CONTROL MOTOR USING STATE SPACE DISCRETE TIME OPTIMAL TRACKING CONTROLLER

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ABSTRACT

Brushless Direct Current (BLDC) motors are widely used for high performance control applications. Conventional PID controller only provides satisfactory performance for set-point regulation. In this paper, a discrete time optimal tracking control of BLDC motor is presented. Modeling of the BLDC motor is expressed in state equation. A discrete time full-order state observer is designed to observe states of BLDC motor. Feedback gain matrix of the observer is obtained by pole assignment method using Ackermann formulation with observability matrix. The state feedback variables are given by the state observer. A discrete time LQ optimal tracking control of the BLDC motor system is constructed to track the velocity of the BLDC motor to the reference based on the designed observer. Numerical results are shown to prove that the performance of the proposed controller.

Keywords: *Sate Space, Optimal Control, Discrete Time System, Tracking Control, modeling*

1. INTRODUCTION

The disadvantages of DC motors emerge due to the employment of mechanical commutation since the life expectancy of the brush construction is restricted. Furthermore, mechanical commutators lead to losses and contact uncertainties at small voltages and can cause electrical disturbances (sparking). Therefore, Brushless Direct Current (BLDC) motors have been developed. BLDC motors do not use brushes for commutation; instead, they are electronically commutated. BLDC motors are a type of synchronous motor. This means that the magnetic field is generated by the stator and the rotor which rotates at the same frequency so that the BLDC motor do not experience the "slip" that is normally seen in induction motors. In addition, BLDC motor has better heat dissipation characteristic and ability to operate at higher speed [1]. However, the BLDC motor constitutes a more difficult problem in terms of modeling and control system design due to its multi-input nature and coupled nonlinear dynamics.

Therefore, a compact representation of the BLDC motor model was obtained in [2]. This model is similar to permanent magnet DC motors. As a result, PID controller can be easily applied to control BLDC motors. So there have been a lot of approaches to search the parameters of optimal PIC controllers to control the BLDC motors, including using iterative learning control [7], using LQR approach [8]. In recent years, researchers had applied another algorithm to enhance high performance system. R. Singh presented DC motor predictive models [5], this research designed optimal controller also. M. George introduced speed control of separated excited DC motor [4]. GUPTA presented a robust variable structure position control of DC motor [6]. These researches focused in continuous time system so that implementation of microcontroller is not convenient.

This paper presents a speed control of BLDC motor using discrete time optimal tracking controller. The model of the BLDC motor is expressed as discrete time equations. The optimal tracking controller based on the estimated states by using discrete time observer is designed to control. The effectiveness of the designed controller is shown via numerical and experimental results.

2. BRUSHLESS DC MOTORS

Unlike a permanent magnet DC motor, the commutation of a BLDC motor is controlled electronically. To rotate the BLDC motor, the stator windings should be energized in a sequence. It is important to know the rotor position in order to understand which winding will be energized following the energizing sequence. Rotor position is sensed using Hall effect sensors embedded into the stator.

The dynamic characteristics of BLDC motors are similar to brushed DC motors. The model of BLDC motor can be represented as [2].

$$T_m = K_t i \tag{1}$$

$$E = K_e \omega \tag{2}$$

$$J\dot{\omega} + b\omega = T_m = Ki \tag{3}$$

$$L\frac{di}{dt} + Ri + K\omega = V \tag{4}$$

where

R : Armature resistance [Ω].

L : Armature inductance [H].

K : Electromotive force constant [Nm/A].

K_t : Torque constant [Nm/A].

K_e : Voltage constant [Vs/rad].

V : Source voltage [V].

ω : Angular velocity of rotor [rad/s].

J : Moment of inertia of the rotor [kgm^2].

b : Damping ratio of the mechanical system [Nms].

In SI unit system, K_t is equal to K_e.

