presented the relationship between Moore family and Morphological operators. We hope that this analysis is useful for a better treatment of images and signals. Algebraic structures play an important role in finding new applications of Mathematical Morphology. We hope that this paper give an edge towards new ideas in this field. Conventional method to process an image is by using Fourier and Discrete Fourier approach. Mathematical Morphology is purely based on sets and algebraic structures. So it is more useful than Fourier operators. Repeated application of Morphological operators in various combinations gives us new operators which are more useful for getting information about the images and signals. So ,in the construction of operators, these ideas are very important. So, such a theoretical frame work and analysis is necessary for improving the efficiency of operators and enriching the theory.

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