

# AREA-TIME OPTIMAL VLSI INTEGER MULTIPLIER WITH MINIMUM COMPUTATIONAL TIME AND WEBER PROBLEM

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## ABSTRACT

The Ill-post problem on a VLSI Design namely area-time optimal VLSI Integer multiplier with minimum computation time can be regarded as a weber problem. The elementary results with extension leading to the classical problem on VLSI integer multiplier can be posed and settled in affirmative.

**Keywords:** *VLSI, Multiplier, Extension, Computation Time*

## 1. INTRODUCTION

The random locational equilibrium problem also known as Weber problem can be stated as follows. There are  $n$  locations (destination or sinks) in the two dimensional Euclidean space. It is required to find the location of a facility so that the sum of the weighted distance from the location of the facility to the destination is minimized.

The problem has been considered by Cooper [1963] when the location of the destinations is deterministic. Katz and Cooper [1974] have presented a scheme numerical solution. Cooper [1974] has considered the problem when the location of the destination is stochastic in nature and the pair of coordinators follows a bivariate distribution. In this case it is recommended to minimise the sum of weighted average of linear distances of the destinations from the location facility. Although Cooper [1974] has presented the general algebraic expression of the equations which would lead to the optimum solution of the problem the numerical solution could be very involved. Wesolowsky[1970] has recommended the use of rectangular distance instead of linear distance and derived the optimum solution when the coordinates of the destination follow certain

known forms of bivariate probability distribution.

Mustafi (1982) has given a simple solution of the problem when the coordinates of the destination follow a bivariate uniform distribution. We present our extension to bivariate normal distribution.

In VLSI theory we have  $AT^2$  optimal problem which aims at minimizing computation time in the integer multiplication. This problem can be considered as a location optimization distance problem. This is studied by Mehlhorn.K and Preparata.F.P in early 1985 as a computational complexity problem.

## 2. A BIVARIATE DISTRIBUTION AND OPTIMISING EQUATION

Suppose  $q$  with coordinates  $(x_0, y_0)$  is the location of the new facility. Let  $P_j$  with coordinates  $(x_j, y_j)$  have a bivariate uniform distribution given by

$$f(x_j, y_j) = 1/ab + 4\alpha_j / ab(x_j - b/2) \dots (1)$$

with  $0 < x_j < a$ ,  $0 < y_j < b$ ,  $-1 < \alpha_j \leq 1$ ,  
 $j=1,2,\dots,n$ .

The  $j^{\text{th}}$  destination is randomly distributed in the rectangle bounded by two axes and the lines  $x=a$  and  $y=b$ . The quantity  $\alpha_j$  is a parameter indicating the degree of dependence. If  $\alpha_j = 0$ ,  $x_j$  and  $y_j$  are independent.

The linear distance between  $p_j$  and  $q$  is given by

$$d_j = [(x_j - x_0)^2 + (y_j - y_0)^2]^{1/2} \dots\dots (2)$$

suppose  $w_j > 0$  ( $i=1,2,\dots,n$ ) is the weight assigned to the  $j^{\text{th}}$  destination. Then the coordinates  $(x_0, y_0)$  of the new facility should be chosen in such a manner that

$$z = \sum_{j=1}^n w_j E(d_j) \text{ is minimised.}$$

The optimum values of  $x_0$  and  $y_0$  are solutions of the equations.

$$\frac{\partial z}{\partial x_0} = \sum_{j=1}^n w_j \int_0^a \int_0^b \frac{(x_0 - x_j)}{d_j} f_j(x_j, y_j) dx_j dy_j = 0$$

---(3)

$$\frac{\partial z}{\partial y_0} = \sum_{j=1}^n w_j \int_0^a \int_0^b \frac{(y_0 - y_j)}{d_j} f_j(x_j, y_j) dx_j dy_j = 0$$

--- (4)

It can be shown that for any bivariate density

$$\frac{\partial^2 z}{\partial x_0^2} > 0; \frac{\partial^2 z}{\partial y_0^2} > 0$$

$$\frac{\partial^2 z}{\partial x_0^2} \cdot \frac{\partial^2 z}{\partial y_0^2} - \left( \frac{\partial^2 z}{\partial x_0 \partial y_0} \right)^2 > 0 \dots\dots\dots (5)$$

The condition given in (5) ensure that  $z$  is minimised if  $x_0$  and  $y_0$  are chosen as a solution of the equation given in (4)

### 3. THE OPTIMUM LOCATION OF FACILITY

Theorem.1 (Mustafi 1982), If the probability density function of the destination ( $j = 1, 2, \dots, n$ ) is a bivariate uniform distribution as given by (1) the optimum location of the new facility should be at

$$x_0 = \frac{a}{2}, y_0 = \frac{b}{2}.$$

Proof:

