CAPACITY CONSIDERATIONS IN MIMO SYSTEMS AND IMPACT OF TRANSMIT-SIDE SPACIAL FADING CORRELATION ON THE PERFORMANCE OF NON-ORTHOGONAL SPACE_TIME CODES

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ABSTRACT

High data rates are a primary concern for wireless system designers today. It is likely that future breakthroughs in wireless communications will be driven mainly by the demand for high data rate applications. Streaming video for example, requires two to three orders of magnitude higher data rates than speech. A simple solution for increasing data rate is to increase channel bandwidth. This however is not usually considered, as it is very costly. Use of multiple element arrays (MEA) at both transmitter & receiver on the other hand can result in the realization of these higher data rates. So far, only partial consideration has been given to the analysis of space-time coding in the practical case of correlated fading. In this work, we will expand newly published results on the performance of space-time codes in the presence of spatial fading correlation, by quantifying the impact of the fading on different classes of space-time codes. Explicitly, we show that in the case of two and above receive antenna, transmit-side correlation has a more brutal impact on the performance of non-orthogonal space-time codes. Orthogonal spacetime codes offer maximum potency against transmit correlation.

Keywords: *Multiple input multiple output MIMO, Capacity, Autonomous Hybrid Power System (AHPS)*

I. INTRODUCTION

In the past years, diverse diversity techniques, amongst them time diversity, frequency diversity, and space diversity, have been used to improve the reliability of communications channels. Space diversity has been progressively more popular since it is a means of improving the reliability without scarifying the spectral efficiency. Initial work concentrated on receive diversity, i.e. the use of multiple antennas at the receiver, an attention on transmit diversity emerged lately. In this framework, the novel notion of space-time coding [1] - [5] sparked widespread interest. In most work on space-time codes, uncorrelated spatial fading was assumed. However, an insufficiently rich scattering environment or too closely spaced antennas can cause the individual antennas to be correlated.

In his paper [6], H. Bolcskei used a physically motivated Rayleigh fading MIMO channel model proposed in [7] that incorporates receive and transmit correlation to derive results on the impact of the correlation on error performance. It /has been shown that that the maximum achievable diversity order is given by the product of the ranks of receive and transmit correlation matrices.

In this paper we will discuss why using multiple antennas at both the transmitter and receiver can be an effective technique for increasing capacity of a wireless link. We will consider a single user Gaussian channel with multiple transmitting and receiving antennas. We will consider the case where there are an equal number of transmitting & receiving antennas and will denote that number by n. The analysis can easily be generalized to any combination of transmit & receive antennas.

We will consider a linear model for the channel in which the received vector **y** & C^n depends on the transmitted vector **x** & C^n via

$$\mathbf{y} = \boldsymbol{H}\mathbf{x} + \mathbf{n}$$

Where $\boldsymbol{H}$ is a n x n complex matrix & **n** is zero mean complex Gaussian noise, with independent, equal variance real & imaginary parts. We will consider $\boldsymbol{H}$ to be a random matrix chosen according to a probability distribution and each use of the channel corresponding to an independent realization of ***H***.

We will consider capacity to be a random variable and the goal will be to find the complementary cumulative distribution function curves. These curves will give us an idea of the increase in capacity that can be achieved using multiple transmit & receive antennas. For the baseline case of a single transmit & receive antenna, it is well known that Shannon's classical capacity formula predicts an increase in capacity of 1 bit/ transmission for a 3 dB increase in SNR, in the high signal-to-noise ratio region. We will find that for independent Rayleigh fading paths between n transmit & receive antenna pairs, this increase is n bits/transmission for a 3 dB increase in SNR, for large n.

II. CHANNEL MODEL:

We consider a channel with input **x** and output **y** = ***H*****x** + **n,** where **n** is the additive white Gaussian noise. The channel matrix **H** is random and independent of both **x** and **n**. It is assumed that entries of **H** are zero mean, uncorrelated Gaussian with independent real and imaginary parts each with variance ½. Equivalently, each entry of **H** has uniformly distributed phase and Rayleigh distributed magnitude, with expected magnitude square equal to unity. This is intended to model a Rayleigh fading channel.

