



# ROBUST MODEL REFERENCE ADAPTIVE PI CONTROL

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## ABSTRACT

The advantage of the Model Reference Adaptive Scheme is to establish robustness with respect to bounded disturbances and unmodeled dynamics. In the model reference adaptive system (MRAS), the desired index of performance is given by the reference model. The tracking error represents the deviation of the plant output from the desired trajectory. The adaptive control without having robustness property may go unstable in the presence of small disturbances or unmodeled dynamics. A new idea is proposed to reduce the time for adaptation in MRAC systems. The idea behind to design Robust model Reference Adaptive PI control system is by adding the control signal from the PI controller to the control signal from modified MRAC. It has been checked with the lateral dynamic model of Boeing 747 airplane. The proposed controller is easy to implement and has superior transient behavior as compared to the standard adaptive controller and modified adaptive controller.

*Index Terms* – Model Reference Adaptive Control, Modified Model Reference Adaptive Control, Control Systems, MATLAB.

## I. INTRODUCTION

In the sense of control theory and engineering, an adaptive controller is an "intelligent" controller that can modify its behavior in response to the variations in the dynamics of the process and the character of the disturbances. An adaptive system is any physical system that has been designed with an adaptive viewpoint.

The Model Reference Adaptive Control System is an adaptive servo system in which the desired performance is expressed in terms of the reference model, which gives the desired response to the reference signal. Robust Control is not considered to be an adaptive system even though it can handle classes of parametric and dynamic uncertainties. The adaptive law introduces a multiplicative non-linearity that makes the closed loop plant nonlinear and often time varying. Because of this, the analysis and understanding of the stability and robustness of adaptive control schemes are challenging. Some of the basic methods used to design adaptive laws are Sensitivity methods, Lyapunov design, and Gradient method and Least square methods based on estimation error and cost criteria [1]-[5]. This method of developing adaptive laws is based on the direct method of Lyapunov and its relationship with positive real functions.

The main characteristics of the simple MRAC schemes are the adaptive laws driven by the estimation error which due to the special form of the control law is equal to the regulation or tracking error. They are derived using the Lyapunov design approach without the use of the normalization and a simple Lyapunov function is used to design the adaptive law and establish boundedness for all signals in the closed loop plant [5]. Under certain assumptions on the plant and reference model, MRAC schemes are designed that guarantee signal boundedness and asymptotic convergence of the tracking error to Zero [2]. These results however provide little information about the rate of convergence and the behavior of the tracking error during the initial stages of adaptation [10-11]. The disadvantage of this MRAC scheme is that it takes some time to adapt and some oscillations will come after a certain period. Hence modified MRAC is designed. In modified MRAC adaptation time is decreased but this scheme also some oscillation will come after a certain period.

The idea behind to design proposed Robust model Reference Adaptive control system is by adding the control signal from the PI controller, to the control signal from modified MRAC. It has been checked with the lateral dynamic model of Boeing 747 airplane and the adaptation time is decreased and has no oscillations will come. The proposed controller is easy to implement and has



superior transient behavior as compared to the conventional adaptive controller.

This sophisticated controller can work well over a wide range of operating conditions. This can be used for continuous adaptation in industrial applications.

**II. STATEMENT OF THE PROBLEM**

Consider the SISO, LTI plant described by the vector differential equation

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p, x_p(0) = x_0 \\ y_p &= C_p^T x_p \end{aligned} \tag{1}$$

where  $x_p \in \mathcal{R}^n; y_p, u_p \in \mathcal{R}^1$  and  $A_p, B_p, C_p$  have the appropriate dimensions. The transfer function of the plant is given by

$$y_p = G_p(s) u_p \tag{2}$$

with  $G_p(s)$  expressed in the form

$$G_p(s) = \frac{k_p Z_p(s)}{R_p(s)} \tag{3}$$

where  $Z_p, R_p$  are monic polynomials and  $K_p$  is a constant. The reference model, selected by the designer to describe the desired characteristics of the plant, is described by the differential equation

$$\begin{aligned} \dot{x}_m &= A_m x_m + B_m r, x_m(0) = x_{m0} \\ y_m &= C_m^T x_m \end{aligned} \tag{4}$$

where  $x_m \in \mathcal{R}^{P_m}$  for some integer  $P_m; y_m, r \in \mathcal{R}^1$  and  $r$  is the reference input which is assumed to be a uniformly bounded and piecewise continuous function of time. The transfer function of the reference model given by  $y_m = W_m(s) r$  is expressed in the same form as (3).

$$W_m(s) = \frac{k_m Z_m(s)}{R_m(s)} \tag{5}$$

where  $Z_m(s), R_m(s)$  are monic polynomials and  $k_m$  is a constant.