Combining (3) and (4) yields

$$LJ\dddot{\theta} + (Lb + RJ)\,\ddot{\theta} + (Rb + K^2)\,\dot{\theta} = KV \tag{5}$$

$\boldsymbol{x_m} = [\omega \quad \dot{\omega}]^T$ is defined as state vector of the BLDC motor. Eq. (5) can be written as

$$\underbrace{\begin{bmatrix}\dot{\omega}\\ \ddot{\omega}\end{bmatrix}}_{\dot{x}_m} = \underbrace{\begin{bmatrix}0 & 1\\ \dfrac{Rb+K^2}{LJ} & \dfrac{Lb+RJ}{LJ}\end{bmatrix}}_{A_m}\underbrace{\begin{bmatrix}\omega\\ \dot{\omega}\end{bmatrix}}_{x_m} + \underbrace{\begin{bmatrix}0\\ \dfrac{K}{LJ}\end{bmatrix}}_{B_m}V$$
$$y_m = \underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{C_m}\underbrace{\begin{bmatrix}\omega\\ \dot{\omega}\end{bmatrix}}_{x_m} \tag{6}$$

where y_m is angle velocity of the rotor of the BLDC motor.

The discrete time system equations of the BLDC motor can be obtained as

$$\boldsymbol{x_m}(k+1) = \boldsymbol{\Phi_m}(T)\boldsymbol{x_m}(k) + \boldsymbol{\theta_m}(T)V(k)$$
$$y_m(k) = \boldsymbol{C_m}(T)\boldsymbol{x_m}(k) \tag{7}$$

where

$\boldsymbol{x_m}(k) \in \Re^{2\times 1}$ is state vector of the BLDC motor at the k^{th} sample time, $y_m(k) \in \Re$ is speed of the rotor of the BLDC motor at the k^{th} sample time,

$$\boldsymbol{\Phi_m}(T) = e^{A_m T} = \boldsymbol{I_3} + \boldsymbol{A_m}T + \frac{1}{2!}\boldsymbol{A_m^2}T^2 + \frac{1}{3!}\boldsymbol{A_m^3}T^3 + \cdots \quad \in \Re^{2\times 2},$$

$$\boldsymbol{\theta_m}(T) = \int_0^T \Phi_m(T-\tau)\boldsymbol{B}d\tau \quad \in \Re^{2\times 1},$$

and $\boldsymbol{C_m}(T) = \boldsymbol{C_m} \quad \in \Re^{1\times 2}$

3. CONTROLLER DESIGN

A) Discrete Time Full-Order State Observer Design

To implement the discrete time optimal tracking controller, the information of all state variables of the system is needed. However, all state variables are not accessible in practical systems [3]. Furthermore, in the system that all state variables are accessible, the hardware configuration of the system becomes complex and the cost to implement this system is very high because sensors to measure all states are needed. Because of these reasons, a discrete time observer is needed to estimate the information of all states of the system. In the case that the output of the system is measurable and the system is full-observable, a discrete time full-order state observer can be designed to observe information of all state variables of the system.

It is assumed that the system (7) is full-observable. The system equations of the discrete time closed loop observer are proposed as follows:

$$\boldsymbol{\hat{x}_m}(k+1) = \boldsymbol{\Phi_m}(T)\boldsymbol{\hat{x}_m}(k) + \boldsymbol{\theta_m}(T)V(k) - \boldsymbol{L}(\hat{y}_m(k) - y_m(k))$$
$$\hat{y}_m(k) = \boldsymbol{C_m}(T)\boldsymbol{\hat{x}_m}(k) \tag{8}$$

where $\boldsymbol{\hat{x}_m}(k) \in \Re^{2\times 1}$ is state vector of the observer at the k^{th} sample time, $\hat{y}_m(k) \in \Re$ is speed of rotor of the observer at the k^{th} sample time, and $\boldsymbol{L} \in \Re^{2\times 1}$ is the feedback gain matrix.

$\boldsymbol{\tilde{x}_m}(k) = \boldsymbol{x_m}(k) - \boldsymbol{\hat{x}_m}(k)$ is defined as the estimated error state vector between the motor and the observer. Subtracting Eq. (8) from Eq. (7), the error state equation can be obtained as

$$\boldsymbol{\tilde{x}_m}(k+1) = [\boldsymbol{\Phi_m}(T) - \boldsymbol{LC_m}(T)]\,\boldsymbol{\tilde{x}_m}(k) = \boldsymbol{A_{cd}}\boldsymbol{\tilde{x}_m}(k) \tag{9}$$

The design objective of the observer is to obtain a feedback gain matrix $\boldsymbol{L}$ such that the estimated error states approach to zero as fast as possible.