Using the form of  $f_j(x_j, y_j)$  as given in (1)

we obtain from (4)

$$\frac{\partial z}{\partial x_0} = \sum_{j=1}^n w_j \int_0^a \int_0^b \frac{(x_0 - x_j)}{[(x_j - x_0)^2 + (y_j - y_0)^2]^{3/2}} \left[ \frac{1}{ab} + \frac{4\alpha_j}{ab} \left( x_j - \frac{a}{2} \right) \left( y_j - \frac{b}{2} \right) \right] dx_j dy_j = T_1 + T_2 \dots\dots\dots (7)$$

$$T_1 = \frac{1}{ab} \int_0^a \int_0^b \frac{(x_0 - x_j)}{[(x_j - x_0)^2 + (y_j - y_0)^2]^{3/2}} dx_j dy_j$$

$$= \frac{1}{ab} \int_0^b \left[ \int_{-x}^{a-x_0} \frac{u_j}{(u_j^2 + (y_j - y_0)^2)^{3/2}} du_j \right] dy_j$$

$$\text{At } x_0 = \frac{a}{2}, y_0 = \frac{b}{2}$$

$$T_1 = -\frac{1}{ab} \int_0^b \left\{ \left[ \frac{a^2}{4} + \left( y_j - \frac{b}{2} \right)^2 \right]^{1/2} - \left[ \frac{a^2}{4} + \left( y_j - \frac{b}{2} \right)^2 \right]^{3/2} \right\} dy_j = 0 \dots\dots\dots (8)$$

$$T_2 = \frac{4\alpha_j}{ab} \int_0^a \int_0^b \frac{\left( x_j - \frac{a}{2} \right) \left( y_j - \frac{b}{2} \right) (x_0 - x_j)}{[(x_j - x_0)^2 + (y_j - y_0)^2]^{3/2}} dx_j dy_j$$

$$\text{At } x_0 = \frac{a}{2}, y_0 = \frac{b}{2}$$

$$T_2 = -\frac{4\alpha_j}{ab} \int_0^a \int_0^b \frac{\left( x_j - \frac{a}{2} \right) \left( y_j - \frac{b}{2} \right)}{\left[ \left( x_j - \frac{a}{2} \right)^2 + \left( y_j - \frac{b}{2} \right)^2 \right]^{3/2}} dx_j dy_j$$

$$= -\frac{4\alpha_j}{ab} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left\{ \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{u_j^2}{(u_j^2 + v_j^2)^{\frac{1}{2}}} du_j \right\} v_j dv_j$$

$$= -\frac{8\alpha_j}{ab} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left\{ \frac{a}{4} \sqrt{\frac{a^2}{4} + v_j^2} - \frac{v_j^2}{2} \log \left( \frac{a}{2} + \sqrt{\frac{a^2}{4} + v_j^2} \right) + \frac{v_j^2}{2} \log \sqrt{v_j^2} \right\} v_j dv_j$$

Since the integrand is an odd functioning, it vanishes. Hence

$$T_2 = 0 \dots \dots \dots (9)$$

In a similar way we can show  $\frac{\partial z}{\partial y_0} = 0$ . This

completes the proof of the theorem.

With the same notation of theorem 1, we states & prove the following result.

Theorem 2. The minimum value of z is given by

$$W \left[ \frac{1}{6} \sqrt{a^2 + b^2} + \frac{b^2}{12a} \log \left( \frac{a}{2} + \frac{1}{2} \sqrt{a^2 + b^2} \right) + \frac{a^2}{12b} \log \left( \frac{b}{2} + \frac{1}{2} \sqrt{a^2 + b^2} \right) \right] \dots \dots \dots (11)$$

Where

$$W = \sum_{j=1}^n w_j \dots \dots \dots (10)$$

Proof: follows by using  $x_0 = \frac{a}{2}, y_0 = \frac{b}{2}$  and the notation of the expectation of continuous random variable. We note that if  $n = 1, w_1 = 1, a = b$ , the minimum expected linear distance is given by

$$= \left[ \frac{1}{6} \sqrt{a^2 + b^2} + \frac{a^2}{12a} \log \left( \frac{a}{2} + \frac{1}{2} \sqrt{a^2 + b^2} \right) + \frac{a^2}{12a} \log \left( \frac{a}{2} + \frac{1}{2} \sqrt{2a^2} \right) - \frac{a^2}{12a} \log \frac{a}{2} - \frac{a^2}{12a} \log \frac{a}{2} \right]$$

$$= \frac{a}{6} \left( \sqrt{2} + \log(\sqrt{2} + 1) \right)$$

$$= .383 a$$

#### 4. AN EXTENSION OF THE PROBLEM

Mustafi (1982) has considered the solution of the problem when the coordinates of the destination follow a bivariate uniform distribution. Here we consider the solution of the problem when coordinates of the destination follow a bivariate normal distribution.

