EXACT SOLUTION OF MAGNETO-HYDRODYNAMIC SYSTEM WITH NON LINEARITY ANALYSIS

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ABSTRACT

Analytical and numerical solution of Navier Stokes equation coupled with Maxwell's equation have been proposed with its experimental prototype model. The MHD system based on two and three dimensional processes were formulated and simulated using MATLAB with iterative approach. Experimental results were obtained by the developed prototype model. Non Linearity issues i.e. turbulence in fluid flow in MHD system were investigated. We have extended the work presented in reference [1], wherein only two dimensional solutions were suggested.

Keywords – *Iterative Solution, Convergence And Turbulence, Nonlinearity, Magneto-Hydrodynamic.*

1. INTRODUCTION

Magneto hydrodynamics (MHD) is the theory of the interaction of electrically conducting fluids and electromagnetic fields. The idea of MHD is that magnetic fields can induce currents in a moving conductive fluid, which create forces on the fluid, and

Also changes the magnetic field itself [4, 5]. The set of equations which describe MHD are a combination of the

Navier Stokes equation and Maxwell's equations of electro magnetism. MHD is a continuum theory, and as such it cannot treat kinetic phenomena, i.e. those in which the existence of discrete particles or a non-thermal velocities distribution are important. The electric currents transmitted in an electrolyte solution interact with the magnetic field to form Lorenz body forces that in turn, drive fluid motion. Lorenz force is the flow in the direction perpendicular to both magnetic and electric fields in conductive fluid. Navier Stokes equation is nonlinear partial differential equation based on Newton's second law. Navier Stokes equation dictates velocity, not position. Hence, MHD is dynamics of conducting fluid in presence of electric and magnetic fields. MHD system behaves linear when realized with no electric field applied [6, 7].

The solution of this system becomes complex when treated in three dimensional modeling [8, 9].

The MHD equations in our problem have been reduced

to two differential equations in two variables stream function (ψ) and magnetic field (B). When mean free path becomes comparable to the flow characteristics length scale i .e. Molecule length scale that is smaller than mean free path, flow deviates from Navier stokes equation. It is quantified by $K_n = \lambda/L$. Solution to Navier Stokes equation is called velocity field or flow field, description of fluid flow at given point in space and time [10, 11]. Once velocity is solved then flow rate and drag can be evaluated. In the absence of steep gradients in fluid properties and when Knudson number increases Navier- Stokes ceases to be valid [12, 13].

Potential applications of this work can be in astronomy, geo-physics, liquid metal cooling of nuclear reactors, and electromagnetic casting of metals, MHD power generation and propulsion [14, 15]. Despite its apparent simplicity, MHD describes a remarkably rich and varied mix of phenomena. In essence, the theory is a marriage between fluid mechanics and electromagnetism. When compared with the extremely detailed particle simple theory, perhaps even crude and the subject is one whose development continues to flourish. The MHD flow is governed by classical fluid dynamics and electromagnetic, including a set of coupled partial differential equations that express the conservation of mass continuity and Navier-Stokes equation joined to the Maxwell's, current continuity and constitutive equations [16]. We have extended this paper with solution of 3D form2D. The paper focuses on solution of stream function, finite difference method, an numerical approach, where work on 2D analysis is presented in equations (1-4). 3D analysis without electric field was formulated with equations (6-7). Solution to applied electric field have been formulated in equations (8- 11). 2D convergence result is shown in figure1. 3D simulation results are given in figure 2-7. Prototype developed system is depicted in figure 8-10. In this paper we have first presented mass conservation equation, second Navier Stokes equation, which is Newton's second law taking into account the force of magnetic field on moving charge. In study of 2D flow, the electrical field effects are neglected [1]. The third set is Maxwell's equation especially to monopole condition along with Ampere's law with the current given by ohm's law in a moving frame (the frame in which the moving particles of fluid is at rest). The mass conservation equation assuming the fluid to be incompressible leads us to express the velocity field as the curl of a velocity vector potential [2, 3]. The curl of the Navier Stokes equation leads to the elimination of pressure, there by leaving with an equation involving only magnetic field, and the fluid velocity field. The curl of the Ampere law equation leads us to another equation relating to the magnetic field to the velocity field.

We have divided our paper in eight main sections. Section 1 presents introductory part. Section2 comprises MHD formulations in 2D . Section 3 consists of 3D analysis without considering electric field [17, 18]. Section 4 has been worked for detailed 3D analysis under the influence electric field effects [19]. Section 5 speaks about practical implementation and measurement of velocity field. Section 6 describes results and figures. Conclusion of this paper has been reported in section 7. Measurement of velocity needs more instrumentation for precision measurements. We have presented v, B, and J as vector fields.

ABBREVIATIONS USED IN THE TEXT

E Electric intensity (Volts per meter)

H Magnetic intensity (ampere per meter)

D Electric flux density (coulomb per square meter)

B Magnetic flux density (Weber per square meter)

J Electric current density (ampere per square meter)

q_v Electric charge density (coulomb per cubic meter)

q Charge electric

σ Conductivity

μ Permeability

ϵ Permittivity

ψ Stream function

$\vec{\Omega}$ *Vortisity*

V *Viscosity*

$\vec{J}$ Current density

MHD Magneto hydrodynamics.

2. MHD FORMULATIONS

The MHD flow is governed by classical fluid dynamics and electromagnetic, including a set of coupled partial differential equations.

let there be a conducting fluid *having* σ conductivity, ρ mass density , p Pressure, $\nu = \eta / \rho$ kinetic viscosity, R Reynolds Number, v velocity of fluid and J= ρ V mass flux density , J = $\boldsymbol{\sigma}$ (E + v x B), where J= Current Density , B is Magnetic Field ,E= Electric Field, F = J X B, where F Lorentz force i.e. force on this current on the fluid per unit volume, Navier Stokes [1] equation is given by the relation for a conducting fluid in an electromagnetic field acquires an extra term J x B caused by the force of the magnetic field on the moving charges.

$$\rho (v . \nabla v + t) = - \nabla p + \eta \nabla^2 v + J \times B \quad (1)$$

Above can be expressed as

$$\rho (\Omega \times v + \frac{\nabla^2 v}{2} + v, t) = - \nabla p + \eta \nabla^2 v + J \times B \quad (2)$$

Where $\Omega = \nabla \times v$ is vorticity.

$$\rho (\Omega \times v + \frac{\nabla^2 v}{2} + v, t) = - \nabla p + \eta \nabla^2 v + J \times B \quad (3)$$

Taking curl of equation (3) to eliminate pressure term

$$(\nabla \times (\Omega \times v) + \Omega, t) = v \nabla^2 \Omega + \nabla \times (J \times B) \quad (4)$$

Evolution of 2D solution of the above equation

As $\vec{v} = \vec{\nabla} \times \vec{\psi}$

$\Omega = \nabla \times v = \nabla \times \nabla \times \psi$

$V = V_x \hat{x} + V_y \hat{y}$ (for 2D flow)

$B = B_z \hat{z}$

$\nabla^2 \Omega = \nabla(\nabla . \Omega) - \nabla \times \nabla \times \Omega$

$\nabla \times (\Omega \times v) = \hat{k} [\frac{\partial}{\partial x}(v_x \frac{\partial v_y}{\partial x} - v_x \frac{\partial v_x}{\partial y}) + \frac{\partial}{\partial y}(v_y \frac{\partial v_y}{\partial x} - v_y \frac{\partial v_x}{\partial y})]$

$\frac{\partial \Omega}{\partial t} = \hat{k} \frac{\partial}{\partial t} [\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}]$