That is, the feedback gain matrix $\boldsymbol{L}$ must be designed such that eigenvalues of $\boldsymbol{A_{cd}}$ exist in unit circle for the system (9) to be stable. By pole assignment method using Ackermann formulation with observability matrix $\boldsymbol{O_m}$, the feedback gain matrix $\boldsymbol{L}$ is obtained as follows [3]:

$$\boldsymbol{L} = \Delta'(\boldsymbol{\Phi_m})\ \boldsymbol{O_m^{-1} e_2^T} = \Delta'(\boldsymbol{\Phi_m}) \begin{bmatrix} \boldsymbol{C_m} \\ \boldsymbol{C_m \Phi_m} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{10}$$

where $\Delta'(\boldsymbol{\Phi_m})$ is desired characteristic equation of the observer, $\boldsymbol{O_m} = [\boldsymbol{C_m} \quad \boldsymbol{C_m \Phi_m}]^T$ is observability matrix, and $\boldsymbol{e_2} = [0 \quad 1]$ is unit vector.

Block diagram of this observer is shown in Fig. 1.

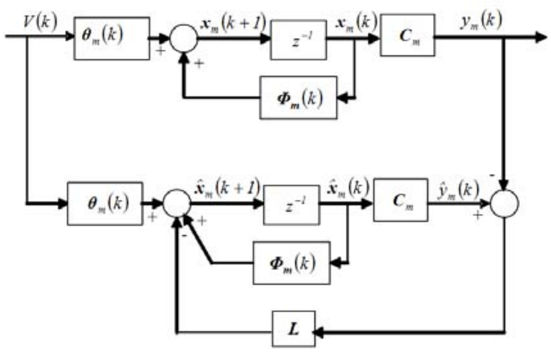

Figure 1. Block diagram of the system with observer.

B) Discrete time optimal controller design based on discrete time full-order state observer

The discrete time state variables equation of the BLDC motor can be rewritten as follows:

$$\begin{aligned} \boldsymbol{x}[(k+1)] &= \boldsymbol{A_d x}(k) + \boldsymbol{B_d u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C_d x}(k) \end{aligned} \tag{11}$$

where $\boldsymbol{x}(k) \in \Re^{2\times 1}$ is state vector, $\boldsymbol{y}(k) \in \Re$ is output, $u(k) \in \Re$ is control input, and $\boldsymbol{A_d} \in \Re^{2\times 2}$, $\boldsymbol{B_d} \in \Re^{2\times 1}$, $\boldsymbol{C_d} \in \Re^{1\times 2}$ are matrices with corresponding dimensions.

An error signal $\boldsymbol{e}(k) \in \Re$ is defined as the difference between the reference input $\boldsymbol{r}(k) \in \Re$ and the output of the system $\boldsymbol{y}(k)$ as follows:

$$\boldsymbol{e}(k) = \boldsymbol{r}(k) - \boldsymbol{y}(k) \tag{12}$$

It is denoted that the incremental control input is $\Delta \boldsymbol{u}(k) = \boldsymbol{u}(k) - \boldsymbol{u}(k-1)$ and the incremental state is $\Delta \boldsymbol{x}(k) = \boldsymbol{x}(k) - \boldsymbol{x}(k-1)$. If the system (11) is controllable and observable, it can be rewritten in the increment as follows:

$$\begin{aligned} \Delta \boldsymbol{x}[(k+1)] &= \boldsymbol{A_d} \Delta \boldsymbol{x}(k) + \boldsymbol{B_d} \Delta \boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C_d x}(k) \end{aligned} \tag{13}$$