**Assumptions**

1.  $Z_p(s)$  is a monic Hurwitz polynomial of degree  $m_p$
2. An upper bound  $n$  of degree  $n_p$  of  $R_p(s)$
3. The relative degree  $n^* = n_p - m_p$  of  $G_p(s)$ , and
4. The sign of the high frequency gain  $k_p$  are known
5.  $Z_m(s), R_m(s)$  are monic Hurwitz polynomials of degree  $q_m, P_m$  respectively, where  $pm \leq n$
6. The relative degree  $n_m^* = P_m - q_m$  of  $W_m(s)$  is the same as that of  $G_p(s)$ , i.e.,  $n_m^* = n^*$ .

**III. STRUCTURE OF AN MRAC DESIGN**

As in Ref. [2, 3&10] the state space realization of the control law is used

The state space realization of the control law:

$$\begin{aligned} \dot{\omega}_1 &= F\omega_1 + gu_p, \omega_1 = 0 \\ \dot{\omega}_2 &= F\omega_2 + gy_p, \omega_2 = 0 \\ u_p &= \theta^{*T} \omega \quad \text{where} \quad \omega_1, \omega_2 \in \mathcal{R}^{n-1} \end{aligned} \tag{6}$$

$$\theta^* = [\theta_1^{*T}, \theta^{*T}, \theta_3^*, C_0^{*T}]^T, \omega = [\omega_1^T, \omega_2^T, y_p, r]^T$$

$\lambda_i$  are the coefficients of

$$\Lambda(s) = s^{n-1} + \lambda_{n-2}s^{n-2} + \dots + \lambda_1(s) + \lambda_0 = \det(sI - F) \tag{7}$$

where  $F$  is an  $(n-1) \times (n-1)$  stable matrix such that  $\det(sI - F)$  is a Hurwitz polynomial whose roots include the zeros of the reference model, and that  $(F, g)$  is a controllable pair. We define the ‘regressor’ vector as  $\omega = [\omega_1, \omega_2, y_p, r]^T$

In the standard adaptive control scheme, the Control  $u$  is structured as  $u_p = \theta^T \omega$

where  $\theta = [\theta_1, \theta_2, \theta_3, c_0]^T$  is a vector of adjustable parameters, and is considered as an estimate of a vector of unknown system parameters,  $\theta^*$

Obtain the state space representation of the overall closed-loop plant by augmenting the state  $x_p$  of the plant (1) with the states  $\omega_1, \omega_2$  of the controller (6) i.e.,

$$\begin{aligned} \dot{Y}_c &= A_c Y_c + B_c c_0^* r, Y_c(0) = Y_0 \\ y_p &= C_c^T Y_c \end{aligned} \tag{8}$$



where

$$A_c = \begin{bmatrix} Ap + Bp\theta_3^* C_p^T & Bp\theta_1^{*T} & Bp\theta_2^{*T} \\ g\theta_3^* C_p^T & F + g\theta_1^{*T} & g\theta_2^{*T} \\ gC_p^T & 0 & F \end{bmatrix},$$

$$B_c = \begin{bmatrix} B_p \\ g \\ 0 \end{bmatrix}$$

$$Y_c = [x_p^T, \omega_1^T, \omega_2^T]^T \in \mathfrak{R}^{np+2n-2}$$

$$C_c^T = [C_p^T, 0, 0]$$

(9)

and  $Y_0$  is the vector with initial conditions. The transfer function from  $r$  to  $y_p$  is given by