We will consider a "quasi-static" analysis that is the channel remains constant during a burst of data transmission and changes randomly from burst to burst. The channel characteristics are not known to the transmitter but the receiver knows the channel perfectly. Since the transmitter does not know the channel, we will assume constant transmit power.

We will only analyze the narrowband case where the bandwidth is taken to be narrow enough that the channel can be treated as flat over frequency. We will assume that the environment has a large number of scatterers so that the Rayleigh fading model is appropriate. The assumption of independent Rayleigh paths is also justified since for antenna elements separated by λ/2, the path losses tend to roughly decorrelate [8].

<u>Capacity Expressions for MIMO systems</u>:

Assuming that the transmitted vector is composed of n statistically independent equal power components each with a Gaussian distribution, the capacity expression can be derived from a general basic formula given as

$$C = \log_2 \frac{\det A_y . \det A_x}{\det A_u} \qquad (1)$$

where $A_x = E[xx^*] = \hat{P}/n . I_n$ [1], A_y = E[yy*] = N $\hat{P}/n . GG^*$ [2] and A_u = E[uu*] where u is the 2n dimensional vector (x, y)'. So A_u has A_x in the northwest corner and A_y in the southeast corner. The remaining two corners are transpose conjugates of each other. The northeast of these is $\hat{P}$/n . G^*. The statistical independence among all components of the 2n dimensional vector (x, y) is what facilitates the explicit computation of A_x, A_y & A_u. This is because all entries are variances and covariances of Gaussians. Using the relation

$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det A . \det(D - CA^{-1}B) \qquad (2)$$

we can write A_u as a multiple of det A_x so det A_x can be cancelled in (1). After the cancellation the numerator in the argument of the logarithm becomes[N . I_n + $\hat{P}$/n . GG^*]. The denominator becomes det[$N^{-1}.I_n$]. Since the product of determinants is the determinant of the product and $\hat{P}^{1/2}$. G = $P^{1/2}$. H and ρ = P/N, the formula for generalized capacity is

$$C = \log_2 \det[I_n + (\rho / n).HH^*] \text{ bps/Hz} \qquad (3)$$

A. A brief discussion of [1]:

This part relies to a great extent on material in [8] so it is fair to mention briefly what I understand from these two papers. Whereas both [8] and [9] discuss capacity limits in a fading wireless channel, [9] approaches the problem from an information theoretic point of view (it also discusses capacity of multi antenna Gaussian channels with fading channels as a special case), while [8] looks at it from a classical perspective.

The paper by Foschini and Gans [8] is motivated by the need to find limits on data rates that can be achieved in a multi element arrangement with antennas at both transmitter and receiver. The channel model assumed is the same as mentioned above. The paper lists capacity expressions for different scenarios like the cases with no diversity, only receive diversity, only transmit diversity, combined transmit and receive diversity (MIMO) and spatial cycling using one transmitter at a time. The last of these cases is where n transmitters and n receivers are used, but only one transmitter is used at a time i.e. cycling through all n transmitters one at a time. It then goes on to derive capacity

[1] We assume that G is the actual channel response and H is the normalized channel response so that $\hat{P}^{1/2}$. G = $P^{1/2}$. H defines the relationship between G and H.

[2] * denotes complex conjugate transpose.

expressions for combined transmit receive diversity cases (the last two cases). Specifically it derives a lower bound on capacity for MIMO channels and analyzes the expression for large number of antennas ($n \rightarrow \infty$). The result indicates that for large n, capacity scales at least linearly with increasing n, and for ρ large is given by $\log_2(\rho/e)$. Using the expressions for capacity, complementary cumulative distribution (ccdf) curves are generated for a number of scenarios. The results are discussed in detail, underlining the capacity benefits that can be achieved using multiple antennas at transmitter and receiver. Capacity benefits from using MIMO are also compared to receive and transmit diversity scenarios and it is shown that MIMO is in fact superior to both transmit and receive diversity in terms of the data rates that it promises. The paper then touches briefly upon 1-D and 2-D codes for systems transmitting in a wireless environment and analyzes the performance of 1-D codes. It then gives a summary of main points and concludes with a list of suggestions for future work specially in antenna theory to realize the use of multi element arrays on transmitters and receivers.

