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FUZZY MINIMAL GENERALIZED CONTINUOUS FUNCTIONS

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ABSTRACT
We have introduced and studied the new class of functions called fuzzy minimal genralized continuous function, a new class of fuzzy closed and fuzzy open maps called fuzzy minimal genralized closed functions

Keywords*:* *Fuzzy Miminal space, fmg-continuous,* fuzzy minimal strongly, perfectly continuous, pre continuous.

1. Introduction

In 1968, Chang [4] introduced fuzzy topological spaces by using fuzzy sets[14]. Since then various authors have contributed to this area. Various results in ordinary topological spaces have been put in the fuzzy settings and alos various departures have been observed. Azad[3] observed that for fuzzy topological space the closure of a product of two fuzzy sets is not equal to the product of their closures. Coining the concept of a fuzzy space "' product related"' ro a fuzzy space, he overcome this departure further introducing the notions of fuzzy regular open(regular closed) set, he defined and studied semi continuous , almost continuous and weakly continuous mappings in the fuzzy settings. α -open (α-closed) sets, preopen(preclosed) sets, strongly semi continuous mappings and precontinuous mappings were introduced by Njastad[10], Mashaour[8], Noiri[11] and Mashhour[8] respectively. Abdulla[1] introduced and studied on fuzzy strong semi continuoity and precontinuous. Nagaveni[9] studied the fuzzy generalized weakly closed sets.

In this paper, We have a certain kind of investigation of fuzzy minimal generalized continuous functions. Further we have studied the properties of these continuous

2. Preliminaries

In this section, We begin by recalling some definitions and properties.
For easy understanding of the material incorporated in this article. We recall some basic some basic definitions and results. For details on the following notions we refer to [4,7,12].

Definition 2.1[2]: Let (X,τ) be a fuzzy topological space. A fuzzy set A in X is said to be fuzzy semi open if $A \leq cl(int(A))$.

Definition 2.2[2]: Let (X,τ) be a fuzzy topological space. A fuzzy set A in X is said to be fuzzy pre open if $A \leq int(cl(A))$

Definition 2.3[2]: Let (X,τ) be a fuzzy topological space. A fuzzy set A in X is said to be fuzzy α -open if $A \leq int(cl(int(A)))$.

Definition 2.4[2]: Let (X,τ) be a fuzzy topological space. A fuzzy set A in X is said to be fuzzy β -open if $A \leq cl(int(cl(A)))$.

The family of all fuzzy semi open fuzzy preopen fuzzy α open and fuzzy β-open set is denoted by FSO(X), FPO(X),$F\alpha$ O(X) and $F\beta$ O(X) repectively and they studied by many authors [5,6,12]. The complement of a fuzzy semi open, fuzzy preopen, fuzzy α -open and fuzzy β -open set is cally fuzzy semi closed fuzzy preclosed, fuzzy α-closed and fuzzy β -closed set respectively.The

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union of all fuzzy semi open fuzzy preopen, fuzzy α -open and fuzzy β -open sets of X contained in A is called fuzzy semi interrior, fuzzy pre interrior, fuzzy α-interior and fuzzy β - interior of A and is denoted by sint(A), pint(A), α -int(A) and β -int(A) respectively.Similarly scl(A), pcl(A), αcl(A) and β cl(A) are defined.

Definiton 2.5[2]: A family μ of fuzzy sets in X is said to be a fuzzy minimal structures on X if α, X $\in \mu$ for any $\alpha \in I$. In this case $(X, \mu)$is called a fuzzy minimal space.

Example 2.6[2]: Let (X, τ) be a fuzzy a fuzzy topological space. Then $\mu = \tau$, Fso(X), Fpo(X), $F\alpha(X)$ and $f\beta$ o(X) are fuzzy minimal structures on X.