#### 5. DISTRIBUTION AND THE OPTIMISING EQUATIONS

Suppose Q with coordinates  $(x_0, y_0)$  is the location of the new facility. Let  $p_j$  with coordinates  $(x_j, y_j)$  be the location of the  $j^{th}$  destination ( $j = 1, 2, \dots, n$ ). It is assumed  $(x_j, y_j)$  has a bivariate normal distribution given by

$$f(x_j, y_j) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp \left( -\frac{1}{2(1-\rho^2)} (x_j^2 - 2\rho x_j y_j + y_j^2) \right)$$

$$-\infty < x_j < \infty$$

$$-\infty < y_j < \infty \dots \dots \dots (11)$$

The linear distance between  $p_j$  and Q is given by

$$d_j = \left[ (x_j - x_0)^2 + (y_j - y_0)^2 \right]^{\frac{1}{2}} \dots \dots \dots (12)$$

Suppose  $w_j > 0 (j = 1, 2, \dots, n)$  is the weight assigned to the  $j^{th}$  destination. Then the coordinates  $(x_0, y_0)$  of the new facility should be chosen in such a way that

$$z = \sum_{j=1}^n W_j E(d_j) \dots \dots \dots (13) \text{ is minimized. The}$$

optimum values of  $x_0$  and  $y_0$  are the solutions of the equations  $\frac{dz}{dx_0} = 0 = \frac{dz}{dy_0} \dots \dots \dots (14)$

For any bivariate density function  $\frac{\partial^2 z}{\partial x_0^2} > 0, \frac{\partial^2 z}{\partial y_0^2} > 0,$

$$\frac{\partial^2 z}{\partial x_0^2} \frac{\partial^2 z}{\partial y_0^2} - \left( \frac{\partial^2 z}{\partial x_0 \partial y_0} \right)^2 > 0 \dots \dots \dots (15)$$

The condition given in (15) ensures Z is minimized if  $x_0$  and  $y_0$  are chosen as solution of the equation given in (14).

## 6. THE OPTIMUM LOCATION OF THE FACILITY

We state and prove our result for the bivariate normal case theorem 3. If the probabilities density function of the destination ( $j = 1, 2, \dots, n$ ) is bivariate normal distribution given by (11) the optimum location of the new facility should be at  $x_0 = 0, y_0 = 0$

Proof: using the form of  $f(x_j, y_j)$  as given in (11)  
We obtain from (14)

$$\frac{\partial z}{\partial x_0} = \sum_{j=1}^n \int_{-\infty}^{\infty} w_j \int_{-\infty}^{\infty} \frac{x_0 - x_j}{[(x_j - x_0)^2 + (y_j - y_0)^2]^{3/2}} \times \frac{1}{2\pi\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} (x_j^2 - 2\rho x_j y_j + y_j^2) \right\} dx_j dy_j$$

----(16)

To evaluate the integral in (16), let us note  
 $u_1^2 - 2\rho u_1 u_2 + u_2^2 = (1-\rho^2)u_1^2 + (u_2 - \rho u_1)^2$   
we may write

$$f(x_j, y_j) = \phi(x_j) \frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{x_j - \rho y_j}{\sqrt{1-\rho^2}}\right) \text{ in}$$

$$\text{which } \phi(u) = \frac{1}{\sqrt{1-\rho^2}} \exp\left(-\frac{u^2}{2}\right) \text{ we thus}$$

determine

$$\frac{dz}{dx_0} = \sum_{j=1}^n \int_{-\infty}^{\infty} w_j \int_{-\infty}^{\infty} \frac{x_0 - x_j}{[(x_j - x_0)^2 + (y_j - y_0)^2]^{3/2}} \phi(x_j) \frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{x_j - \rho y_j}{\sqrt{1-\rho^2}}\right) dx_j dy_j$$

At  $x_0 = 0, y_0 = 0$

$$\frac{dz}{dx_0} = - \sum w_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{x_j}{(x_j^2 + y_j^2)^{3/2}} \phi(x_j) \frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{x_j - \rho y_j}{\sqrt{1-\rho^2}}\right) dx_j dy_j = 0$$

In a similar way

$$\frac{\partial z}{\partial y_0} = 0$$

This complete the proof of the theorem.

With same notation of theorem 3, we state and prove the following result

Theorem 4. Minimum value of the expression for z is less than

$$w(1-\rho^2)^{1/2} / \sqrt{\pi} \text{ where } w = \sum_{j=1}^n w_j$$

Proof: The minimum value of z is

$$\sum_{j=1}^n w_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x_j^2 + y_j^2)^{1/2} f(x_j, y_j) dx_j dy_j \leq w \times E(\text{Max}(x_j, y_j)) = w(1-\rho^2)^{1/2} / \sqrt{\pi}$$

from the known result of bivariate normal distribution [3]

## 7. ILLUSTRATION

The above random equilibrium problem can be utilised to find Area-Time optimal VLSI Integer multiplier with minimum computational time. According to VLSI theory,  $[\log n, \sqrt{n}]$  is the range of computational time for which they may exist an  $AT^2$  optimal multiplier of n-bits integer. We can settle this theoretical question by establishing a class of  $AT^2$  optimal multipliers with computation time  $O(\log n \cdot n^{n/2})$  by posing the problem as 2-dimensional locational equilibrium problem for a facility.

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