PERFORMANCE ANALYSIS OF DIFFERENT M-ARY MODULATION TECHNIQUES IN FADING CHANNELS USING DIFFERENT DIVERSITY

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ABSTRACT

The comparison of symbol error probability(SEP) for different M-ary modulation techniques over slow, flat, identically independently distributed Rician fading channel is presented in this paper. Because fading is one of the major limitations in wireless communication, so modulation technique with diversity is used to transmit message signal efficiently. Exact analysis of symbol error probability for M-ary differentially encoded or differentially decoded phase shift keying(PSK) and coherent M-ary phase shift keying, transmitted over Rician fading channel has been performed with N branch receive diversity using maximal-ratio-combining(MRC) where channel side information known at the receiver. Also the performance of M-ary quadrature amplitude modulation (QAM) over Rician fading channel has been analyzed here. Approximate formula has been used to represent symbol error probability for M-ary quadrature amplitude modulation over a Gaussian channel. The boundary condition for approximation is $M > 4$ and $0 \le SNR \le 30$ dB.

Keywords: *Maximal-Ratio-Combining (MRC), Phase Shift Keying (PSK) Symbol Error Probability (SEP), Signal-to-Noise Ratio (SNR), Quadrature Amplitude Modulation (QAM)*

1. INTRODUCTION

M-QAM is a well known modulation technique use in wireless communication. In wireless communication fading phenomenon is a boundary condition. So the practice for combating fading in wireless communication over such a time varying channel is to use diversity technique. Due to the high spectral efficiency M-QAM is an attractive modulation technique for wireless communication. MDPSK and MPSK are also two modulation technique use in wireless communication. For different values of Rician parameter symbol error probability (SEP) is different. So that performance varies with the change of Rician parameter (when k = 0 then it is called Rayleigh when k = α then it is called AWGN). It is also true for the change of diversity and message signal. Exact analysis of symbol error probability (SEP) has been presented for M-ary differentially encoded/ differentially decoded phase shift keying (MDPSK) and coherent M-ary phase shift keying (MPSK), transmitted over Rician fading channel using *N* branch receive diversity with maximal-ratio-combining (MRC). Our analysis has followed the same track for M-QAM and the simplicity of the SEP expression used has resulted in simple closed form expression of the SEP of *N* order diversity at Rician fading channel. For different conditions this three modulation technique shows different characteristics. The goal of our analysis is to highlight the performance by comparing them in different working conditions.

2. FADING

In wireless communications, fading is deviation or the attenuation that a telecommunication signal experiences over certain propagation media. The fading may vary with time, geographical position and/or radio frequency, and is often modeled as a random process [2].

2.1 Slow Fading

Slow fading arises when the coherence time of the channel is large relative to the delay constraint of the channel. So the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use.

2.2 Flat Fading

Flat fading attenuates or fades all frequencies in a communications in the same amount. In this fading, the coherence bandwidth of the channel is larger than the bandwidth of the signal.

2.3 Rayleigh fading

Rayleigh fading is a statistical model which assumes that the magnitude of a signal that has passed through a transmission medium will vary randomly, or fade, according to a Rayleigh distribution. It is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver.

2.4 Rician fading

Rician fading is a stochastic model for radio propagation anomaly caused when the signal arrives at the receiver by two different paths, and at least one of the paths is changing. Rician fading occurs when one of the paths, typically a line of sight signal, is much stronger than the others.

3. DIVERSITY

3.1 Transmit Diversity

Transmit diversity is radio communication using signals that originate from two or more independent sources that have been modulated with identical information-bearing signals and that may vary in their transmission characteristics at any given instant. It can help overcome the effects of fading, outages, and circuit failures.

3.2 Linear Diversity

Linear diversity combining involves relatively simple weighted linear sums of multiple received signals. Linear diversity combining is the only kind generally applicable for distortion less reception of analog transmission.

4. SYSTEM PERFORMANCE MEASURES

4.1 Average Signal-to-Noise Ratio

Probably the most common and best understood performance measure characteristic of a digital communication system is *average SNR*, where the word *average* refers to statistical averaging over the probability distribution of the fading. In simple mathematical terms, if γ denotes the instantaneous SNR at the receiver output which includes the effect of fading, then the average SNR [5]

$$\bar{\gamma} = \int_0^{\infty} \gamma \cdot p_\gamma(\gamma)\, d\gamma \; \ldots\ldots\ldots\ldots (1)$$

Where p_γ (γ) denotes the probability density function (PDF) of γ.

