FUZZY SWING-UP AND STABILIZATION OF REAL INVERTED PENDULUM USING SINGLE RULEBASE

SINGH VIVEKKUMAR RADHAMOHAN, MONA SUBRAMANIAM A, DR. M.J.NIGAM

Indian Institute of Technology Roorkee, Roorkee, India-247667

ABSTRACT

In this paper a real time control for swing-up stabilization of inverted pendulum is designed using Fuzzy logic. In the model proposed here a single rulebase is used to control both position and angle simultaneously during both swing-up and stabilization. The proposed fuzzy control scheme successfully fulfills the control objectives and also has an excellent stabilizing ability to overcome the external impact acting on the pendulum system. The effectiveness of this controller is verified by experiments on a simple inverted pendulum with fixed cart length.

KEYWORDS : *Fuzzy control, Inverted Pendulum, Swing-up Stabilization*

1. INTRODUCTION

Inverted pendulum being an inherently unstable system is often used as a benchmark for verifying the performance and effectiveness of a control algorithm. Inverted pendulum is a combination of various research areas like robotics, control theory, computer control etc. The inverted pendulum consists of a motor controlled carton to which a pendulum is freely pivoted. The control task is to swing the pendulum from vertically downward position to vertically upward position and to keep the pendulum in vertically upright position once it reaches there. The cart should also be placed at its initial position. All this is to be done just by moving the cart back and forth on the rail without crossing the limit switches.

In literature we can find a number of control algorithms for swing-up stabilization. Wei et al. [1] presented a nonlinear control strategy by decomposing the control law into a sequence of steps. Chung and Houser [2] proposed a nonlinear state feedback control law to regulate the cart position as well as the swinging energy of the pendulum. Zhao and Spong [3] applied a hybrid-control strategy, which globally asymptotically stabilizes the system for all initial conditions. Their method does not take the rail length restriction into account. Chatterjee *et al.* [4] proposed an energy based control law that swings up and stabilizes a cart-pendulum system with restricted travel and restricted control force in simulations as well as in real-time experiments. However, a major problem with their method is to design suitable potential wells and coefficients using intuition and time consuming iterations. All these methods require an extensive knowledge of system dynamics.

However the use of fuzzy logic does not require extensive knowledge of the system. This feature becomes of major importance when dealing with complex non-linear systems. Moreover, the dynamic modeling of systems show a dependency on their mechanical parameters, subject to lifetime modifications (friction factors affected by the abuse of joints), and on their dynamical parameters that vary with the performed task. These considerations also give advantage of fuzzy control methods over other non-linear methods. A number of techniques using fuzzy logic have been proposed. Wong [5] adopted the genetic algorithm to tune all the membership functions of a fuzzy system in order to keep an inverted pendulum upright. Yamakawa [6] designed a high-speed fuzzy controller hardware system and used only seven fuzzy rules to control the angle of an inverted pendulum. But these methods do not take the rail length into consideration. Matsuura [7] and Yasunobu [8] both used the information of the cart to build a set of 49 fuzzy rules for conducting the virtual target angle, and then used the virtual target angle and the information of the pendulum to construct another set of 49 fuzzy rules for total stabilization. Most of the fuzzy control algorithms used for swing-up and stabilization use a large number of membership functions. The parameters of these membership functions are difficult to decide. Moreover, large number of membership functions makes it difficult to determine the rules in rulebase.

Here we propose to use a single fuzzy rule base for angle and position control. This rule base takes care of swing up and stabilization as well. The design of this rule base uses 3x3 rules or in other words 9 rule rulebase which is a relatively simple design. With less number of membership functions and rules the number of parameters is reduced considerably. We have combined the position error and angular error to obtain a single error signal 'e', similarly the differential position error and differential angular error are combined to obtain a single differential error signal 'de'. These signals are given to the fuzzy controller thereby, eliminating two different fuzzy controllers being used for the purpose. With this the number of tuning parameters is reduced as well. We use switches to change the gains for swingup and stabilization, just by changing the gains we are able to change the mode between swing up and stabilization. During swing up mode we induce energy into the system by triggering only those rules which increase the energy into the inverted pendulum. This process is continued until the pendulum reaches 30^o from the final vertically upright position, the stabilizer mode of control comes into effect at this point and a handover is made. In the stabilization mode the pendulum is maintained in upright position with minimum error.