$\nabla \times \nabla \times \Omega = \hat{k} [[-\frac{\partial^3 v_y}{\partial x^3} + \frac{\partial^3 v_x}{\partial^2 x \partial y} - [\frac{\partial^3 v_y}{\partial^2 y \partial x} - \frac{\partial^3 v_x}{\partial y^3}]]$

$\nabla \times J \times B = \hat{k} (\frac{\partial v_x B_z^2}{\partial x} - \frac{\partial v_y B_z^2}{\partial y})$

$\nabla \times B = (\mu J + \mu \in E, t)$;

On neglecting displacement current

$J = \frac{\nabla \times B}{\mu}$

We have force per unit volume due to electromagnetic field as

$F = \mu^{-1}(\nabla \times B) \times B$

$= \mu^{-1} B . \nabla B - \frac{1}{2\mu}(B^2)$

We thus get Navier stroke's equation as

$(\nabla \times (\Omega \times v) + \Omega, t) = v \nabla^2 \Omega + \frac{1}{\mu p} \nabla \times (\Omega B \times B)$

Where $\Omega B = (\nabla \times B) = \mu J$

If $v_x = \frac{\partial \psi}{\partial y}$, $v_y = -\frac{\partial \psi}{\partial x}$

The equation (4) can be reduced in terms of two variable i.e. B Magnetic field and ψ Stream function.

$\hat{k} [-\frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial \psi}{\partial y} \frac{\partial^3 \psi}{\partial x^3} - \frac{\partial^2 \psi}{\partial y \partial x} \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \psi}{\partial y} \frac{\partial^3 \psi}{\partial^2 y \partial x} + \frac{\partial^2 \psi}{\partial y \partial x} \frac{\partial^2 \psi}{d x^2} + \frac{\partial \psi}{\partial y} (\frac{\partial^3 \psi}{\partial y \partial^2 x}) + \frac{\partial^2 \psi}{\partial y \partial x} \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial \psi}{\partial x} \frac{\partial^3 \psi}{\partial y^3}] + \hat{k} \frac{\partial}{\partial t} [-\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}] =$

$v \hat{k} [-[\frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^4 \psi}{\partial x^2 \partial y^2}] - [-\frac{\partial^4 \psi}{\partial y^2 \partial x^2} - \frac{\partial^4 \psi}{\partial y^4}]] + \sigma \hat{k} [\frac{\partial}{\partial x} [\frac{\partial \psi}{\partial y} B_z Z] + \frac{\partial}{\partial y} [\frac{\partial \psi}{\partial x} B_z 2]$

(5)

As per Maxwell's Ampere's law

$\nabla \times E = - B, t$

Taking curl on both sides and solving

$(\nabla^2 B - \mu \sigma B, t) + \mu \sigma \nabla \times (v \times B) = 0$

When fluid velocity and magnetic field are functions of x, y, t. then

$\nabla^2 B_0 - \mu \sigma B, t + \mu \sigma \nabla \times (V \times B) = 0$

Here v_x, v_y, B_z are functions of x, y, t

$(\nabla \times V \times B)_x = (V \times B)_{y,z} - (V \times B)_{z,y}$

$(\nabla \times V \times B)_y = (V \times B)_{z,x} - (V \times B)_{x,z}$

$(\nabla \times V \times B)_z = (V \times B)_{y,x} - (V \times B)_{x,y}$

Evolution of 2D MHD system can be found under assumed initial and final conditions. Taking initial input as Gaussian pulse = $e^{-(t-t0)/} \sigma^2$

$(\nabla^2 . B - \mu \sigma B, t) + \mu \sigma \nabla \times (v \times B) = 0$

$(\nabla^2 . B - \mu \sigma \frac{\partial B}{\partial t}) + \mu \sigma \nabla \times (v \times B) = 0$

Solving the above equation

$\nabla . B = 0$,

$\nabla \times E = - B, t$

$\nabla \times B = \mu \sigma (E + v \times B)$

$\nabla \times B = \hat{\imath} (\frac{\partial B_z}{\partial y}) - \hat{\jmath} (\frac{\partial B_z}{\partial x})$

$\nabla \times \nabla \times B = \hat{k} (-\frac{\partial^2 B_z}{\partial x^2} - \frac{\partial^2 B_z}{\partial y^2})$

$V \times B = \hat{\imath} (v_y B_z) - \hat{\jmath} (v_x B_z)$

$\nabla \times (V \times B) = \hat{k} (-\frac{\partial v_x B_z}{\partial x} - \frac{\partial v_y B_z}{\partial y})$

(6)

Hence final reduced equation is given below in B and ψ variables.

$\hat{k} (\frac{\partial^2 B_z}{\partial x^2} - \frac{\partial^2 B_z}{\partial y^2}) - \mu \sigma \hat{k} \frac{\partial B_z}{\partial t} - \mu \sigma \hat{k} [\frac{\partial}{\partial x} (\frac{\partial \psi}{\partial y} B_z) - \frac{\partial}{\partial y} (\frac{\partial \psi}{\partial x} B_z)] = 0$

(7)

Taking initial magnetic field B_{z0} as Gaussian and initial electric field negligible, we shall solve this problem for Magnetic field, B and Stream function ψ taking them as function of x, y and. From above equation, we observe that velocity of conducting fluid is varying whenever magnetic field is changed and vice versa. Solution of above equation (7) has been worked out by iteration method. As equation (7) has been reduced to only two variables in stream function ψ and magnetic field B. We can solve this equation by Finite Difference Method numerically for computing stream function ψ and their corresponding magnetic field B by iterative solution. Now substituting the value of magnetic field obtained in the problem equation and solving it for new set of stream function ψ, we shall continue the same approach till convergence. Here we can assume initial stream function ψ as $2n\pi\ radian$ and then proceed to evaluate their corresponding value magnetic field B. We have also worked this problem with experimental and analytical methods. It has been found [11] from results that computed results are in close proximity. We can get much better results if simulated for 3 D solution but process may be complex.

Convergence plot for iterative solution of Magnetic field and Stream function have been shown in figure 1. We have obtained two sets of equations after optimization and iterative solution have been worked with numerical technique for convergence using MATLAB.

Figures 2-7 clearly depicts solution to this problem. We have reduced system to two variables as seen from equation (5) and (7). Equations(1-25) presents the complete solution to this problem. We have observed that velocity variation is dependent on magnetic field and electric field during experimental results. we have taken plot for stream function and magnetic field under certain boundary conditions and initial conditions. Dimensions and material of electrodes, conductivity of fluid, dimensions of channel and working chamber with inlet and outlet, gap between electrodes and position of magnets producing cross field make the difference in velocity of fluid. Fluid velocity found to be increasing when any of the field is increasing during experiments. These results in line with numerical results obtained. Bubble formation due to electrolysis has observed in the conducting fluid when operated with DC source which produces retardation effect. During investigations bubble formation got reduced when MHD system operated with AC source and fluid velocity enhances significantly. MHD system has been observed bidirectional capability on electric field reversal. Heating of fluid takes place because of electric current in conducting fluid which need to be investigated for control of temperature rise. Life of the MHD system can be significantly large as compared to system having moving parts, as there are no moving parts in the proposed system.