Definition 2.7[2]: A fuzzy set $A \in I^X$is said to be a fuzzy m-open set if$B^c \in \mu$. we get
$m - int(A) = \vee U : U \leq A, U \in \mu$
$m - cl(A) = \wedge F : A \leq F, F^c \in \mu$

Remark 2.8: Choosing one of τ, Fso(X), Fpo(X), Fα(X) and Fβ o(X) instead of μ then m-int(A) would be int(A), sint(A) pint(A), $\alpha(A)$ and $\beta(A)$ respectively . similarly m-cl(A) is equal to cl(A),scl(A), pcl(A), $\alpha cl(A)$ and $\beta cl(A)$ respectively.

Proposition 2.9 [2]: For any two fuzzy sets A and B

- (a) m-int(A) $\leq$A and m-int(A) = A is a fuzzy m-open set. specially m-int($\alpha 1_X$) =$\alpha 1_X$ for all $\alpha \in I$.
- (b) $A \leq m - cl(A)$ and $A = m - cl(A)$ if A is fuzzy m-closed set.Specially m-cl($\alpha 1_X$) = $\alpha 1_X$ for all $\alpha \in I$.
- (C) $m - int(A) \leq m - int(B)$ and $m - cl(A) \leq m - cl(B)$if $A \leq B$
- (d) $m - int(A \cap B) = m - int(A) \cap (m - int(B)$ and $m - int(A) \cup (m - int(B)) \leq m - int(A \cup B)$
- (e)$m - cl(A \cup B) = m - cl(A) \cup (m - cl(B) and m - cl(A \cap B) \leq (m - cl(A)) \cap (m - cl(A))$
- (f) $m - int(m - int(A)) = m - int(A)$and $m - cl(m - cl(B)) = m - cl(B)$
- (g) $[m - clZ(A)]^c = m - int(A^c)$ and $[m - int(A)]^c = m - cl(A^c)$

3.Fuzzy minimal generalized continuous

In this section we have introduced the new class of definition fuzzy minimal genaralized continuous function (fmg-Continuous).Also we studied some of its properties.

Definition 3.1: Let (X, m_X) and (Y, m_Y) be fuzzy minimal space. A map $f : (X, m_X) \rightarrow (Y, m_Y)$ is said to be fuzzy minimal generalized continuous functions (fmg-continuous) if the inverse image of every fuzzy minimal open set in $(Y, m_Y)$is fmg-open in (X, m_X).

Theorem 3.2: If a map $f : (X, m_X) \rightarrow (Y, m_Y)$ from a fuzzy minimal space (X, m_X) into a fuzzy minimal space (Y, m_Y) is fuzzy minimal continuous then it is fmg-continuous but not conversely.

Proof: Let γ be a fuzzy minimal open set in fuzzy minimal space (Y, m_Y). Since f is fuzzy minimal continuous, $f^{-1}(\gamma)$ us fuzzy minimal open in (X, m_X). As every fuzzy minimal open set is fmg-open. We have $f^{-1}(\gamma)$ is fmg-open in (X, m_X). Therefore f is fmg-continuous.

Remark 3.3: The following example shows that the converse of the above theorem need not be true.

Example 3.4 Let $(X, m_X) = (Y, m_Y) = \{a, b, c\} : I = [0, 1]$and the function $\alpha, \beta, \gamma : (X, m_X) :\rightarrow [o, 1]$ defined as

$$\alpha(x) = \begin{cases} 1 & X = a \\ 0, & otherwise \end{cases}$$

$$\beta(x) = \begin{cases} 1 & X = b \\ 0, & otherwise \end{cases}$$

$$\gamma(x) = \begin{cases} 1 & X = b, c \\ 0, & otherwise \end{cases}$$

consider $m_X = \{1, 0, \alpha\}$. and $m_Y = \{0, 1, \beta, \gamma\}$. Now (X, m_X) and (Y, m_Y) are fuzzy minimal spaces. Define $f : (X, m_X) \rightarrow (Y, m_Y)$ by $f(a) = b$ $f(b) = c$ $f(c) = a$ then f is fmg-continuous but not fuzzy minimal continuous as the inverse image of the fuzzy minimal open set γ in (Y, m_Y) is $\gamma : (X, m_X) \rightarrow I$defined as

$$\gamma(x) = \begin{cases} 1 & X = b, c \\ 0, & otherwise \end{cases}$$

which is not fuzzy minimal open in (X, m_X).

