UNKNOWN INPUT SLIDING-MODE OBSERVER DESIGN FOR A DRUM-TYPE BOILER

[1]A. A. MODARRES, [2]H. MOMENI

[1]Ph.D.Stud., Department of Electrical Engineering, Tarbiat Modarres University, Tehran, Iran -14115

[2]Prof., Department of Electrical Engineering, Tarbiat Modarres University, Tehran, Iran -14115

E-mail: modarres@mapna.com, momeni_h@modares.ac.ir

ABSTRACT

In this paper we propose a gain matrix selection procedure for a class of full order unknown input observers (UIO). These kinds of observers employ sliding-mode principles to force the observation error into the sliding surface. Then, the error starts a sliding motion along the surface until it asymptotically converges to zero. Although this method exhibits good performance, there is not any explicit and direct method to compute the observer gains. In our approach, we consider the problem of finding the observer gains in the terms of linear matrix inequalities (LMIs). The simulation results for an industrial drum type boiler demonstrate the effectiveness of the proposed approach.

Keywords: Sliding Mode Observer, Unknown input, Boiler, LMIs.

1. INTRODUCTION

Observers are used to estimate the plant's states using only input and output information. The observer was first proposed and developed by Luenberger in 1966 [1]. In recent years, the observer design for uncertain linear systems in which their uncertainties could be considered as unknown inputs, has attracted more attention. This kind of observer plays an important role in robust model-based fault detection [2, 3]. Another fundamental application of the unknown input observer is to construct the control for systems in which all states are not physically available or measuring is very expensive [4]. In this regard, observer architectures utilizing the concept of sliding-modes have been proposed by several authors [4-6]. Other methods of unknown input observer design for linear systems could be found in [7-9].

This paper builds on the works of Zak et al. [5 and 10], which are relevant to the sliding-mode observer and has been attractive to many authors [11-12]. The mentioned method consists of a discontinuous term to enable the observer to reject unknown input effects. The discontinuous term is designed to drive the trajectories of the observer so that the estimation state error vector is forced onto and subsequently remains on a surface in the error space. This motion is referred to as sliding mode. When a sliding mode is achieved the system will experience a reduced-order motion that is insensitive to unknown input.

However, the method proposed by Zak et al. invariably requires a symbolic manipulation package to solve the synthesis problem that is formulated. Although Tan and Edwards [11] employed an LMI approach to solve the problem of finding the observer gain matrices for the sliding-mode observer, the mentioned method used sate transformation, and it requires not only finding transformation matrices but also changing coordinates to obtain the canonical form. Therefore it is indirect and more or less complex.

Considering these facts, in the present work we propose an LMI approach to find gain matrices of the sliding-mode observer, without any necessity for symbolic manipulation package or finding transformation matrices. In this way, the problem will be stated as standard forms of LMIs, and only an LMI-toolbox [13] is required to solve.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the linear time-invariant system described by:

$$\dot{x} = Ax + Bu(t) + Dv(t) \qquad (1)$$

$$y = Cx \quad (2)$$

where $x \in R^n, u \in R^{m_1}, v \in R^{m_2}$ and $y \in R^p$ are the state vector, the known input vector, the unknown input vector and the output vector of the system, respectively.

A, B, C and D are known constant matrices of appropriate dimensions. The unknown input can be a combination of unmeasurable or unmeasured disturbances, unknown control action and unmodeled system dynamics.

We will assume the followings to be valid:

A_1 The rank (CD) = rank (D) = m_2.

A_2 The pair (A, C) is detectable. Thus we may find a matrix $L \in R^{n\times p}$ such that the spectrum of $(A - LC)$ is completely contained in the open left-half plane.

Now consider the following definition and lemma as preliminaries.

Definition [14]: For a real symmetric matrix $\Theta \in S^{2k\times 2k}$ (S is the set of symmetric matrices), the set of complex numbers

$$L_p = \left\{ z \in C \mid \begin{pmatrix} I \\ zI \end{pmatrix}^* \Theta \begin{pmatrix} I \\ zI \end{pmatrix} < 0 \right\}$$

is called an LMI region. If Θ is partitioned according to

$$\Theta = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}$$

then an LMI region is defined by those points $z \in C$ for which $Q + zS + \bar{z}S^T + \bar{z}Rz < 0$.