4.2 Average Symbol Error Probability

Another performance criterion and undoubtedly the most difficult and also most revealing about the nature of the system behavior is the average symbol error probability (SEP). The primary reason for the difficulty in evaluating average SEP lies in the fact that the conditional (on the fading) SEP is, in general, a nonlinear function of the instantaneous SNR, the nature of the nonlinearity being a function of the modulation/detection scheme employed by the system. The average SEP can be written as [6]

$$P_s(E) = \int_0^{\infty} P_s\left(\frac{E}{\gamma}\right) \cdot p_\gamma(\gamma)\, d\gamma \; \ldots\ldots\ldots (2)$$

Where, $P_s(E/\gamma)$ is the conditional SEP.

5. M-ARY MODULATION TECHNIQUE

In an M-ary signaling scheme, we may send one of M possible signals, $s_1(t)$, $s_2(t)$, ……., $s_M(t)$, during each signaling interval of duration T. For almost all applications, the number of possible signals $M=2^n$, where n is an integer. The symbol duration $T = nT_b$, where T_b is the bit duration. These signals are generated by changing the amplitude, phase, or frequency of a carrier in M discrete steps. Different kinds of M-ary modulation technique like MPSK, MDPSK and M-QAM, each of which offers virtues of its own [8].

5.1 M-ary Phase Shift Keying

An M-ary phase-shift-keyed (M-PSK) signal occurs when Θ(t) takes on equiprobable values $\beta_i = \frac{(2i-1)\pi}{M}$, i =1, 2 ...M, in each symbol interval Ts. As such, Θ(t) is modeled as a random pulse stream, that is,

$$\Theta(t) = \sum_{n=-\infty}^{\infty} \theta_n p(t - nt_s) \ldots\ldots(3)$$

Where Θ_n is the information phase in the n'th symbol interval nTs ≤ t ≤ (n+1) Ts ranging over the set of M possible values β_i as above and p_t is again a unit amplitude rectangular pulse of duration T_s seconds.

The conditional probability of symbol error for coherent MPSK is given as follows [4]

$$P_s(E/\gamma) = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} \exp\left[-\gamma \sin^2\left(\frac{\pi}{M}\right).\sec^2\theta\right] d\theta \ldots (4)$$

So the probability of symbol error for MPSK over Rician fading channels with Rician parameter K and diversity N as follows

$$P_s(E) = \frac{1}{\pi}.\left(\frac{N+K}{\bar{\gamma}}\right)^N . \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} \frac{\exp\left[-\frac{K.\sin^2\left(\frac{\pi}{M}\right).\sec^2\theta)}{\frac{N+K}{\bar{\gamma}} + \sin^2\left(\frac{\pi}{M}\right).\sec^2\theta}\right]}{\left(\frac{N+K}{\bar{\gamma}} + \sin^2\left(\frac{\pi}{M}\right).\sec^2\theta\right)^N} d\theta \ldots\ldots(5)$$

The probability of symbol error for coherent MPSK for Rayleigh fading with diversity N is obtainable from (5) by the substitution K=0 in (5), which is given by

$$P_s(E) = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} \frac{1}{\left(1+\frac{\bar{\gamma}}{N}.\sin^2\left(\frac{\pi}{M}\right).\sec^2\theta\right)^N} d\theta \ldots\ldots(6)$$

when K approaches infinity, (5) reduces to (4).

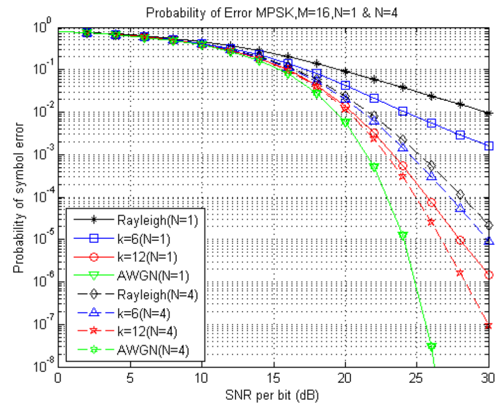

Figure 1: SEP for MPSK over Rician, Rayleigh and AWGN fading channel for different values of K and M=16; solid line for N=1, dashed line for N=4.

5.2 M-ary Differential Phase Shift Keying

MDPSK is the non-coherent version of the MPSK. It eliminates the need for a coherent reference signal at the receiver by combining two basic operations at the transmitter: (1) differential encoding of the input signal and (2) phase shift keying – hence, the name, M-ary differential phase-shift keying (MDPSK).

