ON PROPOSING SCHEME FOR REDUCTION OF PAPR IN OFDM SYSTEM UNDER EQUALIZATION

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) system gives more attentions due to its high data rate and its capability to combat multi-path interference. In this paper, two aims will be studied. First aim is to simulate OFDM system with equalization using Minimum Mean Square Error (MMSE) and Zero Forcing (ZF). Simulation results demonstrate the validity of the analytical results over Stanford University Interim (SUI) channels and AWGN, which are not clarified until now. Also, QAM constellations of this system are shown under different sub-carriers and modulation techniques over different SUI channels to illustrate these results. Other aim is to suggest a proposed scheme of OFDM system with equalization over SUI channels to decrease Peak to Average Power Ratio (PAPR). This proposed scheme combines conventional technique and μ compander technique. Optimum μ will be used in this proposed technique. Comparisons will be made between the system that is used proposed technique and the system without reduction. Simulation results show that the proposed technique is more efficient for reduction of PAPR.

Keywords: *OFDM, SUI channel, compander, clipping, MMSE and ZF.*

1. INTRODUCTION

Wireless communications are generally subject to severe multi-path fading channels that can seriously degrade the system performance. Counteracting the frequency selectivity of multi-path channels by multiplexing information on different orthogonal carriers is the key to the OFDM success. Indeed, if a cyclic prefix is inserted between successive OFDM symbols, the overall system can be viewed as composed of parallel frequency flat channels [1]. In communication systems, which take advantage of the fast Fourier transform (FFT) processing to realize an OFDM modem, the channel induced distortions can then be compensated by using equalizers. Actually, some researches in [2-6] had been made to investigate the effects of equalizations on the performance of OFDM system. Two types of equalizers will be used here which are MMSE and ZF. First, in this paper, simulation of OFDM system using these equalizers will be made. Simulation results demonstrate the validity of the analytical results over SUI channels under using different sub-carriers and modulation techniques. In fact, SUI multi-path channels [7-9] are widely used for evaluation of broadband wireless metropolitan area networks. Moreover, proposed technique will be suggested on the investigated system here to reduce Peak to Average Power Ratio (PAPR). The increase of PAPR comes from characteristics of OFDM system which could cause poor power efficiency or serious performance degradation to transmit power amplifier [10]. In fact, in order to reduce PAPR of OFDM signals, two groups of methods have been proposed in [11]-[24] and [25]. One group intends to reduce the occurrence of large signals before multi-carrier modulation. The fact is that if the independence among signals on different sub-channels is demolished by coding [11]–[14] or selective mapping [15], or the number of independent sub-channels is decreased by partial transmitting [16], PAPR would be reduced. In these methods, however, either redundancy or computational complexity is relatively high, which may result in large delay or overhead in practical systems [14]. The second group processes the OFDM signals directly in [17]–[24] and [25]. The simplest way is to deliberately clip large signals before D/A conversion. Nevertheless, digital clipping suffers from three problems: in-band distortion, which causes significant performance penalty [17]; out-of-band radiation, which reduces the spectral efficiency [18], [19]; and peak regrowth after D/A conversion [20]. In addition, it is more reasonable to treat clipping noise as a kind of impulsive noise rather than a continual additive Gaussian noise for most realistic case when clipping level is relatively high [21]. Therefore, performance degradation due to the clipping may not be so optimistic as analyzed in previous literatures based on Gaussian assumption [22]. In despite of these facts, still clipping techniques be recently attractive for using [26,27]. Other method is compander technique that has been proposed by Wang *et al.* to reduce PAPR with low complexity [23] and [25]. In this method, the PAPR of OFDM signals is reduced by increasing the average power of signals while keeping the peak unchanged, but this reduction in PAPR may be very limited under certain BER performance constraint. The idea of using this technique has been come from the fact that characteristics of OFDM signals are approximately the same as speech signals. The analysis of this fact is found in [25]. In order to obtain more effective reduction in PAPR, we proposed a simple method based on companding and clipping which combine the advantages of both methods, reducing the PAPR of OFDM signals significantly using optimum μ, the parameter that controls the amount of compression. Comparisons will be made by the system that used proposed technique, each method alone, this means clipping only or companding only, and the system without reduction.

The paper is organized as follows: In Section 2, OFDM system analysis will be introduced. In Section 3, Channel model is shown. Simulation results will be made in Section 4. Conclusions will be done in Section 5.