Organization of the paper is as follows. Section 2 describes the simple inverted pendulum system used in experiment and its parameters. Section 3 describes the modeling of simple inverted pendulum. Section 4 describes the scheme for development of fuzzy logic controller. Section 5 provides experimental results to demonstrate the effectiveness of the fuzzy controller proposed here. Section 6 discusses the conclusion of the experiment.

2. REAL INVERTED PENDULUM SYSTEM

Fig.1 Googol Tech Simple Inverted Pendulum

The pendulum used here is a Googol Inverted pendulum. It consists of a motor driven cart and a pendulum freely pivoted above it, along with sensors and electronic circuit. The pendulum can move freely in the vertical plane. The cart movement is controlled by a D.C. motor which is connected to the motor by grooved rubber belt. Control algorithm and data acquisition are realized with Mathworks Matlab Simulink on a personal computer.

The system parameters are

M cart mass: $M = 1.096$ kg

m rod mass: $m = 0.109$ kg

b friction coefficient of the cart: $b = 0.1$ Nm/s

l distance from the rod axis rotation center to the rod mass center: $l = 0.25$ m

I rod inertia: $I = 0.0034$ kg.m.m

Total rail length is 0.6 m

3. MODELING OF THE INVERTED PENDULUM

After ignoring the air resistance and other frictions, 1-stage linear IP can be simplified as a system of cart and even quality rod, as shown in Fig 2.

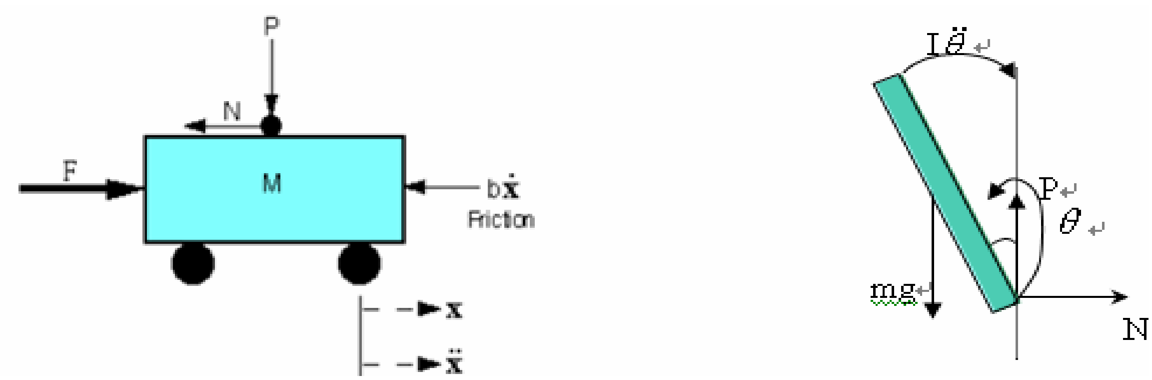


Fig. 2 Cart and Rod Force analysis

Where,

F: force acting on the cart

X: cart position

N: interactive force in the horizontal direction

P: interactive force in the vertical direction

θ: angle between the rod and vertically downward direction

From the forces in the horizontal direction, we obtain:

$$M\ddot{x}=F-b\dot{x}-N$$
(1)

From the force acting on the rod in horizontal direction we get:

$$N=m\frac{d^2}{dt^2}(x+l\sin\theta)$$
(2)

That is

$$N=m\ddot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta$$
(3)

Combining with equation (1), the first dynamic equation is obtained:

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F$$
(4)

To get the second dynamic equation, analyze the force in the vertical direction then we have

$$P-mg=m\frac{d^2}{dt^2}(l\cos\theta)$$
(5)

$$P-mg=-ml\ddot{\theta}\sin\theta-ml\dot{\theta}^2\cos\theta$$
(6)

By moment conservation:

$$-Pl\sin\theta-Nl\cos\theta=I\ddot{\theta}$$
(7)

Combining equation(6)and(7), we get the second dynamic equation:

$$(I+ml^2)\ddot{\theta}+mgl\sin\theta=-ml\ddot{x}\cos\theta$$
(8)

4. CONTROLLER DESIGN

Here we have used fuzzy logic for the design of controller. The controller is split into two stages: swing-up and stabilization. The swing-up routine swings the pendulum from the initial vertically downward position to vertically upward position without crossing the switch limits. The stabilization routine balances the pendulum in vertically upright position at the desired cart position. Swing-up and stabilization routines are discussed below.