Theorem 3.5: A map $f : (X, m_X) \rightarrow (Y, m_Y)$is fmg-continuous if and only if the inverse image of every minimal closed set in fuzzy minimal space (Y, m_Y) is fmg-closed in fuzzy minimal space (X, m_X).

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Proof: Let γ be a fuzzy minimal closed set in a fuzzy minimal space (Y, m_Y). Then γ^c is fuzzy minimal open in a fuzzy minimal space (Y, m_Y). Since f is fmg-continuous. $f^{-1}(\gamma^c)$ is fmg-open in (X, m_X). But $f^{-1}(\gamma^c) = 1 - f^{-1}(\gamma)$ and so $f^{-1}(\gamma)$ is fmg-closed set in(X, m_X).

Conversely assume that the inverse image of every fuzzy minimal closed set in (Y, m_Y) is fmg-closed set in (X, m_X). Let μbe a fuzzy minimal open set in (Y, m_Y). Then μ^c is fuzzy minimal closed set in (Y, m_Y). By hypothesis $f^{-1}(\mu^c) = 1 - f^{-1}(\mu)$ is fmg-closed in (X, m_X) and $f^{-1}(\mu)$ is fmg-open in (X, m_X). Thus f is fmg -continuous.

4.Fuzzy minimal Perfectly continuous, almost continuous Strongly continuous and pre-continuous mappings

In this section , we have introduced the new class of definitions almost continuous, perfectly continuous, strongly continuous and pre-continuous in fuzzy minimal weakly mappings.Also we studied the some of its properties and the relationship between the fmg-continuous functions.

Theorem 4.1: If a function $f : (X, m_X) \to (Y, m_Y)$ is fuzzy minimal perfectly continuous and fmg-continuous then it is fuzzy minimal g-continuous.

Proof: Let $f : (X, m_X) \to (Y, m_Y)$ be fuzzy minimal perfectly continuous and fmg-continuous,μ be a fuzzy minimal open set in (Y, m_Y) then $f^{-1}(\mu)$ is both fuzzy minimal open and fuzzy minimal closed in (X, m_X) as f is fuzzy minimal perfectly continuous. Since $f^{-1}(\mu)$ is fuzzy minimal closed, it is fuzzy minimal semi-closed and f is fmg-continuous implied $f^{-1}(\mu)$ is fmg-open. Thus $f^{-1}(\mu)$ is both fuzzy semi-closed and fmg-open and hence it is fuzzy minimal g-open. Thus f is fuzzy minimal g-continuous.

Definition 4.2: A mapping $f : (X, m_X) \to (Y, m_Y)$ from a fuzzy minimal space (X, m_X) to a fuzzy minimal space (Y, m_Y) is said to be fuzzy minimal almost continuous mapping if $f^{-1}(\mu)$ is fuzzy minimal open in (X, m_X) for each fuzzy minimal regular open set $\mu$of (Y, m_Y).

Theorem 4.3: If $f : (X, m_X) \to (Y, m_Y)$ is fuzzy minimal almost continuous then it is fmg-continuous.

Proof: Assume that f is fuzzy minimal almost continuous and a fuzzy set μ be fuzzy minimal open in (X, m_X). Then $f^{-1}(\mu)$ is fuzzy minimal regular open in (X, m_X). Now $f^{-1}(\mu)$ is fmg-open. Then f is fmg-continuous.

Definition 4.4: A mapping $f : (X, m_X) \to (Y, m_Y)$be a fuzzy minimal spce (X, m_X) to a fuzzy minimal space (Y, m_Y) . Then f is called a fuzzy strongly semi continuous if $f^{-1}(\mu)$is a fuzzy minimal α-open set in (X, m_X) for$\mu \in m_X$.

Theorem 4.5: If f is fuzzy minimal strongly semi-continuous then it is fmg-continuous but not conversely.