2. OFDM SYSTEM ANALYSIS

OFDM system is shown in Figure 1. An IDFT operation is performed by the complex -valued modulation symbols $X = [X(0), \ldots, X(N-1)]^T$ transmitted in the frequency domain. Before equalization, the received frequency domain symbols $Y = [Y(0), \ldots, Y(N-1)]^T$ are given as [28]:

$$Y = HX + Z, \tag{1}$$

where $Z = [Z(0), \ldots, Z(N-1)]^T$ are frequency-domain AWGN samples. Moreover, the frequency-domain channel matrix H, which is a diagonal matrix, is given as:

$$H = \begin{bmatrix} H(0) & 0 & \ldots\ldots\ldots & 0 \\ 0 & H(1) & \ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots & H(N-1) \end{bmatrix} \tag{2}$$

Equation (1) can be rewritten in its expanded form as follows:

$$\begin{bmatrix} y(0) \\ y(1) \\ \ldots\ldots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} H(0) & 0 & \ldots\ldots\ldots & 0 \\ 0 & H(1) & \ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots & H(N-1) \end{bmatrix} \begin{bmatrix} X(0) \\ X(1) \\ \ldots\ldots \\ X(N-1) \end{bmatrix} + \begin{bmatrix} Z(0) \\ Z(1) \\ \ldots\ldots \\ Z(N-1) \end{bmatrix} \tag{3}$$

Assuming Maximal Ratio Combining (MRC) receiver, the channel equalization matrix is simply conjugate transpose H^H of the channel matrix H. The estimates of received modulation symbols in the frequency domain are given by:

$$\hat{X} = H^H(HX + Z) = H^H HX + Z' \tag{4}$$

where $Z' = H^H Z$ and the matrix $H^H H$ is expressed as:

$$H^H H = \begin{bmatrix} |H(0)|^2 & 0 & \ldots\ldots\ldots & 0 \\ 0 & |H(1)|^2 & \ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots & |H(N-1)|^2 \end{bmatrix} \tag{5}$$

The channel matrix H is always orthogonal because all non-diagonal elements of the matrix $H^H H$ are zero. This is the case even when the channel gains $H(0), H(1), \ldots, H(N-1)$ are different as is the case in a multi-path frequency-selective channel. The received modulation symbols estimates are then donated by:

$$\begin{bmatrix} \hat{X}(0) \\ \hat{X}(1) \\ \ldots\ldots \\ \hat{X}(N-1) \end{bmatrix} = \begin{bmatrix} |H(0)|^2 & 0 & \ldots\ldots\ldots & 0 \\ 0 & |H(1)|^2 & \ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots & |H(N-1)|^2 \end{bmatrix} \begin{bmatrix} X(0) \\ X(1) \\ \ldots\ldots \\ X(N-1) \end{bmatrix} + \begin{bmatrix} Z'(0) \\ Z'(1) \\ \ldots\ldots \\ Z'(N-1) \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \hat{X}(0) \\ \hat{X}(1) \\ \ldots\ldots \\ \hat{X}(N-1) \end{bmatrix} = \begin{bmatrix} |H(0)|^2 X(0) \\ |H(1)|^2 X(1) \\ \ldots\ldots \\ |H(N-1)|^2 X(N-1) \end{bmatrix} + \begin{bmatrix} Z'(0) \\ Z'(1) \\ \ldots\ldots \\ Z'(N-1) \end{bmatrix} \tag{7}$$

where

$Z'(k) = H^*(k)Z(k), k = 0,1,2,\ldots\ldots(N-1).$

The modulation symbol estimate on the k^{th} subcarrier from Equation (7) is given by:

$$\hat{X} = |H(k)|^2 X(k) + Z'(k) \tag{8}$$

We note that there is no inter-subcarrier or inter-modulation-symbol interference in OFDM. Therefore, the SNR on the k^{th} subcarrier for an OFDM system using MRC receiver is given by:

$$\gamma_{OFDM-MRC}(k) = \frac{P\,|H(k)|^2}{\sigma^2} = \gamma |H(k)|^2 \tag{9}$$

where $\gamma = \dfrac{P}{\sigma^2}$ is the ratio of signal power to the AWGN noise power.