4.1. Swing-up routine

In the method proposed here we aim to swing up the pendulum from vertical downward position to a $\pm 30^{\circ}$ of the vertical position. While swinging up care is taken that the pendulum does not cross the rail limits as it is a limited rail length situation. The basic strategy is to move the cart in such a way that energy is slowly pumped into the pendulum. This is achieved by satisfying a particular mathematical condition derived from the mechanical energy equations while designing the rule base of fuzzy control system.

The total mechanical energy of the pendulum and its derivative $\dot{E}$ are given by equation [9].

$$E = \frac{1}{2} m L^2 \dot{\theta}^2 + mgL(1-\cos\theta) \quad (9)$$

$$\dot{E} = ml\,\dot{\theta}(\cos\theta)\,\ddot{x} \quad (10)$$

From the above equation it follows that energy E can be pumped or removed from the system by changing the sign of $\dot{E}$.

Now $\dot{E}$ is positive if $\ddot{x} > 0$ and $\dot{\theta}\cos\theta > 0$ or $\ddot{x} < 0$ and $\dot{\theta}\cos\theta < 0$

Similarly $\dot{E}$ is negative if $\ddot{x} < 0$ and $\dot{\theta}\cos\theta > 0$ or $\ddot{x} > 0$ and $\dot{\theta}\cos\theta < 0$

So to pump the energy into the system we design our rulebase such that $\dot{E}$ is positive most of the times and never negative. Energy is pumped into the pendulum slowly and the pendulum is swung up to nearly vertically upright position.

Error(e) \ Error differential(de)	N	Z	P
N	N	N	Z
Z	N	Z	P
P	Z	P	P

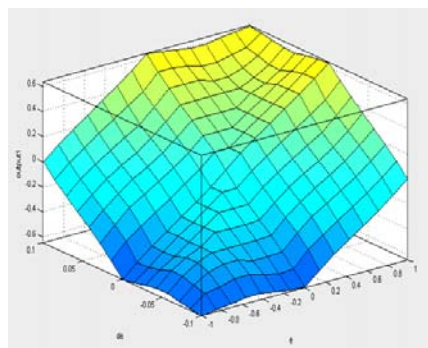


Fig. 3 Fuzzy Rulebase and Control surface

However one has to keep in mind the length of the rail x while swinging up the pendulum. Thus the $\ddot{x}$ generated should be generated by keeping x in mind.

Instead of using a separate rulebase for controlling the position of pendulum, we subtract the position error x from the angle error θ and position velocity error $\dot{x}$ from the angular velocity error $\dot{\theta}$.

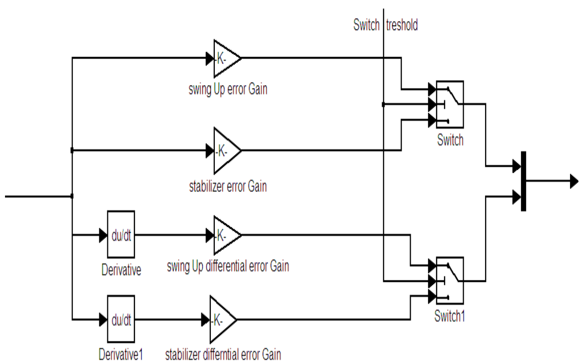


Fig. 4 Switching mechanism between swing-up and stabilization

The gains of $\dot{x}$ are so adjusted that $\dot{x}$ values remain very small in the center and becomes significant only at the edges such that it changes the sign of $\ddot{x}$ to avoid cart from crossing limit switches.

4.2. Stabilization

Once the pendulum is swung within $\pm 30^{\circ}$ of the vertical position the fuzzy stabilization controller takes over from swing up controller. The proposed fuzzy stabilization controller uses two input variable e and $\dot{e}$. Here e is the difference of pendulum angle θ and cart position x , $\dot{e}$ is the difference of cart velocity $\dot{x}$ from the pendulum angular velocity $\dot{\theta}$.Instead of using a separate rule base for stabilization the gains are switched to another value, while the same fuzzy rule base is used.

5. EXPERIMENTAL SETUP

The control algorithm in the above section was applied to a real time inverted pendulum. Fig 5 shows the simulink model of the system used in experiment.