3. 3D MHD SOLUTION WITHOUT ELECTRIC FIELD

From Navier Stokes equation we get

$$(\vec{\Omega} \times \vec{v} + \frac{1}{2}\nabla v^2 + \vec{v},\ t) = -\frac{\vec{\nabla} p}{\rho} + V\,\nabla^2 \vec{v} + J \times \vec{B} \quad (11)$$

Where $\vec{\Omega} = Vortisity$, $V = Kinetic\ Viscosity$, $\psi = Steam\ function$

$J = \sigma(\vec{v} \times \vec{B})$; Current density; as we know

$$(\vec{\nabla} \times (\vec{\Omega} \times \vec{v}) + \vec{\Omega}, t) = V\,\nabla^2\Omega + \sigma\vec{\nabla} \times [(\vec{v} \times \vec{B}) \times \vec{B}] \quad (12)$$

We can write

$$(\vec{v} \times \vec{B}) \times \vec{B} = (\vec{v} . \vec{B})\ \vec{B} - B^2 \vec{v} \quad (13)$$

$$\vec{\nabla} \times \vec{v} = 0$$

$$\vec{\nabla} .B \quad B = \vec{\nabla} \times A$$

$$\vec{\nabla} . \vec{\nabla} \times \psi = 0$$

$$\vec{\nabla} . \psi = 0$$

$$\nabla^2 f = -\vec{\nabla} . \vec{\psi}$$

$$\vec{\Omega} = \vec{\nabla} \times \vec{v}$$

$$\vec{v} = \vec{\nabla} \times \vec{\psi}$$

Hence $\Omega = \nabla \times v = \nabla \times (\nabla \times \psi)$

Or $\nabla . (\nabla . \psi) - \nabla^2 \psi$

As $\psi \longrightarrow \psi_1 = \psi + \vec{\nabla} f$

$$\nabla \times \psi_1 = \nabla \times \psi$$

$$\vec{\nabla} \times \vec{\nabla} f = 0$$

$$\nabla . \psi_1 = 0$$

$$\nabla . \psi + \nabla^2 f = 0$$

$$f(\vec{r}) = \frac{1}{4\pi} \int \frac{(\vec{\nabla} . \vec{\psi}\ (\vec{r}'))}{|\vec{r} - \vec{r}'|} d^3r'$$

$$\vec{\Omega} = -\nabla^2 \vec{\psi}$$

$$\vec{\Omega} \times \vec{v} = -\nabla^2 \vec{\psi} \times (\nabla \times \psi)$$

$$\nabla \times (\vec{\Omega} \times \vec{v}) = (v . \nabla)\,\Omega - (\nabla . \Omega)\,v + (\nabla . v)\Omega - (\Omega . \nabla)\,v \quad (14)$$

$$= (v . \nabla)\,\Omega - (\Omega . \nabla)\,v;$$

Because $\nabla . v = 0$

Hence $-(\vec{\nabla} \times \vec{\psi} . \vec{\nabla})\,\nabla^2 \psi + (\nabla^2 \psi . \nabla)(\nabla \times \psi)$

$$-(\vec{\nabla} \times \vec{\psi}) . \vec{\nabla})\,\nabla^2 \psi + (\nabla^2 \psi . \nabla^2)(\nabla \times \psi) - \nabla^2 \vec{\psi}, t = -V\ \nabla^2 \nabla^2 \vec{\psi} + \sigma\,\vec{\nabla}\,[(\vec{v} \times \vec{B}) \times \vec{B}]$$

; also (15)

$\vec{\nabla}$ x $[(\vec{v}$ x $\vec{B})$ x $\vec{B}]$ =

$(\vec{B}.\vec{\nabla})(v \text{ x } \vec{B}) - \vec{\nabla}.(\vec{v} \text{ x } \vec{B}) \text{ x } \vec{B} + (\vec{\nabla}.\vec{B})(v \text{ x B}) - (\vec{v} \text{ x } \vec{B}).\vec{\nabla}\vec{B}$ (16)

As $\vec{\nabla}.\vec{B} = 0$

$\vec{v} = \vec{\nabla} \text{ x } \vec{\psi}$

$\vec{\nabla}.(\vec{v} \text{ x } \vec{B}) = (\vec{\nabla} \text{ x } \vec{v}).\vec{B} - \vec{v}.\vec{\nabla} \text{ x } \vec{B} \text{-} \vec{\Omega}.\vec{B} - \vec{v}.\vec{\nabla} \text{ x B} - \nabla^2 \vec{\psi}.\vec{B}$; (17)

$\vec{\nabla} \text{ x } \vec{B} = \mu J\sigma (\vec{v} \text{ x } \vec{B})$

$\vec{\nabla}.(\vec{v} \text{ x } \vec{B}) = -\nabla^2 \vec{\psi}.\vec{B}$;

Hence $\vec{\nabla}$ x $[(\vec{v}$ x $\vec{B})$ x $\vec{B}]$ =

$(\vec{B}.\vec{\nabla})[(\vec{\nabla} \text{ x } \vec{\psi}) \text{ x } \vec{B}] + (\nabla^2\vec{\psi}.\vec{B})\vec{B} + [(\nabla^2 \vec{\psi} \text{ x} \vec{B}).\vec{\nabla})\vec{B}$ (18)

$\vec{\nabla} \text{ x } \vec{B} = \sigma (\vec{\nabla} \text{ x } \vec{\psi}) \text{ x } \vec{B}$; $\vec{\psi}, \vec{B}$

$\vec{\nabla}(\vec{\nabla}.\vec{B}) - \nabla^2 \text{ B} = \mu\sigma\vec{\nabla} \text{ x } (\vec{v} \text{ x } \vec{B})$; $\vec{\nabla}.\vec{B} = 0$

$\vec{\nabla^2}\vec{B} + \mu\sigma[(\vec{B}.\vec{\nabla})\vec{v} - (\vec{\nabla}\vec{v})\vec{B} + (\vec{\nabla}.\vec{B})\vec{v} - (\vec{v}.\vec{\nabla})\vec{B}] = 0$

$\vec{\nabla^2}\vec{B} + \mu\sigma[(\vec{B}.\vec{\nabla})\vec{v} - (\vec{v}.\vec{\nabla})\vec{B}] = 0$ (19)

Hence

$-[(\vec{\nabla}.\vec{\psi}).\vec{\nabla}]\vec{\nabla^2}\vec{\psi} + (\vec{\nabla^2}\vec{\psi}.\vec{\nabla})(\vec{\nabla} \text{ x } \vec{\psi}) - \vec{\nabla^2}\vec{\psi}, \text{t} = -\text{V}\vec{\nabla^2}\vec{\nabla^2}\vec{\psi} + \sigma(\vec{B}.\vec{\nabla})(\vec{\nabla} \text{ x } \vec{\psi}) \text{ x } \vec{B} + (\vec{\nabla^2}\vec{\psi}.\vec{B})\vec{B} + [(\vec{\nabla^2}\vec{\psi} \text{ x } \vec{B})\vec{\nabla}]\vec{B}$ (20)

The equation (17) has been reduced to optimized solution of having only two variables i.e. magnetic field B and Stream i.e. B and ψ

4. 3D ANALYSIS WHEN ELECTRIC FIELD APPLIED FUNCTION $\boldsymbol{\psi}$.