III. SIMULATIONS:

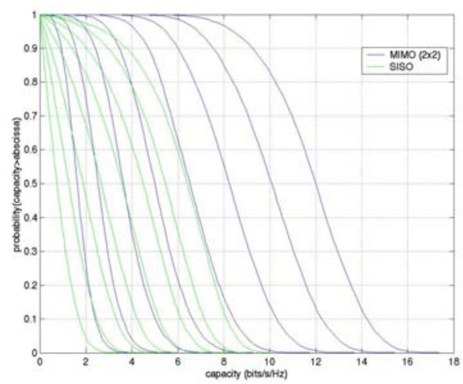
What follows are results of simulations for obtaining complementary cumulative distribution function curves for capacity of a number of MIMO systems, as well as comparison of MIMO systems with transmit and receive diversity.

Explanation of Simulation Results:

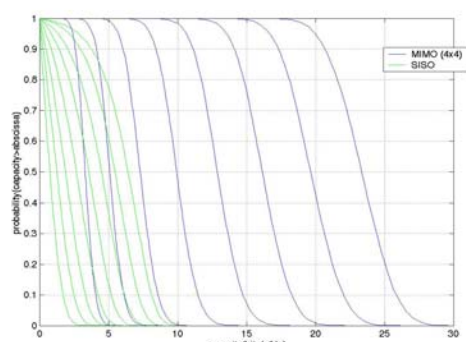
Figures 1. (a), (b) & (c) show the complementary cumulative distribution function curves for three different MIMO systems (having n transmit and receive antennas with n = 2, 4 & 8 respectively). Capacity is measured in bits/s/Hz or equivalently in bits/transmission. The curves have been plotted for SNRs ranging from 0 to 21 dB in steps of 3 dB. The total transmit power has been kept constant. Each figure also includes the baseline case of only one transmit and receive antenna for comparison. The curves clearly underline the capacity advantage of MIMO systems over SISO systems. For example for an average received SNR of 21 dB, the plots predict that for 99% of the channels the capacity is 7, 19 & 42 bits/transmission for n = 2, 4 & 8 respectively whereas it is only about 1 bit/transmission for the single transmit & receive antenna case. The increase in capacity is also substantial for P_{out}'s[3] of 5%, though less than that for 1%. The plots also indicate the capacity advantage of going to a higher n, the number of transmit and receive antennas.

The capacities listed here may at first seem unreasonably high especially for the case of 8 transmit and receive antennas. This is because of the fact that we are not doing a fair comparison. Indeed a better way would be to compare capacity/symbol/dimension for the three different cases to the baseline case of a single transmit & receive antenna. Doing so we find that 7 bits/transmission for n = 2 correspond to 3.5 bits/symbol/dimension, 19 bits/transmission for n = 4 correspond to 4.75 bits/symbol/dimension & 42 bits/transmission for n = 8 correspond to 5.25 bits/symbol/dimension, all considerably higher than the 1 bits/symbol/dimension for the n = 1 case.

Figure 2. shows capacity in bits/symbol/dimension for n = 4 & 8. Again we see the capacity benefits in going to a higher number of transmit and receive antennas at small P_{out}'s.

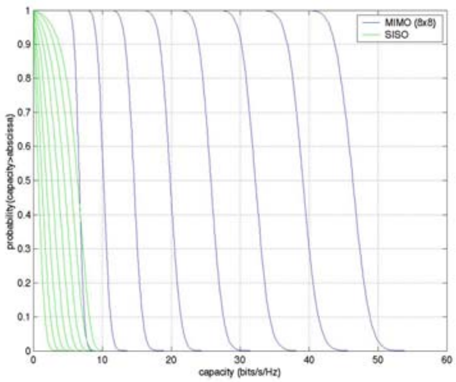


(a)



(b)

[3] P_{out} is the outage probability defined as the probability that capacity is less than the value on abscissa.