The error at the $k+1^{th}$ sample time can be obtained from Eq. (12) as

$$\boldsymbol{e}(k+1) = \boldsymbol{r}(k+1) - \boldsymbol{y}(k+1) \tag{14}$$

Subtracting Eq. (12) from Eq. (14) yields

$$\boldsymbol{e}(k+1) - \boldsymbol{e}(k) = \boldsymbol{r}(k+1) - \boldsymbol{r}(k) - \boldsymbol{y}(k+1) + \boldsymbol{y}(k) \tag{15}$$

Substituting Eq. (13) into Eq. (15) can be reduced as

$$\boldsymbol{e}(k+1) = \boldsymbol{e}(k) + \Delta \boldsymbol{r}(k+1) - \boldsymbol{C_d A_d} \Delta \boldsymbol{x}(k) - \boldsymbol{C_d B_d} \Delta \boldsymbol{u}(k) \tag{16}$$

where $\Delta \boldsymbol{r}(k+1) = \boldsymbol{r}(k+1) - \boldsymbol{r}(k) \in \Re$

It is assumed that future values of the reference input $\boldsymbol{r}(k+1), \boldsymbol{r}(k+2), \cdots,$ cannot be utilized. The future values of the reference input beyond the k^{th} sample time are approximated as $\boldsymbol{r}(k)$. It means that the following is satisfied.

$$\Delta \boldsymbol{r}(k+i) = 0 \qquad \text{for} \quad i = 1,2,\ldots \tag{17}$$

From the first row of Eq. (13) and Eq. (16), the error system can be obtained as

$$\underbrace{\begin{bmatrix} \boldsymbol{e}(k+1) \\ \Delta \boldsymbol{x}(k+1) \end{bmatrix}}_{\boldsymbol{X}(k+1)} = \underbrace{\begin{bmatrix} \boldsymbol{1} & -\boldsymbol{C_d A_d} \\ \boldsymbol{0_{3x1}} & \boldsymbol{A_d} \end{bmatrix}}_{\boldsymbol{A_E}} \underbrace{\begin{bmatrix} \boldsymbol{e}(k) \\ \Delta \boldsymbol{x}(k) \end{bmatrix}}_{\boldsymbol{X}(k)} + \underbrace{\begin{bmatrix} -\boldsymbol{C_d B_d} \\ \boldsymbol{B_d} \end{bmatrix}}_{\boldsymbol{G}} \Delta \boldsymbol{u}(k) \tag{18}$$

where $\boldsymbol{X}(k) \in \Re^{3\times 1}$, $\boldsymbol{A_E} \in \Re^{3\times 3}$, and $\boldsymbol{G} \in \Re^{3\times 1}$.

A scalar cost function of the quadratic form is chosen as

$$J = \sum_{k=0}^{\infty} \left[\boldsymbol{X^T}(k)\ \boldsymbol{Q}\ \boldsymbol{X}(k) + \boldsymbol{\Delta u^T}(k)\ \boldsymbol{R}\ \boldsymbol{\Delta u}(k) \right] \tag{19}$$

where $\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q_e} & \boldsymbol{0_{1\times 2}} \\ \boldsymbol{0_{2\times 1}} & \boldsymbol{0_{2\times 2}} \end{bmatrix} \in \Re^{3\times 3}$ is semi-positive definite matrix, $\boldsymbol{Q_e} \in \Re$, and $\boldsymbol{R} \in \Re$ are positive scalar.

The optimal control signal $\Delta \boldsymbol{u}(k)$ that minimizes the cost function (19) of the system (18) can be obtained as [3]

$$\Delta \boldsymbol{u}(k) = -\left[\boldsymbol{R} + \boldsymbol{G}^{T} \boldsymbol{P}_{I} \boldsymbol{G}\right]^{-1} \boldsymbol{G}^{T} \boldsymbol{P}_{I} \boldsymbol{A}_{E} \boldsymbol{X}(k) \tag{20}$$

where $\boldsymbol{P}$ is semi-positive definite matrix. It is solution of the following algebraic Ricatti equation [3].