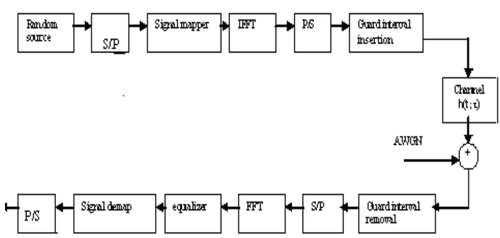

Figure. 1 System Model.

The equalization weight on the k^{th} sub-carrier is given as in case of MMSE [28]:

$$g(k) = \frac{H(k)^*}{|H(k)|^2 + \sigma^2} \tag{10}$$

The estimates of received modulation symbols in the frequency-domain are given by:

$$\hat{X} = G(HX + Z) = GHX + Z' \tag{11}$$

where $Z' = GZ$ and the matrix GH is expressed as follows:

$$GH = \begin{vmatrix} \frac{|H(0)|^2}{|H(0)|^2 + \sigma^2} & 0 & \ldots\ldots & 0 \\ 0 & \frac{|H(1)|^2}{|H(1)|^2 + \sigma^2} & \ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots & \frac{|H(N-1)|^2}{|H(N-1)|^2 + \sigma^2} \end{vmatrix} \tag{12}$$

The modulation symbol estimate on the k^{th} sub-carrier is introduced by:

$$\hat{X}(k) = \frac{|H(k)|^2}{|(H(k)|^2 + \sigma^2} X(k) + \frac{H(k)^*}{|H(k)|^2 + \sigma^2} Z(k) \tag{13}$$

For an OFDM system, the SNR on the k^{th} subcarrier using an MMSE receiver is then given by:

$$\gamma_{OFDM-MMSE}(k) = \frac{P\left(\frac{|H(k)|^2}{|H(k)|^2 + \sigma^2}\right)^2}{\sigma^2\left(\frac{|H(k)|}{|H(k)|^2 + \sigma^2}\right)^2} = \frac{P\,|H(k)|^2}{\sigma^2} = \gamma |H(k)|^2 \tag{14}$$

In OFDM system, the transmission on each sub-carrier is orthogonal and there is no inter-carrier interference. Therefore, MMSE receiver provides the same performance as an MRC receiver in case of OFDM. The equalized weight on the k^{th} sub-carrier in case of a zero-forcing receiver is given as:

$$g(k) = \frac{1}{H(k)} = \frac{H(k)^*}{|H(k)|^2} \tag{15}$$

The modulation symbol estimate on the k^{th} sub-carrier is given by,

$$\hat{X}(k) = X(k) + \frac{H(k)^*}{|H(k)|^2} Z(k) \tag{16}$$

For an OFDM system using a ZF receiver, the SNR on the k^{th} sub-carrier is simply given by:

$$\gamma_{OFDM-ZF}(k) = \frac{P}{\sigma^2\left(\frac{|H(k)|}{|H(k)|^2}\right)^2} = \gamma |H(k)|^2 \tag{17}$$

We note that a ZF receiver provides the same performance as MMSE receiver. Therefore, for 1-tap equalization in OFDM, any of the receivers among MMSE or ZF can be used for achieving the same performance.

$$\gamma_{OFDM-MMSE}(k) = \gamma_{OFDM-ZF}(k)$$

3. CHANNEL MODEL:

SUI channel models are an extension of the earlier work by AT&T Wireless and Erceg et al [7]. In this model, a set of six channels was selected to address three different terrain types that are typical of the continental US [8]. This model can be used for simulations, design, development and testing of technologies suitable for fixed broadband wireless applications [9]. The parameters for the model were selected based upon some statistical models. The parametric view of the SUI channels is summarized in the Table I.

4. SIMULATION RESULTS

To evaluate the performances of OFDM systems using equalizers over SUI channels, OFDM block transmission over different SUI channels will be simulated for evaluating Broadband wireless system in 2-11 GHz bands. The sampling rate was assumed to be 20 Ms/sec. This simulated system employs an OFDM signal with N sub-carriers using different modulation techniques. SUI 1 from train type C (Flat/Light tree density), SUI3 from train type B(Hilly/Light tree density or Flat/moderate tree density), and SUI 5 from train type A(Hilly/moderate to heavy tree density) are assumed for simplicity.

Table I The parametric view of the SUI channels.