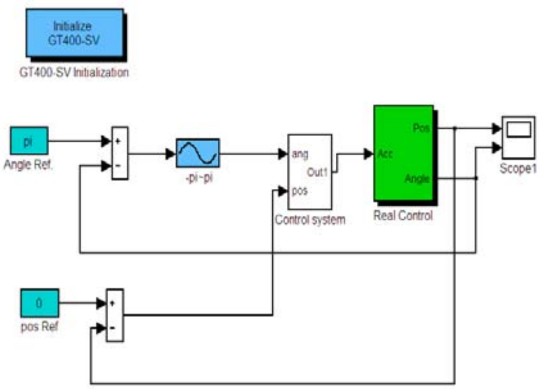
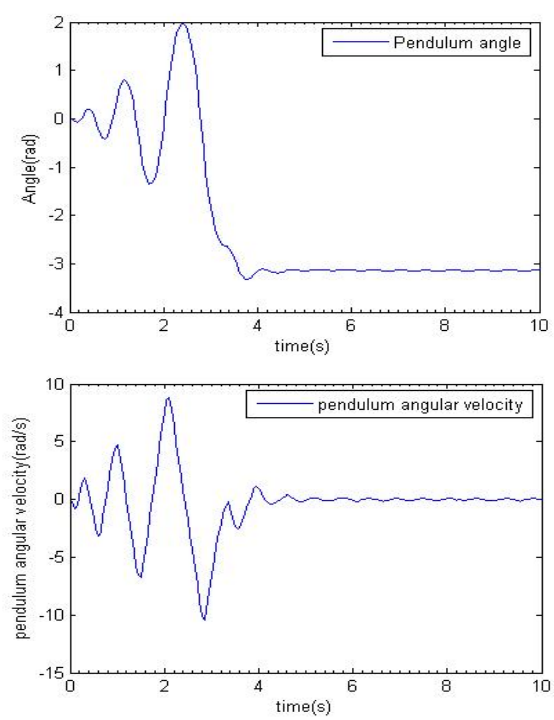


Fig. 5 Simulink model of the overall system

The results obtained are discussed briefly here.Fig.6 shows the angle and position of the pendulum. Initially the pendulum is in vertically downward position. It swings up gradually, responding to the bounded oscillation of the cart. Until 3.35 seconds the swing-up controller is in control. The stabilization algorithm takes over completely for the rest of the time.



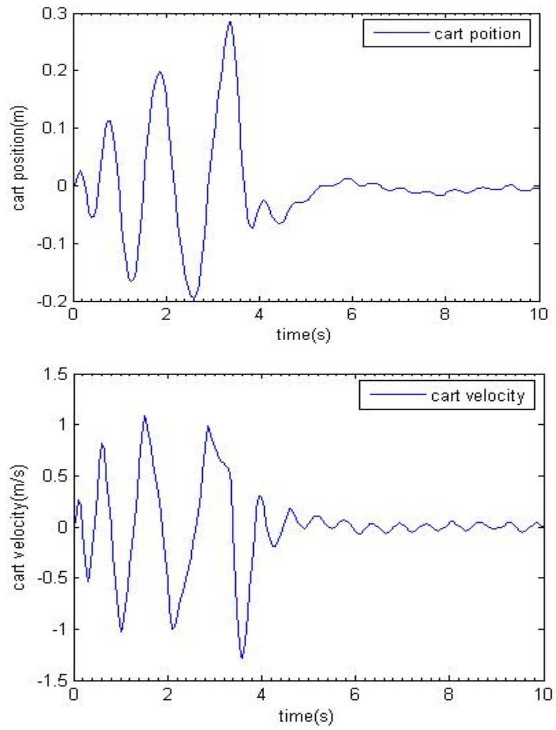


Fig. 6 Swing-up and stabilization of real inverted pendulum with proposed fuzzy controller.

The pendulum is stabilized within 5 s, the cart oscillations are limited from -0.2m to 0.3m during swing up. The pendulum keeps oscillating about the reference while in stabilization mode. However these oscillations are very small about 2 cm for the cart position and the pendulum angle is within 1□, these are basically caused by factors like continuous air disturbances on the rod, non-linear un-modeled dynamics, such as Coulomb friction, pinion backlash, motor dead-zone and magnetic hysteresis, and mechanical imperfections.

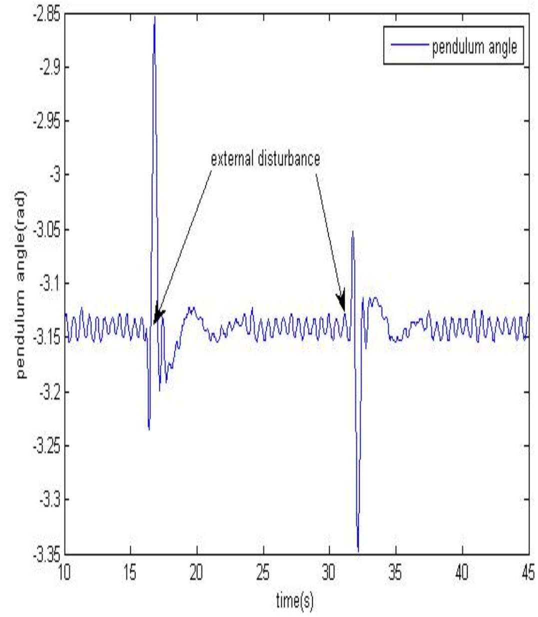


Fig. 7 balancing the pendulum after disturbance is provided.