$$\frac{y_p(s)}{r(s)} = \frac{c_o^* k_p Z_p \Lambda^2}{\Lambda[(\Lambda - \theta_1^{*T} \alpha)R_p - k_p Z_p (\theta_2^{*T} \alpha + \theta_3^* \Lambda)]} = W_m(s)$$

which implies that

$$C_c^T (sI - A_c)^{-1} B_c C_o^* = \frac{c_o^* k_p Z_p \Lambda^2}{\Lambda[(\Lambda - \theta_1^{*T} \alpha)R_p - k_p Z_p (\theta_2^{*T} \alpha + \theta_3^* \Lambda)]} = W_m(s)$$

and therefore,

$$\det(sI - A_c) = \Lambda \left[ (\Lambda - \theta_1^{*T} \alpha)R_p - \left( \frac{k_p Z_p}{R_p} \right) (\theta_2^{*T} \alpha + \theta_3^* \Lambda) \right] = \Lambda Z_p \Lambda_0 R_m$$

It is clear that the eigen values of  $A_c$  are equal to the roots of the polynomials  $\Lambda$ ,  $Z_p$ ,  $R_m$ ; therefore  $A_c$  is a stable matrix. The stability of  $A_c$  and the boundedness of  $r$  imply that the state vector  $Y_c$  in (9) is bounded.

Since  $C_c^T (sI - A_c)^{-1} B_c C_o^* = W_m(s)$ , the reference model may be realized by the triple  $(A_c, B_c C_o^*, C_c)$  and described by the non minimal state space representation

$$Y_m = A_c Y_m + B_c C_o^* r, Y_m(0) = Y_{m0}$$

$$y_m = C_c^T Y_m$$

(10)

where  $Y_m \in \mathfrak{R}^{np+2n-2}$

Letting  $e = Y_c - Y_m$ , to be the state error and  $e_1 = Y_p - Y_m$  the output tracking error, it follows from (6) and (9) that

$$\dot{e} = A_c e, e_1 = C_c^T e$$

i.e., the tracking error  $e_1$  satisfies

$$e_1 = C_c^T e^{A_c t} (Y_c(0) - Y_m(0))$$

Because  $A_c$  is a stable matrix,  $e_1(t)$  converges exponentially to zero. The rate of

convergence depends on the location of the eigen values of  $A_c$ , which are equal to the roots of  $\Lambda(s)Z_p(s)\Lambda_0(s)R_m(s) = 0$ . The rate of convergence by designing to have fast zeros, but it is limited by the dependence of  $A_c$  on the zeros of  $Z_p(s)$ , which are fixed by the given plant.

The main characteristics of the simple MRAC schemes are:

- (1) The adaptive laws are driven by the estimation error, which due to the special form of the control law is equal to the regulation or tracking error. They are derived using the Lyapunov design approach without the use of the normalization.
- (2) A simple Lyapunov function is used to design the adaptive law and establish boundedness for all signals in the closed loop plant.

Let us assume that the relative degree of the plant

$$y_p = G_p(s)u_p = k_p \frac{Z_p(s)}{R_p(s)} u_p$$

(11)

is  $n^* = 1$ . The reference model

$$y_m = W_m(s)r$$

is chosen to have the same relative degree and both  $G_p(s)$ ,  $W_m(s)$  satisfy assumptions 1 to 6, respectively. In addition  $W_m(s)$  is designed to be SPR.

A reasonable approach to follow in the unknown plant parameter case is to replace (6) with the control law

$$\dot{\omega}_1 = F \omega_1 + g u_p, \omega_1(0) = 0$$

$$\dot{\omega}_2 = F \omega_2 + g y_p, \omega_2(0) = 0$$

(12)

$$u_p = \theta^T \omega$$

where  $\theta(t)$  is the estimate of  $\theta^*$  at time  $t$  to be generated by an appropriate adaptive law. We first obtain a composite state space representation of the plant and controller, i.e.,

$$\dot{Y}_c = A_0 Y_c + B_c u_p, Y_c(0) = Y_0$$

$$y_p = C_c^T Y_c$$

$$u_p = \theta^T \omega$$

where  $Y_c = [x_p^T, \omega_1^T, \omega_2^T]^T$

$$A_0 = \begin{bmatrix} Ap & 0 & 0 \\ 0 & F & 0 \\ gC_p^T & 0 & F \end{bmatrix}, B_c = \begin{bmatrix} B_p \\ g \\ 0 \end{bmatrix}$$

$$C_c^T = [C_p^T, 0, 0]$$



and then add and subtract the desired input  $B_c \theta^{*T} \omega$  to obtain

$$\dot{Y}_c = A_0 Y_c + B_c \theta^{*T} \omega + B_c (u_p - \theta^{*T} \omega)$$