Model (Delay / Gain)	L(number of tape)=3 Tap 1	Tap 2	Tap 3	Delay spread
SUI 1	0 μs	0.4 μs	0.8 μs	0.111 μs
	0 dB	-15 dB	-20 dB	
SUI 2	0 μs	0.5 μs	1 μs	0.202 μs
	0 dB	-12 dB	-15 dB	
SUI 3	0 μs	0.5 μs	1 μs	0.264 μs
	0 dB	-5 dB	-10 dB	
SUI 4	0 μs	2 μs	4 μs	1.257 μs
	0 dB	-4 dB	-8 dB	
SUI 5	0 μs	5 μs	10 μs	2.842 μs
	0 dB	-5 dB	-10 dB	
SUI 6	0 μs	14 μs	20 μs	5.240 μs
	0 dB	-10 dB	-14 dB	

BER performances of OFDM system using MMSE and ZF will be investigated in Figures 2 and 3 over SUI 1, 3, 5 channel models and AWGN for 64 QAM and N=256, respectively. From these figures, we conclude that MMSE gives approximately the same results as ZF. This demonstrates the validity of the analytical results which had been proved in Sect. 2. For more illustration, QAM constellation will be shown in Fig. 4 for N=256 and 64QAM. Figures 5, 6 and 7 show the same BER performances of MMSE and ZF but with different modulation technique and sub-carriers. Figures 8 and 9 show Complementary Cumulative Distribution Function (CCDF) and BER performances, respectively over SUI 1. Proposed technique will be added after P/S in the transmitter and before S/P in the receiver. Where μ=16 is used which is optimum at N=256 [25] for compander and clipping ratio=2 for clipper. The calculations of CCDF and PAPR had been made in detail in [29].

From Figure 8, we conclude that the proposed technique gives better CCDF performance than each method alone and the system without reduction. From Figure 9, compander only gives better results than any others. Clipping represented as a hard limitation of all signals which may have large or small levels that is why clipping technique gives worst results than compander. The reason of why combination between compander and clipper give worst BER performance than compander is that signal compression results in spectral regrowth, a form of signal distortion similar to that which occurs with signal limiting (e.g. such as with hard limiters). Besides, the expansion of the compressed signal is occurred at the receiver which amplifies receiver noise. Addition of clipping, which represents as a hard limiter, to compnader increases more distortion. Therefore, the BER performance of proposed scheme becomes worst than using compander only.

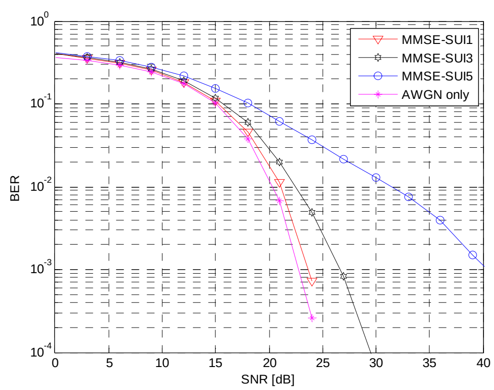

Figure 2. BER vs. SNR (dB) for N=256 and 64 QAM using MMSE.

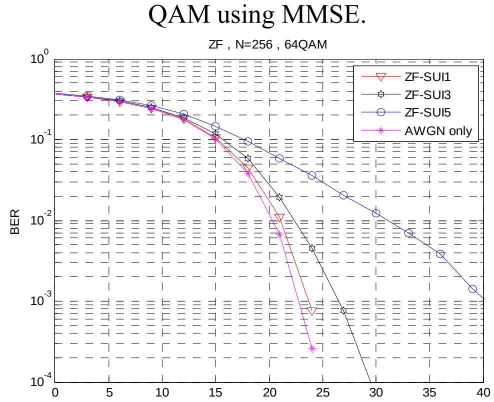

Figure 3. BER vs. SNR (dB) for N=256 and 64 QAM using ZF.

5. CONCLUSIONS

In this paper, two aims had been investigated. First, performance of OFDM system had been simulated under using MMSE and ZF equalizers by using different sub-carriers and modulation techniques over SUI channels which had been demonstrated the validity of analytical results which were described in this paper. Moreover, proposed technique had been suggested to reduce PAPR. This technique combines two methods which are clipping and compander. Optimum parameter is used in our simulations. The results show that proposed technique is an effective technique to reduce PAPR of OFDM system used equalizer over SUI channels.

