

# DIFFERENTIAL EVOLUTION OPTIMIZATION COMBINED WITH CHAOTIC SEQUENCES FOR OPTIMAL DESIGN OF SWITCHED RELUCTANCE MACHINE

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## ABSTRACT

This paper presents Differential Evolution (DE) algorithm combined with chaotic sequences for design optimization of Switched Reluctance Machine (SRM). Two differential evolution approaches based on chaotic sequences using logistic equation are proposed in this work. Stator and rotor pole arc of SRM considered as design variables with the objective of maximizing torque density, maximizing inductance ratio and minimizing copper loss. The feasibility of the proposed method is demonstrated for an 8/6, four-phase, 5 HP, 1500 rpm SRM and compared with classical differential evolution and Genetic Algorithm (GA) method. The results show that the proposed method is effective and robust.

**Keywords:** *Differential Evolution(DE), Switched Reluctance Machine(SRM), Chaotic Sequences, Torque Density, Inductance ratio*

## 1. INTRODUCTION

Switched Reluctance Motors (SRMs) are widely used in various applications due to their inherent simplicity and rugged construction[1,2]. In [2] the fundamentals of SRM design and performance prediction have been described. SRM is intended to operate in deep magnetic saturation to increase the output power density in contrast to traditional motors. Due to the effect of saturation and the variation of magnetic reluctance, the flux-linkage, inductance and torque characteristics are highly nonlinear functions of both rotor position and phase current[3]. These nonlinearities make the design and analysis of SRMs difficult. The conventional design method of a SRM is to maximize the overall static average torque or minimize the torque ripple by using optimal machine geometry. Several design parameters, such as the number of phases, pole arc, bore diameter, air gap, etc., should be tailored according to the requirements of a specific application. From the literature [4], [5] it is evident that torque output as well as the torque ripple are sensitive mainly to stator and rotor pole arcs. Hence this work focuses on pole shape optimization of SRM. In [6], an approach to determine optimum pole arc of SRM to maximize average torque and minimize torque ripple using generalized regression

neural network based optimization is discussed. In [7,8] Genetic Algorithm (GA) have been applied for optimal design of SRM. In this paper, the pole shape optimization of SRM is approached as a constrained nonlinear optimization problem with the objective of maximizing torque density, maximizing inductance ratio and minimizing copper loss. This paper introduces an optimization technique for SRM motivated by two differential evolution approaches based on chaotic sequences using logistic equation [9]. The main advantage of traditional DE based optimization algorithm introduced by [10] over other modern heuristics is finding the true global minimum of a multi modal search space regardless of the initial parameter values, fast convergence, versatility and use of few control parameters. To improve the performance of DE, the use of chaotic sequences combined with DE has been reported in literature and this technique has been successfully applied to various optimization problems [11,17]. The feasibility of the proposed method is demonstrated for an 8/6, four-phase, 5 HP, 1500 rpm SRM and compared with classical differential evolution and Genetic Algorithm (GA) method. The results show that the proposed approach performs better in terms of solution quality, accuracy and convergence time. The organization of paper is as follows. In section

2, the problem formulation is explained, while the DE algorithm with chaos is briefly introduced in Section 3. The performance of the algorithm presented and compared in Section 4. Finally, conclusions are given in Section 5.

## 2. DESIGN OPTIMIZATION OF SRM- PROBLEM FORMULATION

### 2.1 Performance criteria

The structure of 8/6 SRM is shown in figure 1. The three criteria used to evaluate the design of SRM are average torque per volume, copper loss and inductance ratio.

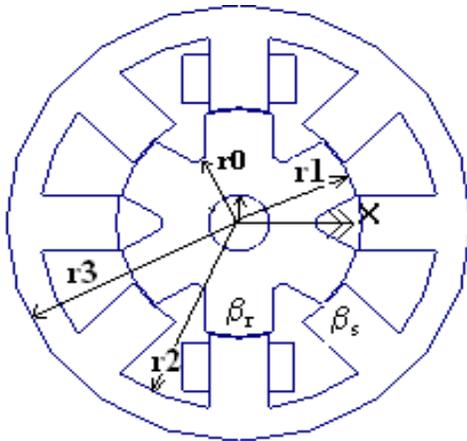


Figure1. Structure of SRM

The computation of average torque is given by

$$T_{ave} = \frac{(W_a - W_u)N_s N_r}{4\pi} \quad (1)$$

$$W_a = \int_0^{I_r} L_a di \quad (2)$$

$$W_u = \frac{1}{2} I^2 L_u \quad (3)$$

where  $I$  represents the rated phase current,  $L_a$  represents the inductance at the fully aligned position and  $L_u$  represents the inductance at the complete unaligned position. A comprehensive program is written in Matlab to compute the difference of co energies at aligned and unaligned position using the procedure described by [1].