If we now absorb the term  $B_c \theta^{*T} \omega$  into the homogeneous part of the above equation, we end up with the representation

$$\dot{Y}_c = A_c Y_c + B_c C_0^* r + B_c (u_p - \theta^{*T} \omega), Y_c(0) = y_0 \quad (13)$$

where  $A_c$  is as defined in (9). Let  $e = Y_c - Y_m$  and  $e_1 = y_p - y_m$  where  $Y_m$  is the state of the non minimal representation of the reference model given by (10), we obtain the error equation

$$\dot{e} = A_c e + B_c (u_p - \theta^{*T} \omega), e(0) = e_0$$

$$e_1 = C_c^T e$$

(14)

Because

$$C_c^T (sI - A_c)^{-1} B_c c_0^* = W_m(s)$$

we have

$$e_1 = W_m(s) \rho^* (u_p - \theta^{*T} \omega)$$

(15)

where  $\rho^* = \frac{1}{C_0}$ , which is in the form of the

bilinear parametric model.

The estimate  $\hat{e}_1(t)$  of  $e_1(t)$  based on  $\theta(t)$ , the estimate of  $\theta^*$  at time  $t$ , is given by

$$\hat{e}_1 = W_m(s) \rho (u_p - \theta^T \omega)$$

(16)

where  $\rho$  is the estimate of  $\rho^*$ . Because the control input is given by

$$u_p = \theta^T(t) \omega$$

It follows that  $\hat{e}_1 = W_m(s) \rho$ ; therefore, the estimation error  $\varepsilon_1, \varepsilon_1 = e_1 - \hat{e}_1$  may be taken to be equal to  $e_1$ , i.e.,  $\varepsilon_1 = e_1$ . Consequently, (16) is not needed and the estimate  $\rho$  of  $\rho^*$  does not have to be generated. Substituting for the control law in (14), we obtain the error equation

$$\dot{e} = A_c e + \bar{B}_c \rho^* \theta^T \omega, e(0) = e_0$$

$$e_1 = C_c^T e$$

(17)

where

$$\bar{B}_c = B_c C_0^* \quad (\text{or}) \quad e_1 = W_m(s) \rho^* \tilde{\theta}^T \omega$$

which relates the parameter error  $\tilde{\theta} \triangleq \theta(t) - \theta^*$  with the tracking error  $e_1$ . Because  $W_m(s) = C_c^T (sI - A_c)^{-1} B_c c_0^*$  is SPR and  $A_c$  is stable, equation (17) is in the appropriate form for applying the SPR-Lyapunov design approach.

We therefore proceed by proposing the Lyapunov-like function

$$V(\bar{\theta}, e) = \frac{e^T P_c e}{2} + \frac{\bar{\theta}^T \Gamma^{-1} \bar{\theta}}{2} |\rho^*|$$

(18)

where  $\Gamma = \Gamma^T > 0$  and  $P_c = P_c^T > 0$  satisfies the algebraic equations

$$P_c A_c + A_c^T P_c = -q q^T - v_c L_c$$

where  $q$  is a vector,  $L_c = L_c^T > 0$  and  $v_c > 0$  is a small constant. The time derivative  $\dot{V}$  of  $V$  along the solution of (17) is given by

$$\dot{V} = -\frac{e^T q q^T e}{2} - \frac{v_c}{2} e^T L_c e + e^T P_c \bar{B}_c \rho^* \tilde{\theta}^T \omega + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} / |\rho^*|$$

Because  $e^T P_c \bar{B}_c = e_1$  and  $\rho^* = |\rho^*| \text{sgn}(\rho^*)$ , we can make  $\dot{V} < 0$  by choosing

$$\dot{\tilde{\theta}} = \dot{\theta} = -\Gamma e_1 \omega \text{sgn}(\rho^*)$$

(19)

which leads to

$$\dot{V} = -\frac{e^T q q^T e}{2} - \frac{v_c}{2} e^T L_c e$$

(20)

Equations (18) and (20) imply that  $V$  and, therefore,  $e, \theta \in L_\infty$ .