We have optimized the above equations in two variables

$\rho(\vec{v}.\vec{\nabla}v + \vec{v}, \text{t}) = -\vec{\nabla}p + \eta\vec{\nabla}^2 v + \vec{J} \text{ x } \vec{B}$

$\rho(\vec{\Omega} \text{ x } \vec{v} + \frac{\vec{\nabla}^2 v}{2} + \vec{v}, \text{t} = -\vec{\nabla}p + \eta\vec{\nabla}^2\vec{v} + \vec{J} \text{ x } \vec{B}$ (21)

Where $\nabla \text{ x } v = \Omega$, $\vec{v} = \vec{\nabla} \text{ x } \vec{\psi}$

$J = \sigma(\vec{E} + \vec{v} \text{ x} \vec{B})$

$(\vec{\Omega} \text{ x } \vec{v} + \frac{\vec{\nabla}^2 v}{2} + \vec{v}, \text{t}) = -\frac{\vec{\nabla}p}{\rho} + \frac{\eta}{\rho}\vec{\nabla}^2 v + \frac{\vec{J} \text{ x } \vec{B}}{\rho}$

$\frac{\eta}{\rho} = v$, $(\vec{v} \text{ x } \vec{B}) \text{ x } \vec{B} = (\vec{v}.\vec{B}).\vec{B} - \vec{B}^2.\vec{v}$

$\vec{\nabla} \text{ x } \vec{v} = 0$ $\vec{\nabla}.\vec{\psi} = 0$

Taking Curl both sides above equation (21)

$\vec{\nabla} \text{ x } (\vec{\Omega} \text{ x } \vec{v}) + \vec{\Omega}, \text{t} = v\vec{\nabla}^2\vec{\Omega} + \frac{\sigma}{\rho}[(-\text{B, t} + \nabla\text{x}(\vec{v} \text{ x } \vec{B})) \text{ x } \vec{B}]$

Where ∇ x E = —B, t

$\Omega = -\nabla^2\psi$

$\vec{\nabla} \text{ x } (-\nabla^2\vec{\psi} \text{ x } \vec{\nabla} \text{ x } \vec{\psi}) - \vec{\nabla}^2\vec{\psi}, \text{t} = -v\vec{\nabla}^2\vec{\nabla}^2\vec{\psi} + \frac{\sigma}{\rho}[\{-\text{B, t} + \vec{\nabla} \text{ x } (\vec{\nabla} \text{ x } \vec{\psi}) \text{ x } \vec{B}\} \text{ x } \vec{B}]$

$\nabla^2 = \triangleq$ (Special operator)

$-\triangleq\vec{\boldsymbol{\psi}}, \text{t} - \vec{\nabla} \text{ x } (\triangleq\vec{\boldsymbol{\psi}} \text{ x } (\vec{\nabla} \text{ x } \boldsymbol{\psi})) = \boldsymbol{v}\triangleq^2\vec{\boldsymbol{\psi}} + \frac{\boldsymbol{\sigma}}{\boldsymbol{\rho}}[\{-\text{B, t} + \vec{\nabla} \text{ x}((\vec{\nabla} \text{ x } \vec{\boldsymbol{\psi}}) \text{ x } \boldsymbol{B})\} \text{ x } \vec{\boldsymbol{B}}]$

(22)

$M_1 = \triangleq\psi, \text{t} = \frac{\delta^2\psi_x}{dt^2} + \frac{\delta^2\psi_y}{dt^2} + \frac{\delta^2\psi_z}{dt^2}$ $[\triangleq = \nabla^2]$

$M_2 = \triangleq\vec{\psi} \text{ x } (\vec{\nabla} \text{ x } \psi) = [\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}] \text{ x } [(\nabla \text{ x } \psi)_x - (\nabla \text{ x } \psi)_y + (\nabla \text{ x } \psi)_z]$

$M_5 = \triangleq^2\vec{\psi} = [\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}][\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}]$

$= \frac{\delta\psi_x{}^2}{\delta x^2}[\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}] + \frac{\delta^2\psi_y}{\delta y^2}[\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}] + \frac{\delta\psi_z{}^2}{\delta z^2}[\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}]$

$M_4 = \nabla \text{ x } M_2 = \nabla \text{ X } [\triangleq\vec{\psi} \text{ x } (\vec{\nabla} \text{ x } \psi)] = 0$ (being curl of scalar)

$= \nabla \text{ X } [\{\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}] \text{ x } [(\nabla \text{ x } \psi)_x - (\nabla \text{ x } \psi)_y + (\nabla \text{ x } \psi)_z\}]$

$M_6 = \nabla$ x (∇ x ψ) x B

$M_8 = [-B,t + M_6]$ x B= [$-B,t +$ ∇ x (∇ x ψ) x B] X B

Hence the resulting equation shall be

$-\boldsymbol{M_1} — \boldsymbol{M_4} = \boldsymbol{\nu}\, \boldsymbol{M_5} + \frac{\sigma}{\rho} \boldsymbol{M_8}$

Hence

$—\triangleq \vec{\psi}$, t $— \vec{\nabla}$ x ($\triangleq \vec{\psi}$ x ($\vec{\nabla}$ x ψ)) = $\nu \triangleq^2 \vec{\psi} + \frac{\sigma}{\rho}$ [{—B, t + $\vec{\nabla}$ x(($\vec{\nabla}$ x $\vec{\psi}$) x $\vec{B}$)} x $\vec{B}$] (23)

∇ X $\left(\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}\right)$ x $[(\nabla \mathbf{x}\, \psi)_x — (\nabla \mathbf{x}\, \psi)_y$ $+ (\nabla \mathbf{x}\, \psi)_z] - \left(\frac{\delta^2\psi_x}{\delta t^2} + \frac{\delta^2\psi_y}{\delta t^2} + \frac{\delta^2\psi_z}{\delta t^2}\right) =$ ν $\left[\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2} \right] \left[\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2} \right]$+ $\frac{\sigma}{\rho}$ [{—B, t + $\vec{\nabla}$ x ($\vec{\nabla}$ x $\vec{\psi}$) x $\vec{B}$} x $\vec{B}$]

Or

$- \left(\frac{\delta^2\psi_x}{\delta t^2} + \frac{\delta^2\psi_y}{\delta t^2} + \frac{\delta^2\psi_z}{\delta t^2}\right) - \nabla$ X $[\{ \frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}\}$ x $\{(\nabla \mathbf{x}\, \psi)_x — (\nabla \mathbf{x}\, \psi)_y + (\nabla \mathbf{x}\, \psi)_z$ }] = ν $[\{\frac{\delta\psi_x{}^2}{\delta x^2} (\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2})\} + \{ \frac{\delta^2\psi_y}{\delta y^2} (\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2})\}$ $+\{\frac{\delta\psi_z{}^2}{\delta z^2} (\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2})\}] + \frac{\sigma}{\rho}$ [{—B, t +

$\hat{\boldsymbol{\imath}}\, \frac{\delta}{\delta y} \{ (\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta\psi_y}) \boldsymbol{B_y} — (\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}) \boldsymbol{B_z} \}$ —

$\frac{\delta}{\delta z} \{ (\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}) \boldsymbol{B_x} — (\frac{\delta\psi_z}{\delta y} — \frac{\delta\psi_x}{\delta z}) \}$

$— \hat{\boldsymbol{\jmath}}\, \{ \frac{\delta B_x}{\delta x} (\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}) + \boldsymbol{B_x} . \frac{\delta}{\delta x} (\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}) — \frac{\delta}{\delta z}$ $\boldsymbol{B_y} (\frac{\delta\psi_z}{\delta x} - \frac{\delta\psi_y}{\delta x}) — \boldsymbol{B_y} (\frac{\delta^2\psi_y}{\delta x\delta z} — \frac{\delta^2\psi_x}{\delta x\delta z}) \}$