$$\boldsymbol{P} = \boldsymbol{Q} + \boldsymbol{A}_{E}^{T} \boldsymbol{P} \boldsymbol{A}_{E} - \boldsymbol{A}_{E}^{T} \boldsymbol{P} \boldsymbol{G}\left[R + \boldsymbol{G}^{T} \boldsymbol{P} \boldsymbol{G}\right]^{-1} \boldsymbol{G}^{T} \boldsymbol{P} \boldsymbol{A}_{E} \tag{21}$$

where $\boldsymbol{Q} \in \Re^{3\times 3}$ is semi-positive definite matrix, and $R \in \Re$ is positive scalar.
By taking the initial values as zero and integrating both side of Eq. (20), the control law $\boldsymbol{u}(k)$ can be obtained as

$$u(k) = K_{1e} \frac{z}{z-1} e(k) + \boldsymbol{K}_{1x} \boldsymbol{x}_{m}(k) \tag{22}$$

where

$\boldsymbol{K}_{1} = [K_{1e} \quad \boldsymbol{K}_{1x}] = \left[\boldsymbol{R} + \boldsymbol{G}^{T} \boldsymbol{P}_{I} \boldsymbol{G}\right]^{-1} \boldsymbol{G}^{T} \boldsymbol{P}_{I} \boldsymbol{A}_{E} \in \Re^{1\times 3}$

Based on the proposed observer (9) and the controller (22), the discrete time optimal controller design based on discrete time full-order state observer can be given as follows:

$$V(k) = K_{1e} \frac{z}{z-1} e(k) + \boldsymbol{K}_{1x} \hat{\boldsymbol{x}}_{m}(k) \tag{23}$$

The discrete time optimal tracking control system of the BLDC motor (7) designed based on the information of states of the system obtained from discrete time closed loop observer (9) is shown in Fig. 2.

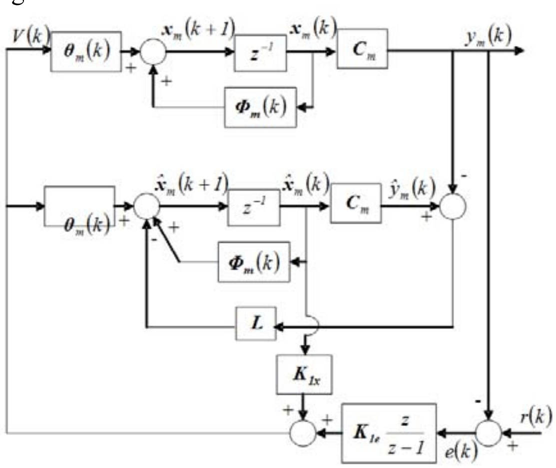

Figure 2. Block diagram of the optimal control of the BLDC motor.

4. NUMERICAL AND EXPERIMENTAL RESULTS

The specification of BLDC motor is shown in Table 1.

Table 1: Specification of BLDC motor

Parameters	Values and units
R	21.2 Ω
K_e	0.1433 V s/rad
D	$1\text{x}10^{-4}$ kg-m s/rad
L	0.052H
K_t	0.1433 kg-m/A
J	$1\text{x}10^{-5}$ kg-m s^2/rad

The effectiveness of the controller (23) as shown in Fig. 2 is verified by the simulation and experimental results.

The BLDC motor is controlled by the optimal tracking controller (23) which is obtained by choosing $R = 1$ and $\boldsymbol{Q} = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. The poles of the system (9) are chosen as $\boldsymbol{\lambda} = [0.375 + \text{j}0.32 \quad 0.375 - \text{j}0.32]$ for fast response. The feedback gain matrix $\boldsymbol{L} = [2.1233 \quad 1809.6]$ is obtained from (10). The simulation results of the designed discrete time optimal tracking controller of BLDC motor designed based on the discrete time full-order state observer are shown in Figs. 3~6.

Fig. 3 shows that discrete time optimal tracking controller of the BLDC motor designed based on the discrete time full-order state observer has good performance. The output of the system converges to the reference input after about 0.03 second, and its overshoot is about 1%. The tracking error of the system is shown in Fig. 5. The control signal input is shown in Fig. 6.