Lemma [14]: All eigenvalues of $A \in R^{n\times n}$ are contained in the LMI region

$$\left\{ z \in C \middle| \begin{pmatrix} I \\ zI \end{pmatrix}^* \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} I \\ zI \end{pmatrix} < 0 \right\}$$

if and only if there exists $P > 0$ such that

$$\begin{pmatrix} I \\ A \otimes I \end{pmatrix}^* \begin{pmatrix} P \otimes Q & P \otimes S \\ P \otimes S^T & P \otimes R \end{pmatrix} \begin{pmatrix} I \\ A \otimes I \end{pmatrix} < 0$$

where * and $\otimes$ refer to transpose conjugate and Kronecker product respectively.

3. SLIDING MODE OBSERVER DESIGN

Consider the described system by (1) and (2). First, we assume that $v(t)$ is bounded, and there is a positive scalar, ρ, such that:

$$\|v(t)\| \le \rho \ \ for\ all \quad t \quad (3)$$

Let $\hat{x}$ be an estimate of x and let e denote the estimation error, that is, $e(t) = \hat{x}(t) - x(t)$.
Then, the vector function $E(.)$ will be defined as below [10]:

$$E(e,\eta) = \begin{cases} \eta \dfrac{FCe}{\|FCe\|} & ; \quad FCe \ne 0 \\ 0 & ; \quad FC = 0 \end{cases} \quad (4)$$

where $\eta \ge \rho$ is a design parameter and $F \in R^{m_2\times p}$ shall be computed properly. Note that in the case of single input single output, $E(e,\eta) = \eta sign(FCe)$.

This shall be considered that, in (4), Ce refers to output error.

$$Ce = C(\hat{x} - x) = \hat{y} - y$$

It could be shown that the state $\hat{x}$ of the dynamical system

$$\dot{\hat{x}} = (A - LC)\hat{x} + Ly + Bu - DE(e,\eta)\, ; \quad (\eta \ge \rho)$$
(5)

is an asymptotic estimation of the state x. Where $L \in R^{n\times p}$ is a gain matrix for observer.

The following theorem implies the necessary condition for existence of observer in form (5). Also, it gives an explicit formula for computation of L and F matrices.

Theorem 1: Suppose we have the described system by (1) and (2), and consider the assumptions A_1 and A_2 are valid. If the following linear matrix inequalities are satisfied:

$$\begin{cases} A^TP + PA - C^TK - K^TC < 0 \\ P = P^T > 0 \end{cases} \quad (6)$$

where K will be defined later, then there exists the observer governed by (5), such that:

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (\hat{x}(t) - x(t)) = 0$$

and the observer gain matrix, L, and matrix F will be computed from:

$$L = P^{-1}K^T \, ; FC = D^T P \tag{7}$$

Proof : To prove the above statement, first we construct the differential equation describing the dynamics of the estimation error e,

$$\dot{e} = \dot{\hat{x}} - \dot{x} = (A - LC)e - Dv - DE(e,\eta) \tag{8}$$

Now, consider the following positive-definite Lyapunov function candidated for equilibrium point $e(t) = 0$ of (8):

$$V(e) = e^T Pe > 0 \tag{9}$$

where P is a symmetric and positive definite matrix. Considering that $E(e,\eta)$ satisfies (5), in [5 and 10], it has been shown that $\dot{V}(e) < 0$ if the following matrix inequality is established:

$$(A - LC)^T P + P(A - LC) < 0 \tag{10}$$

and for some $F \in R^{m_2 \times p}$,

$$FC = D^T P \tag{11}$$

It is clear that (10) is not, simultaneously, an LMI for L and P. We define a matrix K such that:

$$L = P^{-1}K^T \Rightarrow L^T = KP^{-1} \tag{12}$$

Therefore, we use (12) to rewrite (10) as follow:

$$\begin{aligned}
&(A^T - C^T L^T)P + P(A^T - C^T L^T)^T < 0 \Rightarrow \\
&(A^T - C^T KP^{-1})P + P(A^T - C^T KP^{-1})^T < 0 \Rightarrow \\
&(A^T P - C^T K) + P(A - (P^{-1})^T (C^T K)^T) < 0 \Rightarrow \\
&A^T P + PA - C^T K - K^T C < 0
\end{aligned}$$

■

The last inequality is an LMI and could be easily solved by LMI programs. Note that F, in (11), could be computed from this simple formula:

$$F = D^T PC^T (CC^T)^{-1} \tag{13}$$

4. POLE CLUSTERING IN SOME REGIONS

Since many properties of system dynamics can be defined by specifying regions in the complex plane in which eigenvalues of the system lie, we will consider the problem of designing sliding surfaces guaranteeing some regional pole constraints.