$—\hat{\boldsymbol{k}}\, \{ \frac{\delta B_x}{\delta x} . (\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}) + \boldsymbol{B_x} . (\frac{\delta^2\psi_y}{\delta x^2} + \frac{\delta^2\psi_x}{\delta y\delta x}) — \boldsymbol{B_z}$. $(\frac{\delta^2\psi_z}{\delta x\delta y} — \frac{\delta^2\psi_y}{\delta x\delta z}) — \frac{\partial B_z}{\partial y} . (\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}) — \boldsymbol{B_y} (\frac{\delta^2\psi_y}{\delta x\delta y}$ $— \frac{\delta^2\psi_x}{\delta y^2}) + \boldsymbol{B_y} (\frac{\delta^2\psi_x}{\delta y\delta z} — \frac{\delta^2\psi_z}{\delta y\delta x} \}]$ X $\vec{\mathbf{B}}$ (24)

Because curl of scalar quantity is zero, hence

∇ X $[\{ \frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2}\}$ x $\{(\nabla \mathbf{x}\, \psi)_x — (\nabla \mathbf{x}\, \psi)_y + (\nabla \mathbf{x}\, \psi)_z \}]$ = 0

or

$- \left(\frac{\delta^2\psi_x}{\delta t^2} + \frac{\delta^2\psi_y}{\delta t^2} + \frac{\delta^2\psi_z}{\delta t^2}\right) = \nu$ $[\{\frac{\delta\psi_x{}^2}{\delta x^2} (\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2})\} + \{ \frac{\delta^2\psi_y}{\delta y^2} (\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2})\} + \{\frac{\delta\psi_z{}^2}{\delta z^2} (\frac{\delta\psi_x{}^2}{\delta x^2} + \frac{\delta^2\psi_y}{\delta y^2} + \frac{\delta\psi_z{}^2}{\delta z^2})\}]$ + $i\, [B_z(-\frac{\partial B_y}{\partial t} + \frac{\partial B_z}{\partial z}(\psi_{x,z} - \psi_{z,x}) + \frac{\partial(\psi_{x,z} - \psi_{z,x})}{\partial z} B_z - \frac{\partial B_y}{\partial z}(\psi_{y,x} - \psi_{x,y}) - \frac{\partial(\psi_{y,x} - \psi_{x,y})}{\partial z} B_y - \frac{\partial B_y}{\partial x}(\psi_{z,y} - \psi_{y,z}) - \frac{\partial(\psi_{z,y} - \psi_{y,z})}{\partial x} B_y + \frac{\partial B_x}{\partial x}(\psi_{x,z} - \psi_{z,x}) + \frac{\partial(\psi_{x,z} - \psi_{z,x})}{\partial x} B_x)$

$-B_y(-\frac{\partial B_z}{\partial t} + \frac{\partial B_x}{\partial x}(\psi_{y,x} - \psi_{x,y}) + \frac{\partial(\psi_{y,x} - \psi_{x,y})}{\partial x} B_x - \frac{\partial B_z}{\partial x}(\psi_{z,y} - \psi_{y,z}) - \frac{\partial(\psi_{z,y} - \psi_{y,z})}{\partial x} B_z - \frac{\partial B_z}{\partial y}(\psi_{x,z} - \psi_{z,x}) - \frac{\partial(\psi_{x,z} - \psi_{z,x})}{\partial y} B_z + \frac{\partial B_y}{\partial y}(\psi_{y,x} - \psi_{x,y}) + \frac{\partial(\psi_{x,z} - \psi_{z,x})}{\partial y} B_y)]$ — J

$$[B_x(-\frac{\partial B_z}{\partial t} + \frac{\partial B_x}{\partial x}(\psi_{y,x} - \psi_{x,y}) + \frac{\partial(\psi_{y,x} - \psi_{x,y})}{\partial x} B_x - \frac{\partial B_z}{\partial x}(\psi_{z,y} - \psi_{y,z}) - \frac{\partial(\psi_{z,y} - \psi_{y,z})}{\partial x} B_z$$

$-\frac{\partial B_z}{\partial y}(\psi_{x,z} - \psi_{z,x}) - \frac{\partial(\psi_{x,z} - \psi_{z,x})}{\partial y} B_z + \frac{\partial B_y}{\partial y}(\psi_{y,x} - \psi_{x,y}) + \frac{\partial(\psi_{y,x} - \psi_{x,y})}{\partial y} B_y)$

$-B_z(-\frac{\partial B_x}{\partial t} + \frac{\partial B_y}{\partial y}(\psi_{z,y} - \psi_{y,z}) + \frac{\partial(\psi_{z,y} - \psi_{y,z})}{\partial y} B_y - \frac{\partial B_x}{\partial y}(\psi_{x,z} - \psi_{z,x}) - \frac{\partial(\psi_{x,z} - \psi_{z,x})}{\partial y} B_x - \frac{\partial B_x}{\partial z}(\psi_{y,x} - \psi_{x,y}) - \frac{\partial(\psi_{y,x} - \psi_{x,y})}{\partial z} B_x + \frac{\partial B_z}{\partial z}(\psi_{z,y} - \psi_{y,z}) + \frac{\partial(\psi_{z,y} - \psi_{y,z})}{\partial z} B_z)]$+ k $[B_y(-\frac{\partial B_x}{\partial t} + \frac{\partial B_y}{\partial y}(\psi_{z,y} - \psi_{y,z}) + \frac{\partial(\psi_{z,y} - \psi_{y,z})}{\partial y} B_y - \frac{\partial B_x}{\partial y}(\psi_{x,z} - \psi_{z,x}) - \frac{\partial(\psi_{x,z} - \psi_{z,x})}{\partial y} B_x$

$-\frac{\partial B_x}{\partial z}(\psi_{y,x} - \psi_{x,y}) - \frac{\partial(\psi_{y,x} - \psi_{x,y})}{\partial z} B_x + \frac{\partial B_z}{\partial z}(\psi_{z,y} - \psi_{y,z})$ $+\frac{\partial(\psi_{z,y} - \psi_{y,z})}{\partial z} B_z)$

$-B_x(-\frac{\partial B_y}{\partial t} + \frac{\partial B_y}{\partial y}(\psi_{z,y} - \psi_{y,z}) + \frac{\partial}{\partial y}(\psi_{z,y} - \psi_{y,z})B_y - \frac{\partial B_x}{\partial y}(\psi_{x,z} - \psi_{z,x}) - \frac{\partial}{\partial y}(\psi_{x,z} - \psi_{z,x})B_x - \frac{\partial B_y}{\partial x}(\psi_{z,y} - \psi_{y,z}) - \frac{\partial}{\partial x}(\psi_{z,y} - \psi_{y,z})B_y + \frac{\partial B_x}{\partial x}$

$(\psi_{x,z} - \psi_{z,x}) + \frac{\partial}{\partial x} \ (\psi_{x,z} - \psi_{z,x}) \ B_x \ )]$