The motor lamination volume is calculated as

$$V = V_s + V_r \quad (4)$$

where  $V_s$  represents the volume of stator lamination and  $V_r$  represents the volume of rotor lamination. Consequently, the average torque per motor lamination volume is determined as

$$TV = \frac{T_{ave}}{V} \quad (5)$$

The copper loss is computed as

$$P_{cu} = I^2 R_s$$

where  $R_s$  represents the phase resistance.

Torque ripple expected from SRM is evaluated from the torque dips in T-i- $\theta$  characteristics. Torque dip is the difference between the peak torque of a phase and the torque at an angle where two overlapping phases produce equal torque at equal levels of current. This is due to the deficiency of the incoming phase in supplying the necessary torque in those rotor positions [12]. The effect of pole arc variation on mean torque and torque dip can be evaluated from Inductance overlap ratio  $K_L$  given by equation (6). Inductance overlap ratio gives a direct measure of torque overlap of adjacent phases.

$$K_L = 1 - \frac{\epsilon}{\min(\beta_s, \beta_r)} \quad (6)$$

From equation (12) it is evident that by widening the stator and rotor poles, torque overlap can be increased. The higher the  $K_L$ , the lower will be the torque dip and the higher will be the mean torque as well.

### 2.2. Objective function

The three criteria are selected as the design objectives of SRM. The multiobjective problem formulation is given by

$$F_{opt} = (-W_{tv} * \frac{TV}{TV_b} + W_{cu} * \frac{CL}{CL_b} - W_{kl} * \frac{K_L}{K_{Lb}}) \quad (7)$$

where

$$TV_b = \max(TV)$$

$$CL = \min(CL_b)$$

$$K_{Lb} = \max(K_L)$$

$$W_{tv} + W_{cu} + W_{kl} = 1$$

In the above equation  $TV$  denotes average torque per volume,  $CL$  per copper loss and  $K_L$  denotes inductance ratio.  $W_{TV}$ ,  $W_{cu}$  and  $W_{kl}$  represent the weight factors of the average torque per volume, copper loss and inductance ratio.  $TV_b$  represent the base value of average torque per volume,  $CL_b$  represents the base value copper loss and  $K_{Lb}$  represents the base value of inductance ratio. From equation it is seen that the optimization with three objectives is simplified to an optimization function



by using three weight factors. Various weight factors indicate the shares which are taken up by average torque per motor volume, copper loss and inductance ratio in the objective function. In this work a weight factor of 1/3 is considered for all the objectives. Since the torque density and inductance ratio of the machine has to be maximized, the fitness function to minimize is taken equal to minus the average torque per motor volume and inductance ratio.

**2.2. Design constraints**

The following are the constraints are imposed on the design optimization problem according to the rules of feasible triangle[4].

$$\beta_r \geq \beta_s \tag{8}$$

$$\frac{2\pi}{Nr} - \beta_r > \beta_s \tag{9}$$

$$\beta_s > \varepsilon \tag{10}$$

To have a practically feasible and acceptable final design the following performance constraints are imposed.

- (i) Average torque should be greater than 21 N-m.
- (ii) Clearance space between the tips of windings should be greater than 5 mm. The constraints are taken into account by penalizing the fitness proportionally to the constraint violations.

**3. OPTIMIZATION USING DIFFERENTIAL EVOLUTION**

A DE algorithm is a stochastic parallel direct search optimization method that is fast and reasonably robust. DE combines simple arithmetical operators with the classical operators of recombination, mutation and selection to evolve from a randomly generated starting population to a final solution. The fundamental idea behind DE is a scheme whereby it generates the trial parameter vectors. In each step, the DE mutates vectors by adding weighted, random vector differentials to them. If the cost of the trial vector is better than that of the target, the target vector is replaced by the trial vector in the next generation. Price and Storn [13] proposed 10 different variants for DE based on the individual being perturbed, the number of individuals used in the mutation process and the type of crossover used. Each strategy generates trial vectors by adding the weighted difference between other randomly selected members of the population. The general convention used above is DE/x/y/z. DE stands for differential evolution, x represents a

string denoting the vector to be perturbed, y is the number of difference vectors considered for perturbation of x, and z stands for the type of crossover being used exponential or binomial. In this work mutation strategy DE/best/1/bin is used. In this scheme the vector to be perturbed is the best vector of the current population and the perturbation is caused by single difference vector. The optimization procedure of DE/best/1/bin is given by the following steps and procedures

**Step 1: Parameter setup**

The user chooses the parameters of population size, the boundary constraints of optimization variables, the mutation factor (*F*), the crossover rate (*CR*), and the stopping criterion of maximum number of iterations (generations), *Gmax*.