The next Theorem implies the condition that guarantees the rate of convergence for all observed states. This is clear that the observer is not well conditioned, if the time response of observer is greater than the system's response.

Theorem 2: The convergence rate of the sliding-mode observer dynamics described by (5) will be more than a positive real scalar α if and only if there exists a positive definite matrix *P* such that following LMIs are feasible.

$$\begin{cases} A^T P + PA - C^T K - K^T C + 2\alpha P < 0 \\ P = P^T > 0 \end{cases} \tag{14}$$

where in this case L and F are computed from (11) and (12).

Proof: Considering this fact that for reaching a guaranteed rate of convergence, which is more than $\alpha > 0$, it is necessary and sufficient that $\mathrm{Re}(s) < -\alpha$ or $z + \bar{z} + 2\alpha < 0$ and referring to mentioned definition in section II, it is clear that

$$\Theta = \begin{pmatrix} 2\alpha & 1 \\ 1 & 0 \end{pmatrix}$$

is our desired LMI region. Also, from mentioned lemma in section II, it is easily inferred that the necessary and sufficient conditions for reaching to guaranteed rate of convergence α is

$$\begin{pmatrix} I_{n\times n} \\ (A-LC)\otimes 1 \end{pmatrix}^* \begin{pmatrix} P\otimes 2\alpha & P\otimes 1 \\ P\otimes 1 & P\otimes 0 \end{pmatrix} \begin{pmatrix} I_{n\times n} \\ (A-LC)\otimes 1 \end{pmatrix} < 0 \Leftrightarrow$$
$$(A-LC)^T P + P(A-LC) + 2\alpha P < 0 \Leftrightarrow$$

and considering that (12) is valid

$$A^T P + PA - C^T K - K^T C + 2\alpha P < 0$$

■

Now, assume that all poles of observer are required to lie in a disk of radius r and center (q,0).

Theorem 3: All the eigenvalues of the sliding-mode observer dynamics described by (5) will lie in a disk of radius r and center $(q,0)$ if and only if there exists a positive definite matrix P such that following LMIs are feasible.

$$\begin{cases} \begin{pmatrix} -rP & -K^T C - qP + PA \\ -C^T K - qP + A^T P & -rP \end{pmatrix} < 0 \\ P = P^T > 0 \end{cases} \tag{15}$$

where in this case L and F are computed from (11) and (12).

Proof: Considering this fact that the disk of radius r and center (q,0) could be described by:

$$\left\{ z \in C \left| \begin{pmatrix} -r & -q \\ -q & -r \end{pmatrix} + z\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \bar{z}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^T < 0 \right. \right\}$$

and referring to mentioned definition and lemma in section II, it is clear that

$Q = \begin{pmatrix} -r & -q \\ -q & r \end{pmatrix}$, $R = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, and then

$$\begin{pmatrix} I_{2n\times 2n} \\ (A-LC)\otimes I_{2\times 2} \end{pmatrix}^* \begin{pmatrix} P\otimes Q & P\otimes S \\ P\otimes S^T & P\otimes R \end{pmatrix} \begin{pmatrix} I_{2n\times 2n} \\ (A-LC)\otimes I_{2\times 2} \end{pmatrix} < 0$$
$$\Leftrightarrow \begin{pmatrix} -rP & -qP + (A-LC)P \\ -qP + P(A-LC)^T & -rP \end{pmatrix} < 0$$

and considering that (12) is valid

$$\begin{pmatrix} -rP & -K^T C - qP + PA \\ -C^T K - qP + A^T P & -rP \end{pmatrix} < 0$$