$\triangleq \boldsymbol{B} — \boldsymbol{\mu}\ \boldsymbol{\sigma}\ \vec{\boldsymbol{B}}$, t—$\boldsymbol{\mu} \in$ B, t t + $\boldsymbol{\mu}\ \boldsymbol{\sigma}\ \vec{\boldsymbol{\nabla}}$ x ($\boldsymbol{\nabla}$ x $\boldsymbol{\psi}$) x B = 0 (24)

$$R_1 = \triangleq B = \hat{\imath}\left(\frac{\delta^2 B_x}{\delta x^2} + \frac{\delta^2 B_x}{\delta y^2} + \frac{\delta^2 B_x}{\delta z^2}\right) + \hat{\boldsymbol{j}}\left(\frac{\delta^2 B_x}{\delta x^2} + \frac{\delta^2 B_x}{\delta y^2} + \frac{\delta^2 B_x}{\delta z^2}\right) + \hat{\boldsymbol{k}}\left(\frac{\delta^2 B_Z}{\delta x^2} + \frac{\delta^2 B_Z}{\delta y^2} + \frac{\delta^2 B_Z}{\delta z^2}\right)$$

$$R_2 = \mu\ \sigma\ \text{B, t} = \mu\ \sigma\left[\frac{\delta B_x}{dt} + \frac{\delta B_y}{dt} + \frac{\delta B_z}{dt}\right]$$

$$R_3 = \mu \in \text{B,tt} = \mu \in \left[\frac{\delta^2 B_x}{\delta t^2} + \frac{\delta^2 B_y}{\delta t^2} + \frac{\delta^2 B_z}{\delta t^2}\right]$$

$$R_4 = \mu\ \sigma\ \nabla \text{ x } [(\nabla \text{ x } \psi) \text{ x B}]$$

$$= \mu \in \left[\ \hat{\imath}\ \frac{\delta}{\delta y}\left\{\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta\psi_y}\right) B_y — \left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right) B_z\right\} — \frac{\delta}{\delta z}\left\{\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) B_x — \left(\frac{\delta\psi_z}{\delta y} — \frac{\delta\psi_x}{\delta z}\right)\right\}\ \right]$$

$$— \left[\ \hat{\jmath}\ \left\{\frac{\delta B_x}{\delta x}\left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right) + B_x . \frac{\delta}{\delta x}\left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right) — \frac{\delta}{\delta z} B_y \left(\frac{\delta\psi_z}{\delta x} - \frac{\delta\psi_y}{\delta x}\right) — B_y\left(\frac{\delta^2\psi_y}{\delta x\delta z} — \frac{\delta^2\psi_x}{\delta x\delta z}\right)\right\}\ \right]$$

$$— \left[\hat{k}\ \left\{\frac{\delta B_x}{\delta x} . \left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) + B_x . \left(\frac{\delta^2\psi_y}{\delta x^2} + \frac{\delta^2\psi_x}{\delta y\delta x}\right) — B_z . \left(\frac{\delta^2\psi_z}{\delta x\delta y} — \frac{\delta^2\psi_y}{\delta x\delta z}\right) — \frac{\partial B_z}{\partial y} . \left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) — B_y\left(\frac{\delta^2\psi_y}{\delta x\delta y} — \frac{\delta^2\psi_x}{\delta y^2}\right) + B_y\left(\frac{\delta^2\psi_x}{\delta y\delta z} — \frac{\delta^2\psi_z}{\delta y\delta x}\right\}\ \right]$$

(25) Hence $\triangleq$B — $\mu\ \sigma$ B,t — $\mu\ \sigma$ B,tt + $\mu\ \sigma$ ∇ x (∇ x ψ) x B = 0

Or

$$\hat{\imath}\left(\frac{\delta^2 B_x}{\delta x^2} + \frac{\delta^2 B_x}{\delta y^2} + \frac{\delta^2 B_x}{\delta z^2}\right) + \hat{\boldsymbol{j}}\left(\frac{\delta^2 B_x}{\delta x^2} + \frac{\delta^2 B_x}{\delta y^2} + \frac{\delta^2 B_x}{\delta z^2}\right) + \hat{\boldsymbol{k}}\left(\frac{\delta^2 B_Z}{\delta x^2} + \frac{\delta^2 B_Z}{\delta y^2} + \frac{\delta^2 B_Z}{\delta z^2}\right)$$

$$— \boldsymbol{\mu}\ \boldsymbol{\sigma}\left(\frac{\delta B_x}{\delta t} + \frac{\delta B_y}{\delta t} + \frac{\delta B_z}{\delta t}\right) — \boldsymbol{\mu} \in \left(\frac{\delta^2 B_x}{\delta t^2} + \frac{\delta^2 B_y}{\delta t^2} + \frac{\delta^2 B_z}{\delta t^2}\right)$$

$$+ \boldsymbol{\mu}\ \boldsymbol{\sigma}\ [\{\hat{\boldsymbol{\imath}}\ (\boldsymbol{B_y}\ \frac{\delta}{\delta y}\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) + \left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right)\frac{\delta B_y}{\delta y} — \boldsymbol{B_z}\frac{\delta}{\delta y}\left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right) — \left(\frac{\delta x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right)\frac{\delta B_z}{\delta y} — [\boldsymbol{B_x}\frac{\delta}{\delta z}\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) + \left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right)\frac{\delta B_x}{\delta z} — \boldsymbol{B_z} . \frac{\delta}{\delta z}\left(\frac{\delta\psi_z}{\delta y} — \frac{\delta\psi_y}{\delta z}\right) — \left(\frac{\delta\psi_z}{\delta y} — \frac{\delta\psi_y}{\delta z}\right) . \frac{\delta B_z}{\delta z}] — [\ \hat{\boldsymbol{j}}\{\ \frac{\delta B_x}{\delta x}\left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right)$$

$$+ \boldsymbol{B_x} . \frac{\delta}{\delta x}\left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right) — \frac{\delta B_y}{\delta x}\left(\frac{\delta\psi_z}{\delta y} — \frac{\delta\psi_y}{\delta z}\right) — \boldsymbol{B_y}\ \frac{\delta}{\delta x}\left(\frac{\delta\psi_z}{\delta y} — \frac{\delta\psi_y}{\delta z}\right) — \frac{\delta B_y}{\delta z}\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) + \boldsymbol{B_y} . \frac{\delta}{\delta z}\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) — \boldsymbol{B_z} . \frac{\delta}{\delta z}\left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right) — \left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right)\frac{\delta B_z}{\delta z}\}]$$

$$— [\ \hat{\boldsymbol{k}}\ \{\frac{\delta B_x}{\delta x} . \left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) + \boldsymbol{B_x} . \frac{\delta}{\delta x}\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) — \frac{\delta B_z}{\delta x}\left(\frac{\delta\psi_z}{\delta y} — \frac{\delta\psi_y}{\delta z}\right) — \boldsymbol{B_z} . \frac{\partial}{\partial x}\left(\frac{\delta\psi_z}{\delta y} — \frac{\delta\psi_y}{\delta z}\right) — \boldsymbol{B_y}\ \frac{\delta}{\delta y}\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) + \frac{\delta B_y}{\delta y}\left(\frac{\delta\psi_y}{\delta x} — \frac{\delta\psi_x}{\delta y}\right) — \boldsymbol{B_z}\ \frac{\delta}{\delta y}\left(\frac{\delta\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right) — \frac{\delta B_z}{\delta y}\left(\frac{\delta^2\psi_x}{\delta z} — \frac{\delta\psi_z}{\delta x}\right)\}\ ] \quad \textbf{(25)}$$

5. PROTO- TYPE MHD SYSTEM

Magneto hydrodynamic system resulting pumping action in when an electromagnetic field interacts directly with the conducting fluid. The current flow in a direction perpendicular to the direction of magnetic field causes the fluid to experience a force. The force is in a direction perpendicular to both the magnetic field and the current flow. This high conductivity fluid can be used to produce a useful pumping effect. The measurements of bead velocity under the influence of MHD have been difficult due to strong electro kinetic flow in the side-channel. We made visual estimations of bead velocity. Note that no movement in the main channel is observed when only the magnetic field is applied. We have fabricated MHD system using transparent cuboids having two electrodes of zinc plates of 7cms with inlet and out let ports and with an excitation connector for AC / DC operating voltage. We have designed and developed its proto type model and operated on DC and AC supply both. Experiment for flow of conducting fluid based on MHD principle have been realized, video for the same has been prepared. First we have operated our proto type MHD system with 12 V DC car batteries and then same system we tried to operate with 230 V AC supply with rheostat arrangement. We varied electric and magnetic fields one by one and noticed flow of fluid. It was found to be varying proportional to fields as effective result i.e. flow of conducting liquid found to be dependent on Intensity of the magnetic and electric field . Direction of flow fluid also noticed to be varying depending on polarity of the supply. Direction of flow of fluid gets opposite on reversing its polarity.