The conditional probability of symbol error is given by [4]

$$P_s(E/\gamma) = \frac{\sin\frac{\pi}{M}}{2\pi} . \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\exp\left[-\gamma\left(1-\cos\frac{\pi}{M}\cos\theta\right)\right]}{1-\cos\frac{\pi}{M}\cos\theta} d\theta \ldots (7)$$

Now using the following relation given in [3]

$$\int_0^{\infty} x^{\nu}.e^{-\alpha x}.I_{2\nu}(2\beta\sqrt{x})dx = \alpha^{-(2\nu+1)}.\beta^{2\nu}.\exp\left(\frac{\beta^2}{\alpha}\right) \ldots\ldots\ldots(8)$$

The probability of symbol error for MDPSK over Rician fading channels with Rician parameter K and diversity N is shown in (9) [4]

$$P_s(E) = \frac{\sin\frac{\pi}{M}}{2\pi}.\left(\frac{N+K}{\bar{\gamma}}\right)^N$$

$$\cdot\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\exp\left|-\frac{K.\left(1-\cos\left(\frac{\pi}{M}\right).\cos\theta\right)}{\frac{N+K}{\bar{\gamma}}+1-\cos\left(\frac{\pi}{M}\right).\cos\theta}\right|}{\left(1-\cos\left(\frac{\pi}{M}\right)\cos\theta\right).\left(\frac{N+K}{\bar{\gamma}}+1-\cos\left(\frac{\pi}{M}\right).\cos\theta\right)^{N}}d\theta$$

$$\ldots\ldots\ldots\ldots\ldots(9)$$

Where $\bar{\gamma}$ is the mean symbol SNR.

It is shown easily as *K* goes to zero that a substitution of *K* = 0 in (9) yields the probability of symbol error for MDPSK over a Rayleigh fading channel with diversity *N*, i.e.

$$P_S(E) = \frac{\sin\frac{\pi}{M}}{2\pi}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\left(1-\cos\left(\frac{\pi}{M}\right).\cos\theta\right).\left(1+\frac{\bar{\gamma}}{N}\left(1-\cos\left(\frac{\pi}{M}\right).\cos\theta\right)\right)^{N}}d\theta$$

$$\ldots..(10)$$

On the other hand, it is interesting to note, when *K* approaches infinity, that (9) reduces to (7). This means that the system is equivalent to a non diversity reception in AWGN. When *N* = 1 is substituted into (9); a single link analysis is obtained. [4]

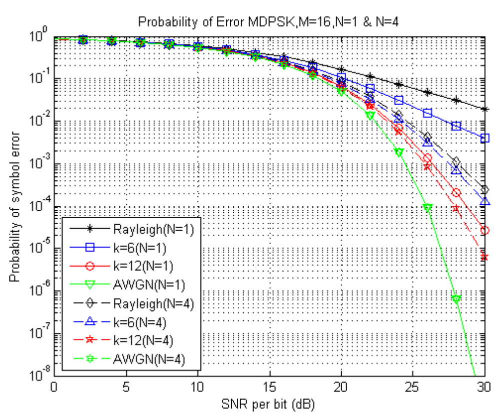

Fig 2: SEP for MDPSK over Rician, Rayleigh and AWGN fading channel for different values of K and M=16; solid line for N=1, dashed line for N=4.

5.3 M-ary Quadrature Amplitude Modulation

In M-QAM modulation scheme, the in-phase and quadrature components are both in-dependently PAM Modulated. The signal constellation for M-QAM consists of a square lattice of message points.

The error probability as a function of *K*, $\bar{\gamma}$ and *N* of the system can be calculated by averaging the conditional probability of error over the *pdf* of γ, i.e.

$$P_S(E) = \int_0^{\infty} P_S(E/\gamma).p(\gamma)d\gamma \ldots(12)$$

Where, *PS* (*E* /γ) is the conditional probability of symbol error.

The probability of symbol error for QAM over a Gaussian channel is given as [1]

$$P_S(E/\gamma) = P_{AWGN}(\gamma) = 0.2.\exp\left[-\frac{1.5.\gamma}{(M-1)}\right]$$

$$.\log_2 M \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

After the substitution of (13) into (12) and putting

$$p_{\gamma}(\gamma) = \left(\frac{N+K}{\bar{\gamma}}\right).\left[\frac{(N+K)\gamma}{K\bar{\gamma}}\right]^{\frac{N-1}{2}}$$

$$.\exp\left[-\frac{(N+K)\gamma+K\bar{\gamma}}{\bar{\gamma}}\right]$$

$$.I_{N-1}\left(2.\sqrt{\frac{K(N+K)\gamma}{\bar{\gamma}}}\right)\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