Proof: Assume that $f : (X, m_X) \to (Y, m_Y)$ is fuzzy minimal strongly semi continuous. Let μ be a fuzzy minimal open set in (Y, m_Y) . Since f is fuzzy minimal strongly semi continuous $f^{-1}(\mu)$ is fuzzy minimal α -open and hence fmg-open in (X, m_X). Thus f is fmg-continuous.

Remark 4.6: The converse of the above theorem need not be true from the following example.

Example 4.7:

Let $(X, m_X) = (Y, m_Y) = \{a, b, c\} : I = [0, 1]$and the function $\alpha, \beta, \gamma : (X, m_X) :\to [o, 1]$defined as

$$\alpha(x) = \begin{cases} 1 & X = a \\ 0, & otherwise \end{cases}$$

$$\beta(x) = \begin{cases} 1 & X = b, c \\ 0, & otherwise \end{cases}$$

$$\gamma(x) = \begin{cases} 1 & X = a, b \\ 0, & otherwise \end{cases}$$

consider the minimal space $m_X = \{0, 1, \alpha, \beta\}$ and $m_Y = \{0, 1, \gamma\}$. Then (X, m_X) and (Y, m_Y) are the fuzzy minimal spaces.Define the function $f : (X, m_X) \to (Y, m_Y)$ by $f(a) = a = f(b)$ and $f(c) = b$.This is a function is fmg-continuous but not fuzzy minimal strongly semi continuous. Since the inverse image of the fuzzy minimal open set. Now the identity map from (X, m_X) into(Y, m_Y) is fmg-continuous but not fuzzy minimal open set α in (Y, m_Y)

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is not fuzzy α-open in (X, m_X).

Definition 4.8:A mapping $f : (X, m_X) \to (Y, m_Y)$be a fuzzy minimal spce (X, m_X) to a fuzzy minimal space (Y, m_Y) . Then f is called a fuzzy pre- continuous if $f^{-1}(\mu)$is a fuzzy minimal pre-open set in (X, m_X) for $\mu \in m_X$.

Theorem 4.9: If a map $f : (X, m_X) \to (Y, m_Y)$is fuzzy minimal pre-continuous then it is fmg-continuous but conversely.

Proof: Let $f : (X, m_X) \to (y, m_Y)$ is fuzzy minimal pre-continuous and μ be a fuzzy minimal open set in (Y, m_Y). Then $f^{-1}(\mu)$ is fuzzy minimal pre-open and hence fmg-open in (X, m_X). Thus f is fmg-continuous.

Remark 4.10: The converse of the above theorem need not be true for the following example.

Example 4.11:

Let $(X, m_X) = (Y, m_Y) = \{a, b, c\} : I = [0, 1]$and the function $\alpha, \beta, \eta\gamma : (X, m_X) :\to [o, 1]$defined as

$$\alpha(x) = \begin{cases} 1 & X = a \\ 0, & otherwise \end{cases}$$

$$\beta(x) = \begin{cases} 1 & X = a, c \\ 0, & otherwise \end{cases}$$

$$\eta(x) = \begin{cases} 1 & X = b, c \\ 0, & otherwise \end{cases}$$

$$\gamma(x) = \begin{cases} 1 & X = c \\ 0, & otherwise \end{cases}$$

If $m_X = \{0, 1, \alpha, \beta\}$ and $m_Y = \{0, 1, \alpha, \gamma\}$be the minmal spaces then (X, m_X) and (Y, m_Y) are fuzzy minimal spaces. consider the function $f : (X, m_X) \to (Y, m_Y)$ by $f(a) = a = f(b)$ and $f(c) = c$.This is a function is fmg-continuous but not fuzzy minimal pre- continuous as the inverse image of the fuzzy minimal open set $\gamma in(Y, m_Y)$ is γ in (X, m_X) which is not fuzzy minimal pre-open.

Remark 4.12: The following implications contained in the following digram are true.

fm strongly continuous ↘ fmg- continuous ↙ fm almost continuous

fm semi continuous ↗ fmg- continuous ↖ fm pre continuous

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