■

5. OBSERVER DESIGN FOR A DRUM-TYPE INDUSTRIAL BOILER

Consider the fourth-order linearized model of a drum-type boiler, which has been presented in [15] as following:

$$A = \begin{pmatrix} 3.7451\text{e-}15 & 7.6548\text{e-}6 & 0 & 0 \\ -4.0887\text{e-}16 & -6.5527\text{e-}5 & 0 & 0 \\ 2.3773\text{e-}16 & 0.00059026 & -0.1426 & 0 \\ 8.1593\text{e-}14 & 0.055355 & -18.216 & -0.083333 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.0015 & -6.9678\text{e-}12 \\ -5.9548\text{e-}5 & 5.9647\text{e-}11 \\ 3.4622\text{e-}5 & 3.2492\text{e-}11 \\ -0.0167 & -1.0733\text{e-}9 \end{pmatrix} \qquad D = \begin{pmatrix} -0.00015 \\ -9.0316\text{e-}5 \\ 5.2512\text{e-}5 \\ 0.0239 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.0500 & -0.0484 & 6.7129 & 0.0500 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The selection of state variables chosen has been suggested by [16].

These variables are total water volume (m3), drum pressure (kg/cm2), steam-water mass fraction in riser and volume of steam in drum (m3). The outputs are chosen as the drum-level (m) and drum pressure (kg/cm2).

The known inputs are feed water flow rate (kg/s) and fuel flow rate (MW), and the unknown input is steam flow rate (kg/s).

First, we compute the eigenvalues of system A. The result is {-0.08333; -0.1426; -6.5527e-5; 0.0}.

Let all eigenvalues of observer lie in a disc of radius r=0.1 and center (-1,0), then the P and K matrices and the observer gains (L and F) will be obtained from (11), (12) and (15) using LMI-toolbox of MATLAB (Version 7.3.0) as below:

$$P = \begin{pmatrix} 4.9362\text{e-}4 & -1.8812\text{e-}5 & 7.3609\text{e-}2 & 5.0936\text{e-}4 \\ -1.8812\text{e-}5 & 8.2680 & -2.7854\text{e-}3 & -1.9371\text{e-}5 \\ 7.3609\text{e-}2 & -2.7854\text{e-}3 & 10.9966 & 7.5997\text{e-}2 \\ 5.0936\text{e-}4 & 10.9966 & 7.5997\text{e-}2 & 5.2568\text{e-}4 \end{pmatrix}$$

$$K = \begin{pmatrix} 0.001285 & 0.026924 \\ 24.50568 & -0.001137 \\ 0.188773 & 3.948654 \\ 0.0013205 & 0.0276447 \end{pmatrix}^T, \quad L = \begin{pmatrix} 2.6956\text{e}4 & 0.026924 \\ -4.4574\text{e-}6 & 2.9639 \\ 7.1854 & 3.2958 \\ -3.6453\text{e}4 & -1.6714\text{e}3 \end{pmatrix}$$

$$F = \begin{pmatrix} 3.5498\text{e-}4 & -7.30161\text{e-}4 \end{pmatrix}$$

Now, we consider these conditions for simulation:

- The known inputs are $u_1 = 25.6 + 2\sin(0.05t)$ and $u_2 = 6.43e7 + 1e7\sin(0.1t)$.

- The unknown input is $w = 25.6 + 5\sin(0.05t + 2)$.

- The initial conditions are {46; 6.8; 0.0408; 3.84} and $\eta = 30$.

Fig. 1 shows the plots of system states and their estimations versus time. In this case the eigen values of observer are {-1.0512; -1.0091+0.0448i; -1.0091-0.0448i; -2.9640} respected to x_1 to x_4.

6. CONCLUSIONS

In this paper it has been shown how linear matrix inequalities can be used to synthesize the gains of a sliding mode observer. A formulation has been presented in a way that there is not any requirement for symbolic manipulation programs and computing transformation matrices. Therefore, this method leads to a direct and explicit formula to obtain observer gains. The numerical example for an industrial drum type boiler illustrates that this approach is effective and easy to implement.