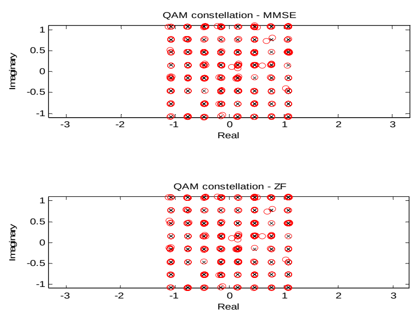

Figure 4 QAM constellation for N=256 and 64 QAM for ZF and MMSE.

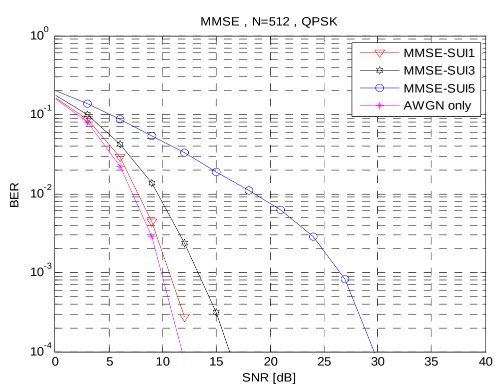

Figure 5. BER vs. SNR (dB) for N=512 and QPSK using MMSE.

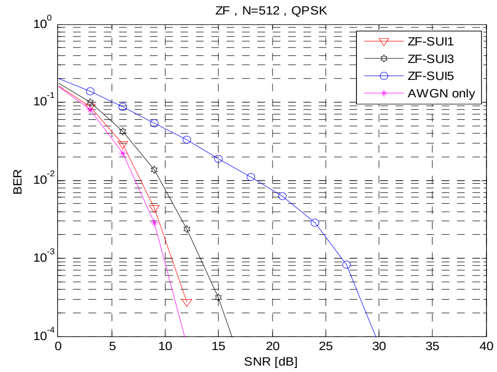

Figure 6. BER vs. SNR (dB) for N=512 and QPSK using ZF.

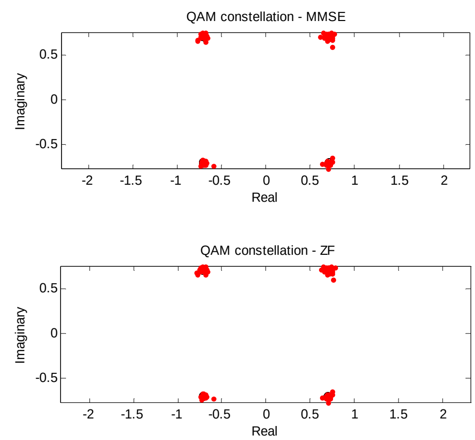

Figure 7 QAM constellation for N=512 and QPSK for ZF and MMSE.

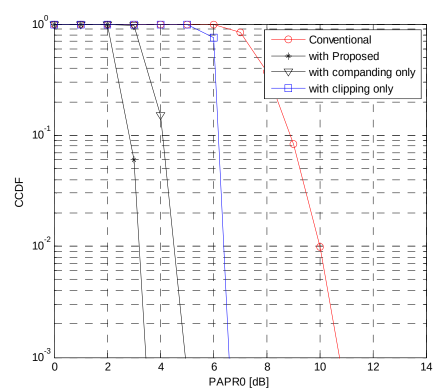

Figure 8 CCDF performance for with and without proposed scheme, clipping alone and compander alone for 16QAM and μ=16 ,N=256 over SUI 1.

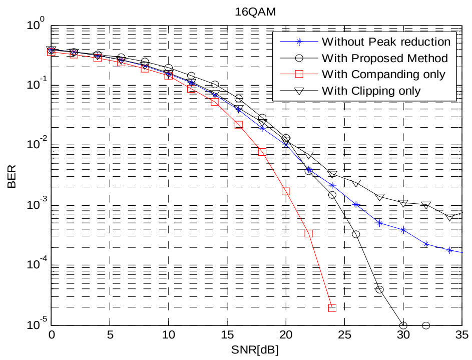

Figure 9 BER performance for with and without proposed scheme, clipping alone and compander alone for 16QAM and μ=16 ,N=256 over SUI 1.

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