The MHD system used NaCl (salty water) as conducting fluid. Two electrodes of zinc strips were connected with 12 V, 20Amp hr battery later with 20V-60V ac supply with a on/off switch. Model was provided with provision of inlet and outlet pipe with two small tanks for storage of conducting fluid. We have operated 12 V DC switch and observed that flow of conducting fluid have taken place. Then with changed input dc supply with AC source varying from 20V to 60 V and observed that there was significant enhancement in the velocity of the fluid as compared to DC source. These results were compared with analytical and numerical methods and found to be very close. More accurate results with data need better instrumentation for necessary set up and process to develop the same is under way.

An electrically conducting fluid (NaCl) using Lorentz force. (Force produced when an electric current is applied across a channel filled with conducting solution in the presence of a perpendicular magnetic field). Flow measurements by mixing salt solution by recording a five seconds movie by mobile video capture camera. The measurements carried out by varying the NaCl concentrations and the experiments were carried out varying both, the magnetic field and electric field. Figure 1 -6 presents numerical results and fig 7 has been for experimental test set up of proto type with inlet, outlet and MHD chamber with two permanent magnets. Fig 8-10 depicts MHD device and Lorentz force direction during magnetic and electric field interaction. Future work in the proto type system may be working towards precise measurements of conducting fluid velocity of gases and fluids, temperature rneasurements inside chamber ,varying magnetic fields at different phases, variations in dimensions of chamber, electrodes, inlet and outlet of this system and document all relevant parameters.

6. FIGURES

We have used initial value of magnetic field B0 as Gaussian function.

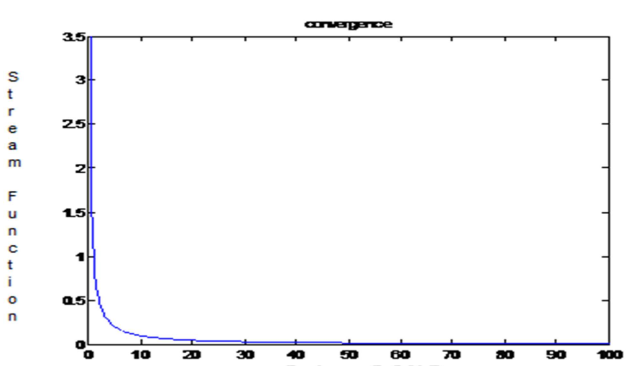

Figure 1 convergence plot in stream function and field

The equation (7) has been reduced to optimized solution of having only two variables i.e. magnetic field B and
stream function ψ and similar solution can be extended for equation . This resulting equations were solved by finite difference method numerically, we have taken initial conditions of stream function to some suitable value such as 2 pie radian and magnetic field as Gaussian function to compute Cartesian coordinates x, y, z computed stream function, results are presented below

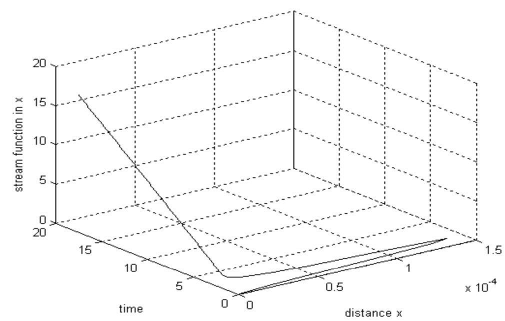

Figure2. 3D simulated results of stream function indicate heavy turbulence in x component

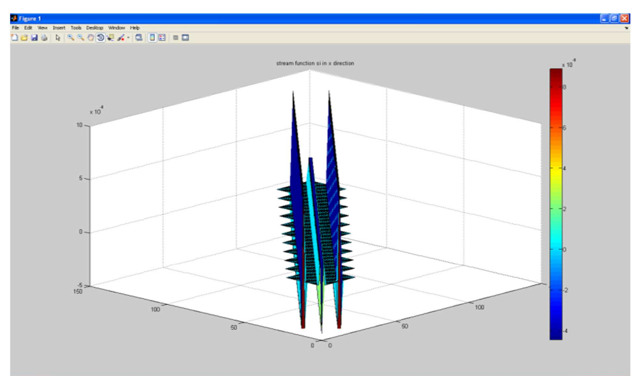

Figure 3 Plot stream function in x direction , when all other parameters made variable

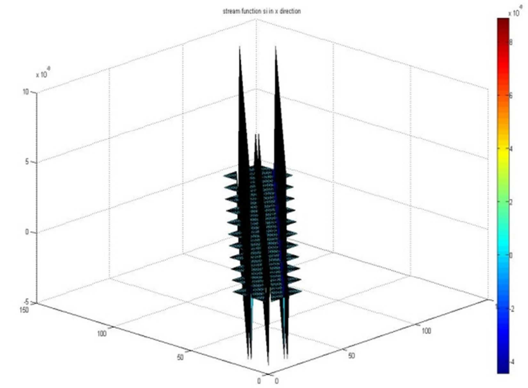

Figure 4 Stream function showing more turbulence in central part

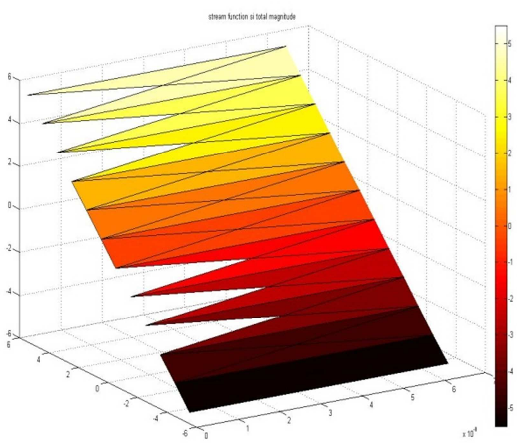

Figure 5 Magnitude of total stream function when x, y, z, t are varying.