(c)

Figure 1, Average received SNR ranging from 0 to 21 dB in steps of 3 dB. (a) Two antennas at both transmitter and receiver(blue curves). Single antenna at both transmitter and receiver (red curves). (b) same as (a) but with four transmit and receive a

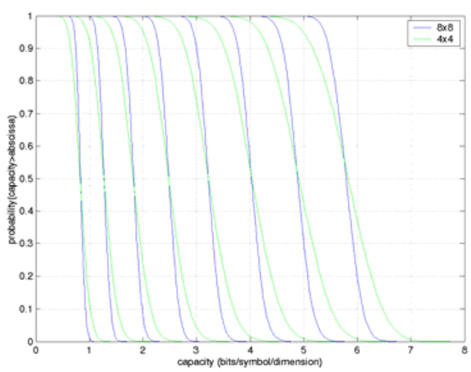
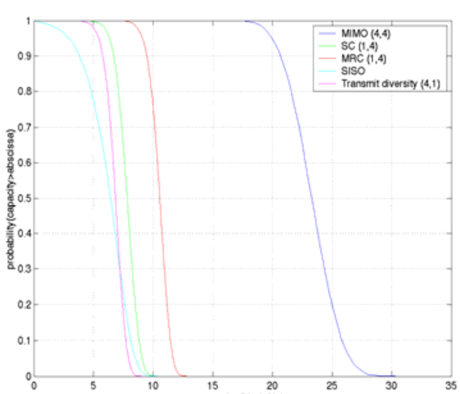
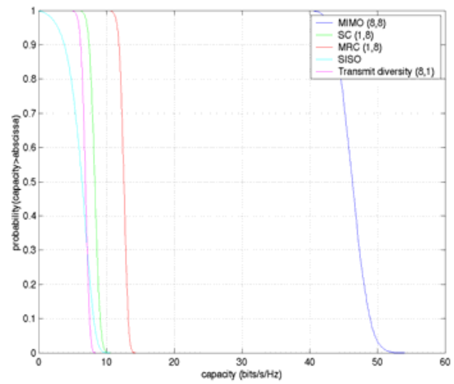


Figure 2 Figure 2. Capacity in bits/symbol/ dimension for (8x8) & (4x4) MIMO systems.

IV. MIMO VS. DIVERSITY:



(a)



(b)

Figure 3 .(a), (b) A comparison of MIMO vs. diversity

Figure 3(a). shows a comparison between using 4 antennas both at the transmitter and receiver, using a single transmitter and 4 antennas at the receiver for diversity and using four transmitters and a single antenna at the receiver, again for diversity. The curves are plotted for an average received SNR of 21 dB. Both selection combining and maximal ratio combining are considered for receive diversity. The curve farthest to the left is the capacity curve for a single transmit & receive antenna. The three curves in the middle are the diversity curves. It may be assumed that the curves for diversity represent the data rates that can be achieved for an average received SNR of 21 dB. The rightmost curve is the capacity curve for a (4,4) MIMO system.

Looking at the curves, it becomes obvious that MIMO is superior to diversity in terms of data rates that can be achieved at least theoretically (though with the use of strong coding schemes we can get data rates close to capacity). Also the benefits of MIMO become more pronounced as we increase n (fig 3 (b)).

A. Simulation Setup for correlation performance

In order to assess the performance of the space time codes, we implemented a simulation testbed in Matlab. For simplicity, we investigated codes for two transmit antennas. We build both space-time trellis codes and Alamouti-type block codes as the coding mechanism. The trellis code has either 4, 8, or 16 states.

The Rayleigh fading channel model is implemented according to [6]. We simulated the performance of a given code under the assumption that the channel would stay constant over a number of symbols (this period is called a “frame”). Then, it would randomly change to a different realization. A number of these Monte-Carlo runs are performed, and then the symbol error rate and the frame error rate are computed as the average over all channel realizations. In this paper, we define frame error rate as the rate of completely error-free frames. The antenna correlation coefficient ρ_T is used as the parameter in the latter case.