**Step 2: Initialization of the population**

Set generation *G* = 0. Initialize a population of *NP* individuals with random values generated according to a uniform probability distribution in the *D* dimensional problem space. These initial values are chosen randomly within user defined bounds.

**Step 3: Evaluation of the population**

Evaluate the fitness value of each individual of the population.

**Step 4: Mutation operation (or differential operation)**

Mutation is an operation that adds a vector differential to a population vector of individuals. For each target vector a mutant vector is produced using the following formula

$$v_i(t+1) = x_{best}(t) + F \cdot (x_{i,2}(t) - x_{i,3}(t)) \tag{11}$$

In the above equation, *i* = 1, 2, . . . , *N* is the individuals index of population, *x<sub>i</sub>(t)* stands for the position of the *i*-th individual of population of real-valued *n*-dimensional vectors, *v<sub>i</sub>(t)* stands for position of the *i*-th individual of a mutant vector, *F* is a real parameter, called mutation factor, which controls the amplification of the difference between two individuals so as to avoid search stagnation. The mutation strategy perturbs the best vector of the current population by single difference vector. The two individuals *x<sub>i,2</sub>(t)* and *x<sub>i,3</sub>(t)* are randomly selected and the difference vector is calculated.

**Step 5: Crossover operation**

To increase the potential diversity of the population a crossover operator is used. DE uses two kinds of



cross over schemes namely “Exponential” and “Binomial”. In this work binomial crossover is used. In this crossover scheme, the crossover is performed on each of the D variables whenever a randomly picked number between 0 and 1 is within the CR value. The scheme may be outlined as

$$u_{i,j}(t) = v_{i,j}(t) \text{ if } (\text{rand}(0,1)) < CR$$

$$= x_{i,j}(t) \text{ else....} \quad (12)$$

In this way for each trial vector  $\vec{X}_i(t)$  an offspring vector  $\vec{U}_i(t)$  is created.

**Step 6: Selection operation**

Selection operator is used to determine which one of the target vector and the trial vector will survive in the next generation. The selection process may be outlined as

$$\vec{X}_i(t+1) = \vec{U}_i(t) \text{ if } f(\vec{U}_i(t)) \leq f(\vec{X}_i(t))$$

$$= \vec{X}_i(t) \text{ if } f(\vec{X}_i(t)) < f(\vec{U}_i(t))..(13)$$

where f is the function to be minimized. If the new trial vector yields a better value of the fitness function, it replaces its target in the next generation otherwise the target vector is retained in the population. Once new population is installed, the process of mutation, recombination and selection is replaced until the optimum is located, or a specified termination criterion is satisfied, e.g., the number of generations reaches a preset maximum Gmax. At each generation, new vectors are generated by the combination of vectors randomly chosen from the current population (mutation). The upcoming vectors are then mixed with a predetermined target vector. This operation is called recombination and produces the trial vector. Finally, the trial vector is accepted for the next generation if it yields a reduction in the value of the objective function.

**3.1 Differential evolution with chaotic approaches**

The three vital control parameters of DE are the population number, the mutation factor and the crossover rate. The speed and robustness of the search are affected with the variation of these parameters. The difficulty in the use of DE arises in view of the fact that the choice of these is mainly based on empirical evidence and practical experience [14,17]. DE’s parameters usually are constant throughout the entire search process. However, it is difficult to properly set control parameters in DE. The application of chaotic sequences in mutation factor design is a powerful strategy to diversify the DE population and improve

DE’s performance in preventing premature convergence to local minima[17]. The application of chaotic sequences can be a good alternative to provide the search diversity in stochastic optimization procedures. Due to the ergodicity property, chaos can be used to enrich the searching behavior and to avoid being trapped into local optimum in optimization problems. In this paper, to enrich the searching behavior and to avoid being trapped into local optimum, chaotic dynamics is incorporated into the DE. In this context, two chaotic DE approaches are proposed. Proposed different chaotic DE approaches have used the well-known logistic equation, which exhibits the sensitive dependence on initial conditions, for determining the mutation factor. The logistic equation is defined as follows

$$y(k) = \mu * y(k-1) * (1 - y(k-1)) \quad (14)$$

Where k is the sample and  $\mu$  is the control parameter,  $0 < \mu \leq 4$ . The behavior of the system represented by equation (14) is greatly changed with the variation of  $\mu$ . The value of  $\mu$  determines whether ‘F’ stabilizes at a constant size, oscillates between a limited sequence of sizes, or behaves chaotically in an unpredictable pattern. And also the behavior of the system is sensitive to initial value of ‘F’ [10]. Equation (14) is deterministic, displaying chaotic dynamics when  $\mu=4$  and  $y(1) \notin \{0,0.25,0.5,0.75,1\}$ .