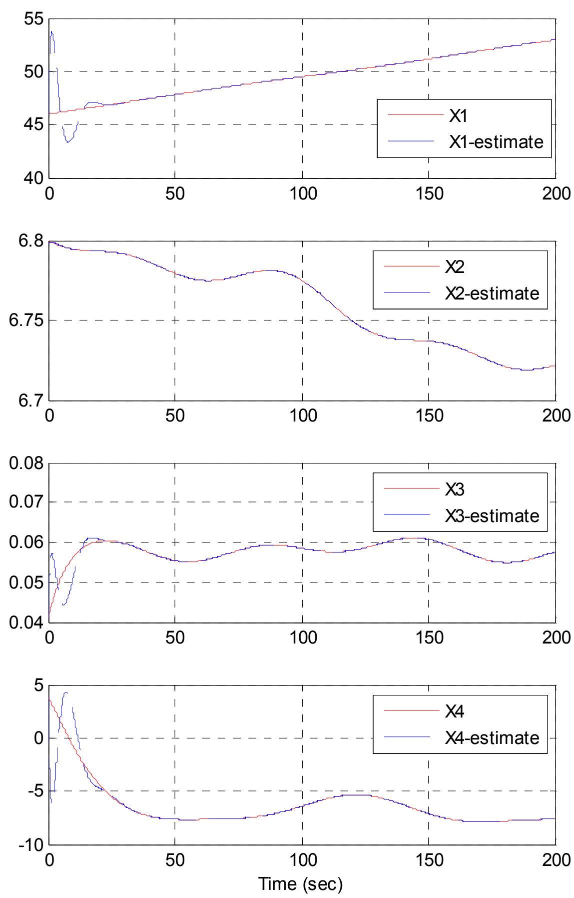

Fig. 1: Plots of x_i 's and their estimates versus time for a drum type boiler.

REFERENCES

[1] D. G. Luenberger, "Observers for multivariable systems". *IEEE Transactions on Automatic Control*, Vol. 11, No.2, pp. 190-197, 1966.

[2] C. Edwards, S. K. Spurgeon, and R. J. Patton, "Sliding mode observers for fault detection and isolation", *Automatica*, Vol. 36, No. 4, pp. 541-553, 2000.

[3] B. Jiang, M. Staroswiecki, and V. Cocquempot, "Fault estimation in nonlinear uncertain systems using robust/sliding-mode observers", *IEE Proceeding on Control Theory and Applications*, Vol. 151, No. 1, pp. 29-37, 2004.

[4] P. Q. Ha, "Dynamic output feedback sliding-mode control using pole placement and linear functional observers", *IEEE transactions on Industrial Electronics*, Vol. 50, No. 5, pp. 1030-1037, 2003.

[5] B. L. Walcott, and S. H. Zak, "State observation of nonlinear uncertain dynamic systems", *IEEE Transactions on Automatic Control*, Vol. 32, No. 2, pp. 166-170, 1987.

[6] B. Walcott and S. H. Zak, "Combined observer-controller synthesis for uncertain dynamical systems with applications" , *IEEE Transactions on Systems, Man and Cybernetics*, Vol. 18, No. 1, pp. 88-104, 1988.

[7] M. Darouach, M. Zasadzinski, and S. J. Xu, "Full-order observers for linear systems with unknown inputs", *IEEE Transactions on Automatic Control*, Vol. 39, No. 3, pp. 606-609, 1994.

[8] W. Chen and M. Saif, "Unknown input observer design for a class of nonlinear systems: and LMI approach", *Proceeding of ACC*, Minneapolis, Minnesota, USA, pp. 834-838, 2006.

[9] S. Hui and S. H. Zak, "Low-order unknown input observers", *Proceeding of ACC*, Portland, OR, USA, pp. 4192-4197, 2005.

[10] S. Hui and S. H. Zak, "Observer design for systems with unknown input", *International Journal of Applied Mathematics and Computer Science*, Vol.15, No. 4, pp. 431-446, 2005.

[11] C. P. Tan and C. Edwards, "An LMI approach for designing sliding mode observers", *Proceedings of the 39th IEEE Conference on Decision and Control*, Sydney, Australia, pp. 2587-2592, 2000.

[12] H. H. Choi and K. S. Ro, "LMI based sliding-mode observer design method", *IEE Proceeding on Control Theory and Applications*, vol. 152, No. 1, pp. 113-115, 2005.

[13] P. Gahinet, A. Nemiroviskii, A. J. Laub, and M. Chilali, "The LMI control toolbox", *Proceedings of the 33th IEEE Conference on Decision and Control*, Lake Buena, Vista, FL, USA, pp. 2038-2041, 1994.

[14] C. Scherer and S. Weiland, "Linear Matrix Inequalities in Control", Delf university, The Netherlands, DISC, 2005.

[15] K. Mrunalini, P. Kundu and K. K. Dutta, "State space model for drum boiler system", *IE(I) Jurnal-EL*, pp. 260-266, 2006.

[16] K. J. Astrom and R. D. Bell, "Drum boiler dynamics", *Automatica*, Vol. 36, pp. 363-378, 2000.