Because  $e = Y_c - Y_m$  and  $Y_m \in L_\infty$  we have  $Y_c \in L_\infty$  which implies that  $y_p, \omega_1, \omega_2 \in L_\infty$ . Because  $u_p = \theta^T \omega$  and  $\theta, \omega \in L_\infty$  we also have  $u_p \in L_\infty$ . Therefore all the signals in the closed-loop plant are bounded. It remains to show that the tracking error  $e_1 = y_p - y_m$  goes to zero as  $t \rightarrow \infty$ .

From (18) and (20) we establish that  $e$  and therefore  $e_1 \in L_2$ . Furthermore, using  $\theta, \omega, e \in L_\infty$  in (17) we have that  $\dot{e}, \dot{e}_1 \in L_\infty$ . Hence,  $e_1, \dot{e}_1 \in L_\infty$  and  $e \in L_2$ , which, imply that  $e_1(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

The MRAC scheme guarantees that:

- (i) All signals in the closed-loop plant are bounded and the tracking error  $e_1$  converges to zero asymptotically with time for any reference input  $r \in L_\infty$ .
- (ii) If  $r$  is sufficiently rich of order  $2n$ ,  $\dot{r} \in L_\infty$  and  $Z_0(s), R_p(s)$  are relatively coprime, then the parameter error  $|\tilde{\theta}| = |\theta - \theta^*|$  and the tracking error  $e_1$  converge to zero exponentially fast.

#### IV. ROBUST ADAPTIVE CONTROL

The model reference adaptive scheme designed for a disturbance free plant model may go unstable in the presence of small disturbances. The adaptive laws that are to be



robust with respect to a wide class of plant model uncertainties are developed which are referred as Robust Adaptive Laws. These Robust adaptive laws are combined with control laws to generate robust adaptive control schemes [23,26]

Under certain assumptions on the plant and reference model, MRAC schemes are designed that guarantee signal boundedness and asymptotic convergence of the tracking error to zero. These results however provide little information about the rate of convergence and the behaviour of the tracking error during the initial stages of adaptation. If the reference signal is sufficiently rich, the exponential convergence and more information about the asymptotic and transient behaviour of the scheme can be inferred. In most situations, the use of sufficient rich reference inputs without violating the tracking objective, the transient and asymptotic properties of MRAC Schemes in the absence of rich input signals are very crucial. A phenomenon known as “bursting”, where the tracking error, after reaching steady state behavior, bursts into oscillations of large amplitude over short intervals of time, have often observed in simulations. Bursting cannot be excluded unless the reference signal is dominantly rich and/or an adaptive law with a dead zone is employed. Bursting is one of the annoying phenomena in adaptive control in simulations and the cause of bursting could be the computational error, which acts as a small bounded disturbance. To eliminate bursting error occurs by modified MRAC schemes are designed

In modified MRAC scheme, the control law design is modified to one that takes into account the fact that the plant parameters are not exactly known and reduces the effect of the parametric uncertainty on stability and performance as much as possible. This control law, which is robust with respect to parametric uncertainty, can then be combined with an adaptive law to enhance the stability and performance. Illustrate this design methodology for the plant given as follows:

The Plant is taken as following equation:

$$\dot{x} = ax + bu + d$$

$$\dot{x}_m = -x_m + r$$

Reference Model taken as

Let choose a control law that employs no adaptation and meets the control objective of stability and tracking as close as possible even though the plant parameters  $a$  and  $b$  are unknown.

Consider the control law:  $u = \bar{\theta}_0 x + \bar{c}_0 r + u_\alpha$

where  $\bar{\theta}_0, \bar{c}_0$  are constants that depend on some nominal known values of  $a, b$ , if available

$$\text{Choose } u_\alpha = -\frac{s+1}{\tau s} e_1 u$$

where  $\tau > 0$  is a small design constant.