B. Diversity order of Codes

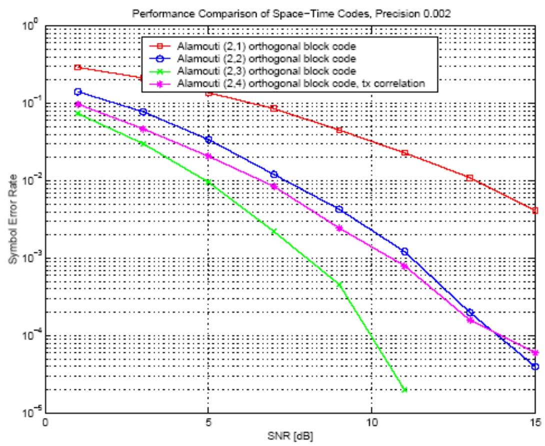


Figure 4 Varying diversity order of space-time codes

For the i.i.d. channel case, we could demonstrate the well-known result that the diversity order, i.e. the slope of the SER (Symbol Error Rate) curve, is equal to the product of number of transmit and receive antennas.

It can be seen in Fig. 4 that the (2Tx, 1Rx) (i.e. two transmit antennas, one receive antenna) case has the lowest SER slope, i.e. the lowest diversity order of 2. The (2Tx, 2x) has diversity order of 4, its error fallow is steeper.

The (2Tx, 3Rx) case has even steeper fallow with diversity order 6. Also included in this graph is what happens when some of the antennas are correlated: In the (2Tx, 4Rx) case, the transmit correlation is set to 1.

That way, we lose transmit diversity altogether, and the overall diversity order becomes 4. Indeed, the slope is the same as in the (2Tx, 2Rx) case.

These results equally apply to the space-time trellis codes or to any other class of space-time codes, they would only have different coding gain, i.e. the curves are shifted up or down.

C. Performance under Correlation

The goal of our project was to show the disproportionate performance degradation of non-orthogonal space-time codes under transmit correlation. For that reason, we did not plot the error rate against the SNR, but against the transmit antenna correlation coefficient. It was observed that in general for values less up to ρ_T = 0.8 the performance of both block and trellis codes degraded slightly, but without any advantage of block codes. The predicted faster degradation of non-orthogonal code performance happened only above ρ_T = 0.8.

A not immediately obvious phenomenon can be observed with respect to the frame error rate: In the case of large frame sizes, the symbol error rate performance of block codes is better, while at the same time the frame error rate performance of the trellis codes is superior. This effect can be explained by the fact that the trellis codes perform a ML sequence detection. Once a symbol is detected in error, it is very likely that a whole sequence of neighboring symbols is also erroneous. Therefore, the errors will appear rather "bursty", while with block codes the errors are more evenly spread out. These different error event characteristics explain that with trellis codes, the probability of receiving an error-free block can be higher than for block codes, while the overall symbol error performance is nevertheless worse. Hence, there is no simple relation between symbol error and frame error rate plots, so we provided both in our figures.

For a fixed SNR value, we compared trellis and orthogonal block code performance in the region of ρ_T = 0.6,…, 1.0. The number of transmit antennas was held constant at two, while the number of receive antennas was varied from 1 to three, yielding more and more diversity advantage in the i.i.d. case.

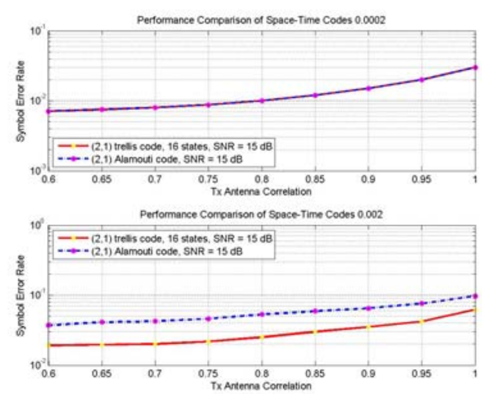


Figure 5 Performance of (2Tx, 1Rx) space time codes

For the case of one receive antenna at SNR = 15 dB, we found no significant performance differences. As can be seen in Fig. 2, the two curves run parallelly.