we get

$$P(\gamma) = \int_0^{\infty}\left(\frac{N+K}{\bar{\gamma}}\right).\left[\frac{(N+K).\gamma}{K.\bar{\gamma}}\right]^{\frac{N+1}{2}}$$

$$.\exp\left(-\frac{(N+K).\gamma+K.\bar{\gamma}}{\bar{\gamma}}\right)$$

$$.I_{N-1}\left(2.\sqrt{\frac{K.(N+K).\gamma}{\bar{\gamma}}}\right)$$

$$.0.2.\exp\left[-\frac{1.5.\gamma}{(M-1)}.\log_2 M\right].d\gamma..(15)$$

Using the following relation given in [3]

$$\int_0^{\infty} x^{v}.e^{-\alpha x}.I_{2v}(2\beta\sqrt{x})dx$$

$$= \alpha^{-(2v+1)}.\beta^{2v}.\exp\left(\frac{\beta^2}{\alpha}\right)\ldots\ldots(16)$$

The probability of symbol error for M-QAM over i.i.d. Rician fading channels with Rician parameter *K*, diversity N, and mean symbol SNR, $\bar{\gamma}$, is given as follows

$$P(\gamma) \approx$$

$$0.2.\log_2 M.\left[\frac{(N+K).(M-1)}{(N+K).(M-1)+1.5.\bar{\gamma}}\right]^{N}$$

$$. \exp\left[-\frac{K.1.5\,\bar{\gamma}}{(N+K).(M-1)+1.5.\bar{\gamma}}\right] \ldots (17)$$

On comparing the SEP expressions for MDPSK and MPSK, available in the literature [4] with our SEP expression (17), obtained using mathematical analysis for M-QAM over slow, flat, i.i.d Rician fading channels when MRC is applied at the receiver, it can be easily seen that SEP expressions for MDPSK and MPSK are in the integral form whereas our SEP expression for M-QAM is in simple closed form and contains only exponential functions. On substituting, N = 1 in equation (17), we found

$$P(\gamma) \approx 0.2.\log_2 M.\left[\frac{(1+K).(M-1)}{(1+K).(M-1)+1.5.\bar{\gamma}}\right] . \exp\left[-\frac{K\,1.5\,\bar{\gamma}}{(1+K).(M-1)+1.5.\bar{\gamma}}\right] \ldots (18)$$

Moreover, it can also be shown that as K goes to zero that a substitution of K=0 in (17) yields the probability of symbol error for M-QAM over a Rayleigh fading channel with diversity N

$$P(\gamma) = 0.2.\log_2 M.\left[\frac{N.(M-1)}{N.(M-1)+1.5.\bar{\gamma}}\right]^N \ldots (19)$$

It can be easily shown that as K goes to infinity, that a substitution of K=∞ in (17) yields the probability of symbol error for M-QAM over a Gaussian channel as

$$P_{AWGN}(\gamma) = 0.2.\exp\left[-\frac{1.5.\gamma}{(M-1)}\right].\log_2 M \ldots (20)$$

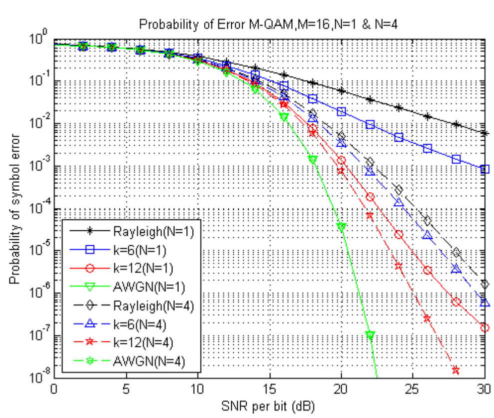

Fig 3: SEP for M-QAM over Rician, Rayleigh and AWGN fading channel for different values of K and M=16; solid line for N=1, dashed line for N=4.

6. PERFORMANCE COMPARISON OF MDPSK, MPSK AND M-QAM

The Comparison curves of three modulation techniques using equation (5), (9) and (17) (MPSK, MDPSK and M-QAM) are given below:

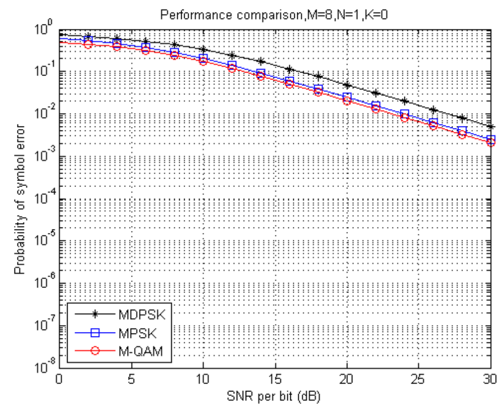

Fig 4: SEP for MDPSK, MPSK, M-QAM for M=8, N=1, K=0.