Choose  $\tau Z$  to  $0 < \tau < \frac{1}{|\bar{\theta}_0|}$  satisfy,

which implies the closed loop transfer function is stable and tracking error  $e_1$  will converge exponentially fast to the residual set whose size reduces to zero as  $\tau \rightarrow 0$ . and therefore all signals in the closed plant are bounded. In the absence of the disturbances, i.e.,  $d=0$ , the modified scheme guarantees that  $e_1 \rightarrow 0$  as  $t \rightarrow \infty$ . The significance of the modified adaptive control scheme is that tracking error can be made small by choosing small design parameter  $\tau$ . By proper choice of  $\tau$ , bursting can be reduced and improve the tracking error performance of the adaptive control scheme

## V. PI CONTROLLER

A standard PI controller is two term controller, whose transfer function is generally written in the parallel form given by equation or the ideal form given by

$$G(s) = K_p + \frac{K_i}{s}$$

$$G(s) = K_p \left(1 + \frac{1}{T_i s}\right)$$

(21)

The proportional term-providing an overall control action is proportional to the error signal through the all pass gain factor. The integral term - reducing steady state errors through low frequency compensation is by an integrator.

## VI. ROBUST MODEL REFERENCE ADAPTIVE PI CONTROL

In this section a Robust Model Reference Adaptive PI Control (MRAPIC) design is introduced. The block diagram of the proposed Robust Model Reference Adaptive PI Control (Robust MRAPIC) is as shown in the figure 1.

In proposed Robust MRAPIC the control signal from modified MRAC signal is added to



the another control signal from PI controller and then given to the Plant. The system forms a closed loop plant and the error is taken from the difference of the plant output to the reference model input. The input to the PI system is the error. The idea behind using PI Controller is it minimizes the steady state error and improves the steady state performance. The gain of PI controller is tuned according to the plant.

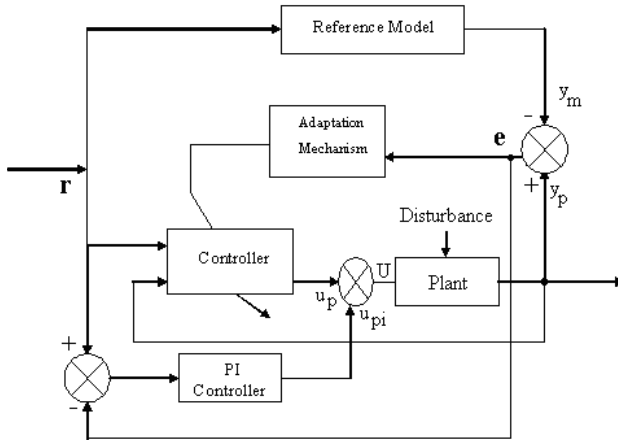


Fig. 1. Proposed Model Reference Adaptive PI Control

The control law is  $U = u_p + u_{pi}$  (22)

$$u_p = \theta_1 \omega_1 + \theta_2 \omega_2 + \theta_3 y_p + c_0 r + u_\alpha$$

$$\text{Choose } u_\alpha = -\frac{s+1}{\tau s} e_1 u$$

where  $\tau > 0$  is a small design constant and  $u_{pi}$  is PI controller output

The adaptive law is given by

$$\dot{\theta} = -\Gamma e_1 \omega \text{ and } \theta(0) = \theta_0$$

$$\text{Where } e_1 = y_p - y_m,$$

$$\theta = [\theta_1, \theta_2, \theta_3, c_0]^T \text{ and } \omega = [\omega_1, \omega_2, y_p, r]^T$$

where  $\Gamma = \text{diag}(\gamma_i)$  for some  $\gamma_i > 0$ , is a positive definite matrix and obtain decoupled adaptive law

$$\dot{\theta}_i = -\gamma_i e_1 \omega_i, \quad i = 1, 2, \dots, 4$$

## VII. COMPUTER SIMULATION

In this section, result of computer simulations for Conventional MRAC, modified MRAC and the proposed Robust MRAPIC Method is reported. The results show the effectiveness of the proposed Robust MRAPIC scheme and

reveal its performance superiority to the Conventional MRAC technique and modified MRAC.