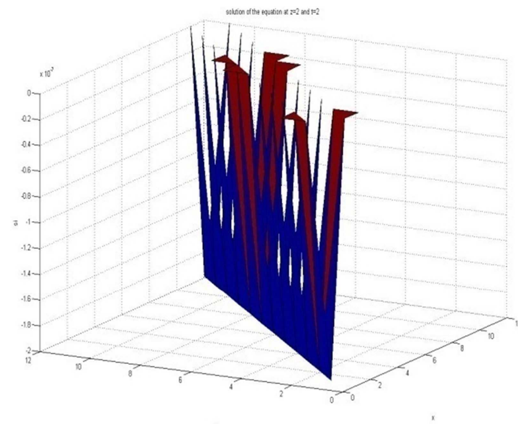

Figure 6 Solution for stream function, when z=2,t=2

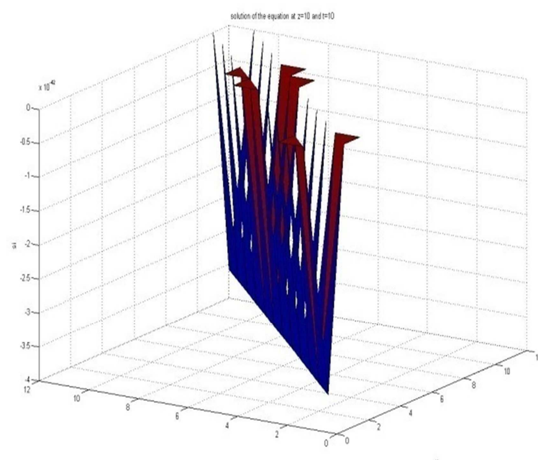

Figure 7 Heavy Turbulence observed

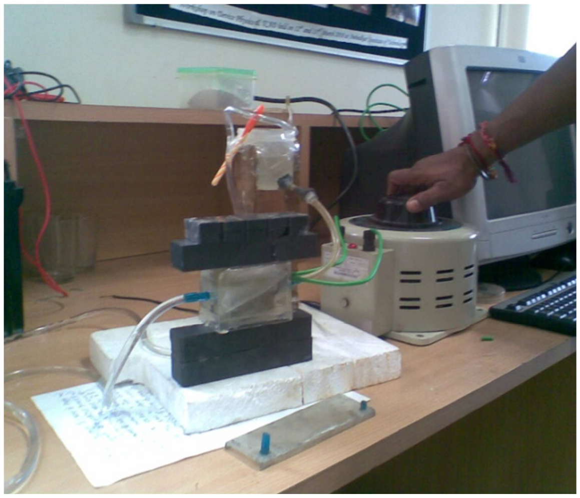

Figure 8 MHD prototype system with complete setup

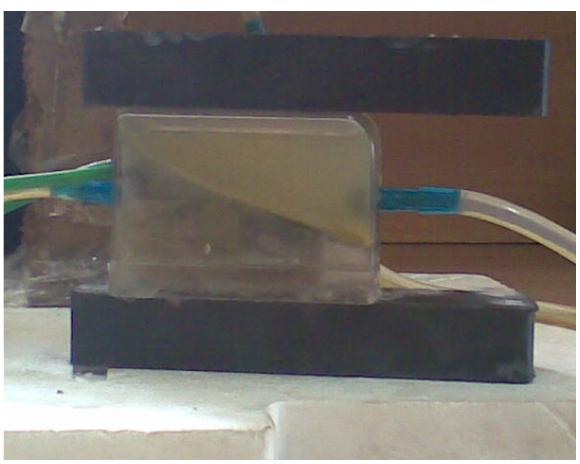

Figure 9. MHD system of 8.5 cm x6 cm x 4.5cm Chamber with inlet and outlet .75cm, turbulence observed at centre.

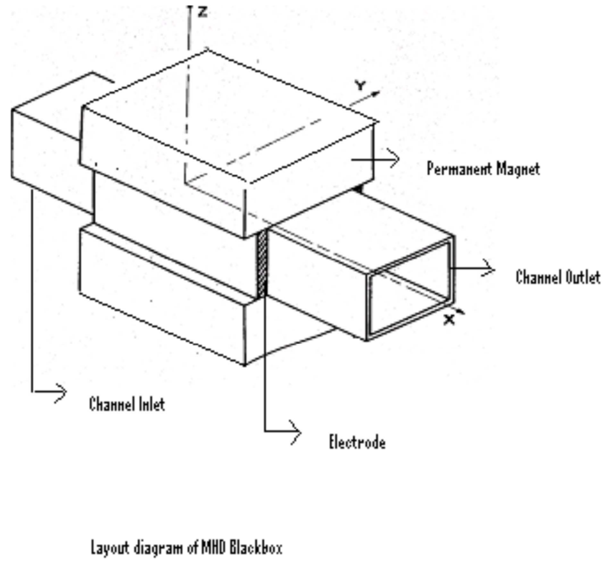

Figure 10 showing various parts and nomenclature of proto type system

The measurement of bead velocity under the influence of MHD has been difficult due to strong electro kinetic flow in the side-channel. We made visual estimations of bead velocity. Note that no movement in the main channel is observed when only the magnetic field is applied.

We have fabricated MHD system using transparent cuboids 2x3x2 cubic cm having two electrodes of zinc plates of 5 cm with inlet port and out let port of 8 mm connected in circular loop through a water reservoir with two rectangular magnets of 8 cm each, and with an excitation to electrodes through connector from AC / DC source voltage is shown in the diagram. We have designed and developed this proto type model and operated first on with DC supply, and later on with AC source. Different velocities of conducting fluid were recorded. Experimentation for flow of conducting fluid based on MHD principle have been realized. For DC source, we have operated our proto type MHD system with 12 V DC car batteries. And later, we operated this prototype MHD on 230 V AC supply with rheostat in series. We varied electric and magnetic fields one by one and noticed difference in velocity of fluid flow. It was observed to be varying proportional to both type of fields viz flow of conducting liquid found to be dependent on Intensity of the magnetic and electric field. Than we reversed the excitation polarity and observed that direction of flow fluid also changing depending on the polarity used. Direction of flow of fluid getting opposite on reversing its polarity.

The MHD system used NaCl (salty water) as conducting fluid. Two electrodes of zinc strips were connected with 12 V, 20 Amp hr battery later with 20V-60V ac supply with a on/off switch. It has been observed that central part of the electrode in black box produces more bubble formation, which is an indication intense turbulence. Model was provided with provision of inlet and outlet pipe with two small tanks for storage of conducting fluid . We have operated 12 V DC switch and observed that flow of conducting fluid have taken place. Then with changed input dc supply with AC source varying from 20V to 60 V and observed that there was significant enhancement in the velocity of the fluid as compared to DC source.

7. DISCUSSION AND CONCLUSION

It is evident from numerical simulations and experimental results that high electric field and magnetic field produces turbulence in central part of the stream function. We proposed this paper solution of two dimensional and three dimensional MHD system, with the results of analytical, numerical and experimental methods. All three types of results are in sync. Precision measurements on the issues of non linearity have been computed. Non linearity gets dominated when system is subjected to high Electric fields and Magnetic fields [19]. Velocity of fluid found to be dependent on both types of fields proportionally. In our work, we have proposed model of MHD system with relevant mathematical formulations. Simulation analysis has been carried out using MATLAB. Corresponding prototype implementation has been worked out , its functionality has been achieved and bead velocity of the conducting .were found to be varying depending on input. Detailed analysis on 3D modelling has been worked with significant turbulence effects which have been more in central part of the stream function. The obtained results also confirm a directly influence of the external excitation and the chosen geometrical dimensions of the MHD. Velocity field and magnitude of flow rate was found to be more significant with AC source than to DC source, also formation of bubbles are less in AC source as compared to DC. Figure 2-10 shows that more turbulence is observed in central part when z is varied. Also when z and t increases stream function decreases i.e. there is very low speed at later time periods and over distant z , the effect of variation of all components of x, y, z, t shows that stream function is concentrated in central part of the region.

It can be used in Microwave propulsion, Satellite propulsion, Space weather, MEMS Development, Micro pumps, Micro fluidic devices, Sensors and Actuators, Precision Switch, Stirring of fluids, MHD Antenna and Desert Cooler water Sprinkler etc.

ACKNOWLEDGEMENT

My Director Prof Asok De and Director NSIT Prof Raj Senani who inspired me for this research enriched with all necessary resources required. I am indebted to my wife Sujata, daughter Monica and son Rahul for giving the sufficient time and conducing environment to work towards this research.

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