This observation can be explained by the fact that under transmit correlation, nonorthogonal code designs degrade the diversity order slightly beyond the drop to d = MR. In the case of MR = 1, we have reached the lowest possible diversity order. The performance cannot degrade any further, therefore it is impossible to see a difference in the performance of orthogonal and non-orthogonal codes.

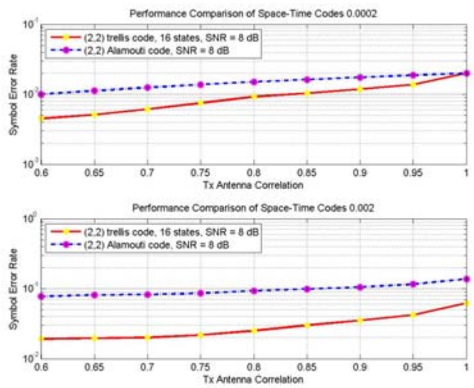


Figure 6 Performance of (2Tx, 2Rx) space time codes

However, in cases like this, where diversity order breaks down completely, matters become complicated by several issues, one of them being that union bound concepts do not apply any more in fading channels. To our understanding, there are still various phenomena in the one receive antenna case that cannot be entirely explained.

When moving to two antennas, the predicted effect becomes clearly visible (Fig. 6). In terms of frame error rate, the advantage of the trellis code shrinks from about 4.5 at ρ_t = 0.6 to 2 at ρ_t = 0.999.

With three receive antennas at SNR = 5.5 dB, the effect becomes even more pronounced (Fig. 7).

We also investigated the performance of the different versions of spacetime trellis codes. In particular, we compared the 8-state and 16-state codes.

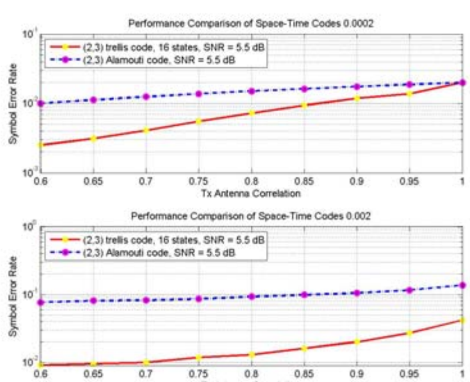


Figure 7 Performance of (2Tx, 3Rx) space time codes

It can be seen from Figs. 7 and 8 that the 16-state code is more powerful to begin with, but its performance degrades to about the same level as that of the 8-state codes in high correlation. Therefore, the observed effect becomes more pronounced with the better 16-state code.

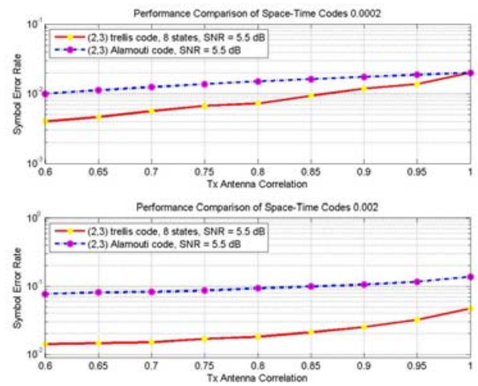


Figure 8 Performance of (2Tx, 3Rx) space time codes

V. CONCLUSIONS:

While it is well known that receive diversity, especially MRC, offers significant capacity improvement over single antenna reception, in contrast, the simultaneous use of transmit and receive diversity greatly increases the capacity over what is possible with only transmit or receive diversity. Use of multiple antennas greatly increases the achievable rates on fading channels if the channel parameters can be estimated at the receiver and if the path gains between different antenna pairs behave independently. The second of these requirements can be met by using strong coding strategies. The first requirement, however, is difficult to fulfill and can be justified in certain communication scenarios and not in others. A more realistic assumption is that of a slowly varying channel instead of channel state information at the receiver. We have shown that non-orthogonal space-time codes suffer from more performance degradation under transmit-side correlated fading than their orthogonal counterparts. These results were previously predicted by theory. The more receive antennas are used, the more pronounced the effect becomes. It is to be believed that the relative performance hit that non-orthogonal codes take under transmit correlation will also increase with the number of transmit antennas. This could be subject of future research.

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