As an example, the system taken for the simulation is the Lateral Dynamic Model of a Boeing 747 airplane.

The liberalized model of the Lateral Dynamics of Boeing 747 can be described as

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = x_2(t) = y_r(t)$$

$$x(t) = [\beta, y_r, P, \Phi]^T$$

$$B = [b]$$

where  $\beta$  is the side-slip angle,  $y_r$  is the yaw-rate,  $p$  is the roll rate,  $\Phi$  is the roll angle,  $y$  is the system output which is the yaw rate in this case, and  $u$  is the control input vector.

From the data provided in horizontal flight at 40,000 ft and nominal forward speed 774 ft/s, the Boeing 747 lateral perturbation dynamics matrices are as follows:

The transfer function for the Lateral Dynamic Model of a Boeing 747 airplane System is given by

$$A = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ -0.598 & -0.115 & 0.0318 & 0 \\ -3.05 & 0.388 & -0.465 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.01 \\ -0.5 \\ 0.2 \\ 0 \end{bmatrix}$$

$$y_p(s) = \frac{-0.5s^3 - 0.2608s^2 - 0.1223s - 0.05832}{s^4 + 0.6358s^3 + 0.9389s^2 + 0.5116s + 0.003674} r(s)$$

$$w_m(s) = \frac{1}{(s+3)}$$

$$(23)$$

$$\text{and } \Lambda(s) = \frac{1}{(s+1)^3}$$

The initial value of the parameters are chosen as  $\theta(0) = [0.794, 2.4772, 2.2306, -1.6715]^T$ . The simulation is done for  $\Gamma = \text{diag}\{1 \ 0 \ 0 \ 1\}$ . The simulation was carried out with MATLAB for time duration  $t \in [0, 30]$  s and The input is chosen as  $r(t) = 10 + 11 \sin(0.7t)$

The adaptive laws and control schemes developed are based on a plant model that is free of disturbances, noise and unmodelled dynamics. These schemes are to be implemented on actual plants that most likely deviate from the plant models on which their design is based. An actual plant may be infinite dimensional, nonlinear and

its measured input and output may be corrupted by noise and external disturbances [1]. It was shown using Conventional MRAC that adaptive scheme designed for a disturbance free plant model and may go unstable in the presence of small disturbances. The disturbances added the Conventional MRAC has some oscillations at the peak of the signal. For the above example, the disturbance is considered as a random noise signal. Figure 2 and 3 describes output and error for the Conventional MRAC.

Figure 4 and 5 describes the output and error for the modified MRAC method. In modified MRAC the oscillation will be reduced and this adaptation time is decreased. Figure 6 and 7 describes the output and error for the proposed Robust MRAPIC Method On the contrary, the proposed method has much less error than conventional method in spite of disturbance.

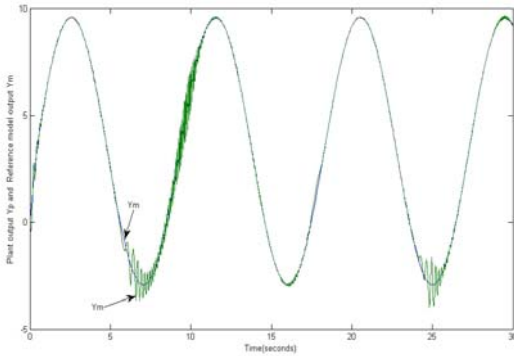


Fig. 2. Plant output  $y_p$  and Reference model output  $y_m$  for the conventional MRAC.

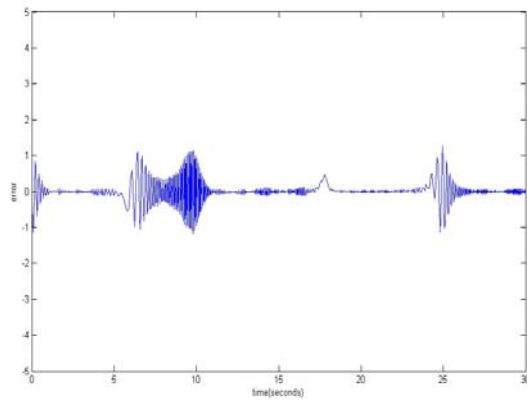


Fig.3. Error for the Conventional MRAC.