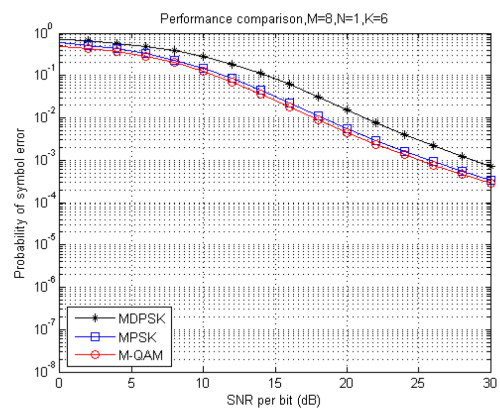

Fig 5: SEP for MDPSK, MPSK, M-QAM for M=8, N=1, K=6.

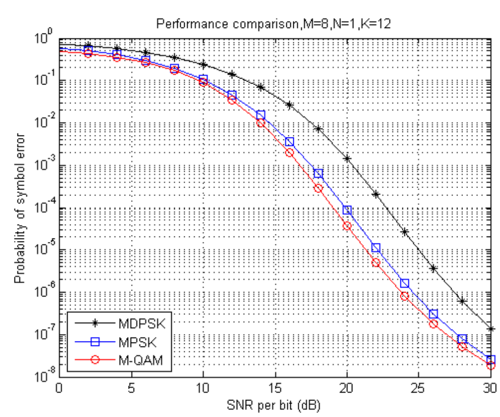

Fig 6: SEP for MDPSK, MPSK, M-QAM for M=8, N=1, K=12.

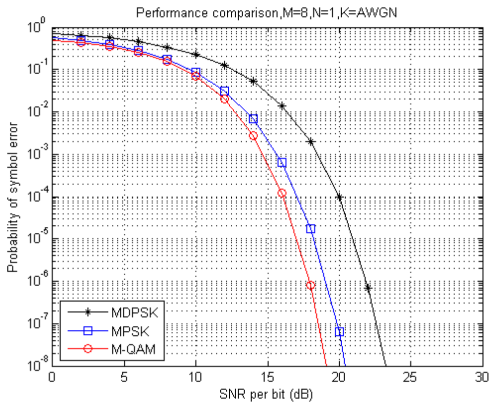

Fig 7: SEP for MDPSK, MPSK, M-QAM for M=8, N=1, K= α (AWGN)

From Fig 4 - 7 for M=8 & N=1, considering 15 dB SNR for M-QAM, at K=0 the SEP is approximately .06294, at K=6 the SEP is approximately .02551, at K=12 the SEP is approximately .004325 and at K=∞ the SEP is approximately .0005691.

Again for considering 15 dB SNR for MPSK, at K=0 the SEP is approximately .07522, at K=6 the SEP is approximately .03185, at K=12 the SEP is approximately .007458 and at K=∞ the SEP is approximately .002029.

Again for considering 15 dB SNR for MDPSK, at K=0 the SEP is approximately .1409, at K=6 the SEP is approximately .08284, at K=12 the SEP is approximately .04237 and at K=∞ the SEP is approximately .02697.

So for M=8, N=1 and all values of K, M-QAM is better than MPSK followed by MDPSK.

7. CONCLUSION

In this paper, the comparison between MDPSK, MPSK and M-QAM for different fading channel are shown here. By analyzing the graphical representation of SEP of these three modulation technique we found that the symbol error probability of M-QAM was lower than other two modulation techniques. For M-QAM modulation with *N* branch diversity, assuming channel information is known, with maximal-ratio-combining (MRC) over Rician fading channel we obtained a simple closed form expression for SEP Eq. (17). Our expression is correct for all vales of *K*, *N*, and *M* > 4 with 0 < SNR < 30 dB. We found that M-QAM modulation technique is most suitable technique for combating fading in wireless communication for its lower SEP. SEP should keep low to transmit data perfectly from transmitter to receiver. For efficient transmission of information in wireless communication the M-QAM modulation technique shows the better performance for its lower SEP.

Our work could be extended for different types of combining with the availability or unavailability of CSIT. Also in the thesis we have assumed that the fading is i.i.d so the work presented here can be extended to the case of correlated fading. Another possible extension to our work would be to obtain exact bit-error rates of M-QAM over slow, flat, Rician fading channels when linear diversity combining is applied to combat degradation due to fading.

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