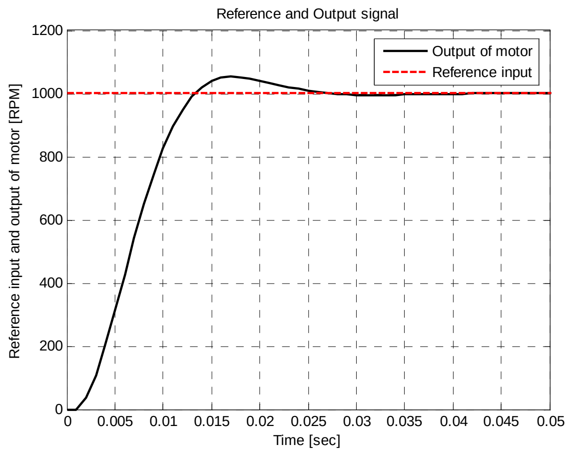

Figure 3. Reference input and output of system using optimal controller.

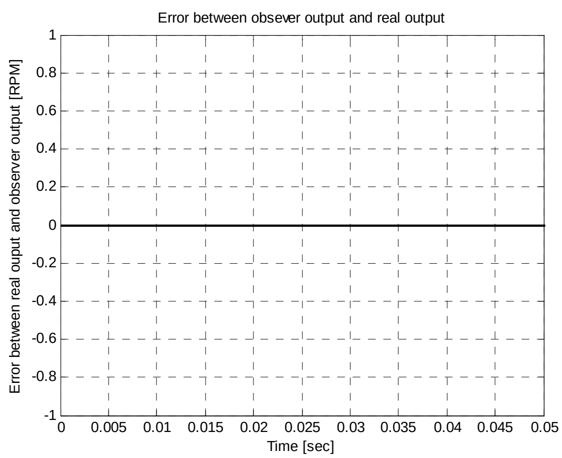

Figure 4. Error between estimated output of observer and output of motor.

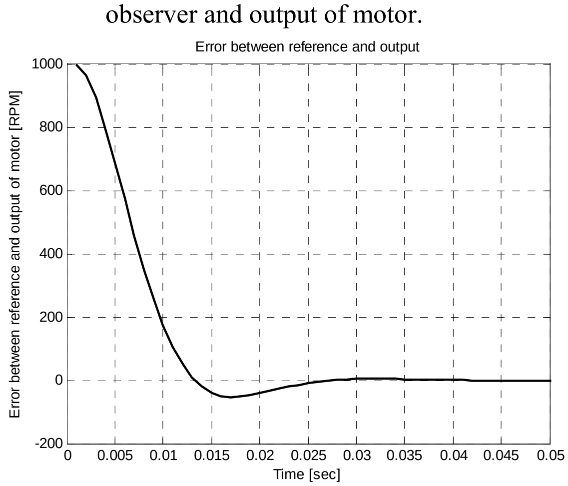

Figure 5. Tracking error of system using discrete time optimal controller.

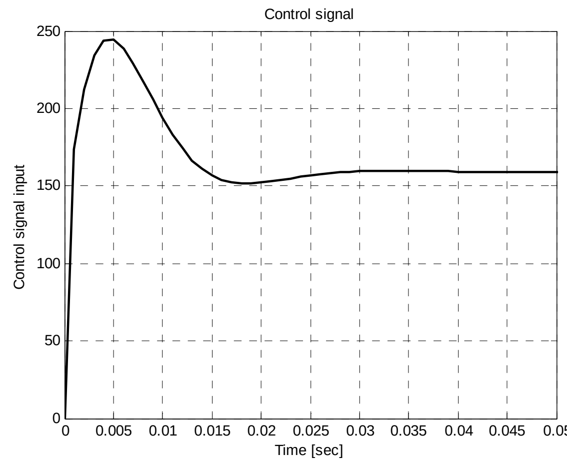

Figure 6. Control signal input using discrete time optimal controller.

5. CONCLUSION

In this paper, a discrete time optimal tracking control system for BLDC motor based on a full-order observer has been applied and investigated to control speed of BLDC motor. Performance of the optimal tracking controller is analyzed. The effectiveness of the designed controller is shown by the simulation results.

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