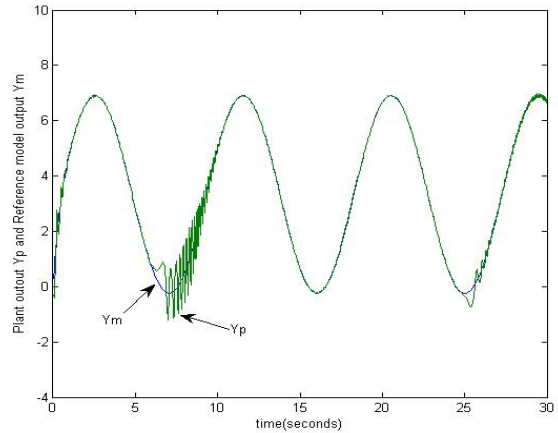


Fig. 3. Plant output  $y_p$  and Reference model output  $y_m$  for the modified MRAC.

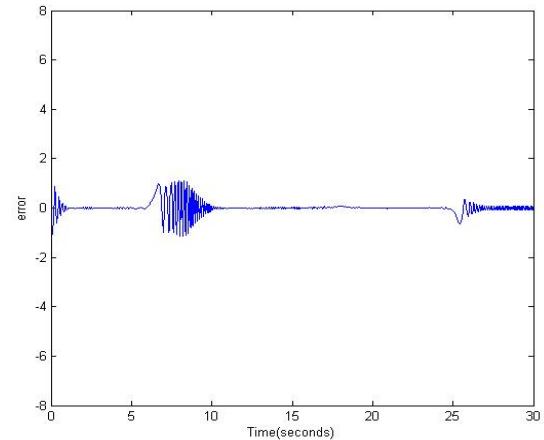


Fig. 4 Error for the modified MRAC.

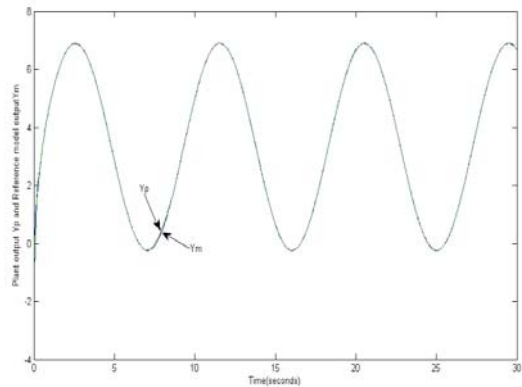


Fig. 4. Plant output  $y_p$  and Reference model output  $y_m$  for the proposed Robust MRAPIC

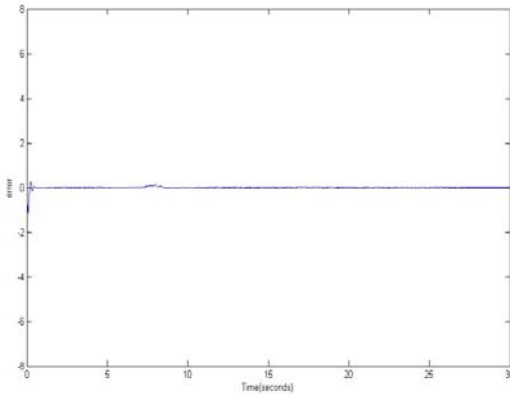


Fig. 5. Error for the proposed Robust MRAPIC

## VIII. CONCLUSION

From the above Simulation Results of the plant considered, the performance is improved by using PI Controller with modified MRAC and the tracking error has become zero within 1 second and no oscillations have occurred. The plant Output has tracked with the reference model output. This method is the Better than the Conventional MRAC and modified MRAC scheme. From the above responses, the efficiency is increased for proposed Robust MRAPIC. The future research would deserve focusing on the proposed controller to unmodeled dynamics and delays. Also this scheme can be extend to discrete time model reference adaptive control system.

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