The two chaotic DE (CDE) approaches in combination of chaotic sequences are described as follows

**CDE1 approach:** The parameter ‘F’ of (11) is modified by the formula (15) through the following equation:

$$v_i(t+1) = x_{best}(t) + y(k) \cdot (x_{i,2}(t) - x_{i,3}(t)) \quad (15)$$

**CDE2 approach:** The parameter ‘F’ of (11) is modified by the formula (16) through the following equation:

$$F = y(k) * \left| \exp\left(\frac{-G}{G_{max}}\right) \right| \quad (16)$$

**4. RESULTS**

The performance of the proposed method is tested on a 5HP motor. The specifications of the sample motor are given in Appendix 1. The algorithm is coded in Matlab and executed using a Pentium IV based PC as the test platform. During the process the following parameter setting is used for

traditional DE: Population size=30, Crossover constant =0.7, Scaling Factor for Mutation=0.8, maximum iteration Itermax = 100 Upon execution of the algorithm, an optimal structure with the configuration  $\beta_s=21.91$  and  $\beta_r=24.08$  is obtained. The performance parameters of the optimal motor design are given in table1. From the table it is clear that there is significant improvement in torque density and inductance ratio.

Table 1 Results of Optimal Design

	Initial Design	Optimal Design
Stator Pole arc	18 deg	21.91deg
Rotor Pole arc	22 deg	24.08 deg
Average Torque	23.14 Nm	29.31Nm
Torque Density	1252 Nm/m <sup>3</sup>	1456 Nm/m <sup>3</sup>
Inductance ratio	0.1667	0.3154
Copper Loss	183 W	190 W
Torque dip	8.87 Nm	4.46 Nm

Table 2 Convergence results of 20 runs using GA, DE and CDE approaches

	GA	DE	CDE1	CDE2
Best Solution	0.3010	0.3011	0.3011	0.3011
Worst Solution	0.0059	0.1582	0.2703	0.2717
Mean	0.2532	0.2989	0.3000	0.3000
Standard Deviation	0.0893	0.0118	0.00392	0.0042

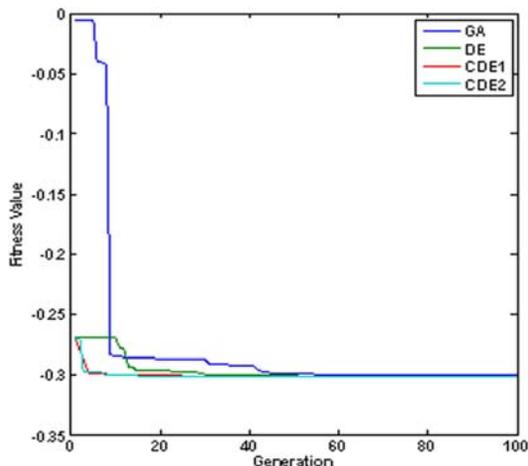


Figure2. Convergence characteristics of different optimization methods

#### 4.1 Comparative studies

In order to verify the robustness of the algorithms, simulations were carried out for 20 independent runs. The results of statistical comparison are summarized in table2. It is seen that in terms of mean and standard deviation CDE1 approach performed significantly better. The performance of the optimization technique in terms of convergence is shown in fig2. From the figure it is evident that the convergence characteristics of chaotic DE approaches are better.

#### 5. CONCLUSION

In this paper, two DE approaches based on chaotic sequences using logistic equation to adapt the mutation factor for design optimization of SRM are proposed. The objective of the proposed chaotic DE approaches is to maximize torque density, maximize inductance ratio and minimize copper loss. The optimized geometry was exposed to finite-element calculation. The optimal machine produced an average torque of 28.96Nm with a torque dip of 4.46 Nm. The results of finite-element calculation confirm the application of optimization procedure for SRM design. Both the traditional DE and the CDE approaches were successfully applied to design optimization of SRM. The CDE approaches can be used as promising optimization methods for solving SRM design problems.

#### APPENDIX 1

##### Design Data of the machine

Machine configuration	8/6
Power output	5 hp
Stator pole arc	18 degrees
Rotor pole arc	22 degrees
Air gap length	0.5 mm
Outer stator diameter	190 mm
Bore diameter	100.6 mm
Stack length	200 mm
Shaft diameter	28 mm
Speed	1500 rpm
Height of stator pole	32.7 mm
Height of rotor pole	19.8 mm
Turns per phase	154
Rated current	13 A



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