Fig.7 shows the plot of θ during stabilization when momentary external disturbances are applied on the rod. Momentary disturbances of magnitude 17^0 and -11.5^0 are provided to the pendulum by hitting it at time t=16 s and t=32s respectively. Each time the pendulum angle is stabilized in less than 3s. Fig.8 shows the results of the fuzzy controller after changing the reference position. The proposed fuzzy controller is able to stabilize the pendulum system and steady state is reached.

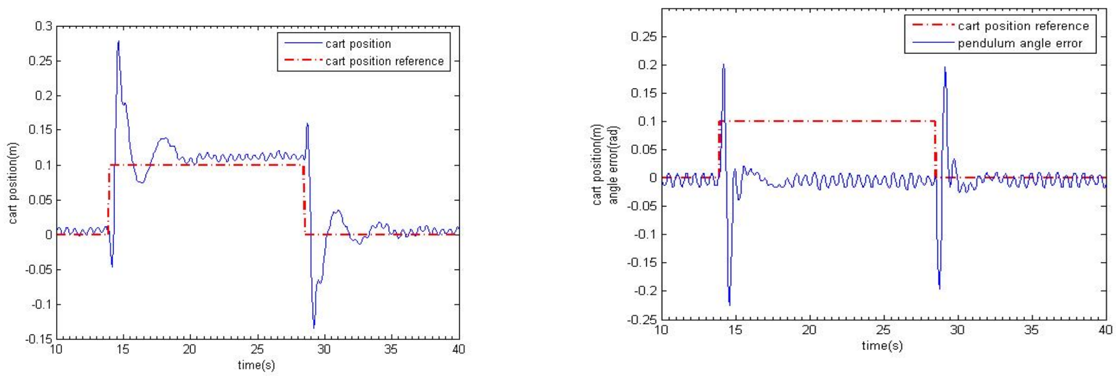


Fig. 8 balancing the pendulum after changing the reference cart position

6. CONCLUSION

Experimental results show that the pendulum can be swung-up very conveniently by controlling its energy. During swing up energy is pumped into the pendulum until it reaches that of the steady state upright position value. While swinging cart is kept within rail limits. Once the pendulum is within a specified range of the upright position a stabilization strategy is used to maintain the pendulum in the upright position. Experimental results show that this strategy of ours is better than that proposed by [4] [10] [12]. Moreover the stabilization strategy is robust and can overcome any external disturbances.

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BIOGRAPHY:

Singh Vivekkumar Radhamohan received his Bachelor of Engineering degree (Electronics and Communication Engineering) from Bhavnagar University, Bhavnagar India, in 2008. Currently, he is pursuing an M.Tech degree specialized in Control and Guidance from Indian Institute of Technology, Roorkee India. His area of research includes Robotic Control, Optimization and Neuro-Fuzzy control.

Mona Subramaniam A receivied his Bachelor of Engineering degree (Electronics and Communication Engineering) from Visvesvaraya Technological University, India, in 2006. Currently, he is pursuing an M.Tech degree specialized in Control and Guidance from Indian Institute of Technology, Roorkee India. His area of research includes Robotic Control, Fuzzy control and Genetic algorithm.

M.J. Nigam received his B.Tech degree in Electronics and Communication Engineering from Regional Engineering College (now NIT), Warangal(AP),India in 1976, M.E. degree in Electronics and Communication Engineering with specialization in Control and Guidance in 1978 and the research work leading to the award of Ph.D degree in Electronics and Computer Engineering in 1992 from University of Roorkee (now IIT Roorkee). From 1980-1982, he served as a faculty member in the Department of Electronics Engineering, Institute of Technology, Banaras Hindu University, India. In 1982, he joined as a faculty of University of Roorkee (now IIT Roorkee), Roorkee, India in the Department of Electronics and Computer Engineering and became Reader in 1989, Associate Professor in 1996 and Professor in 2009. His main research interests are digital image processing, robotics and filtering techniques. A number of research papers in the above areas are published /presented in international journals and conferences.