SEMI BLIND ESTIMATION FOR MULTIUSER MIMO CHANNEL USING ORTHOGONAL SPACE TIME BLOCK CODING

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ABSTRACT

The problem of estimating the Channel State Information (CSI) of several transmitters that communicate with a single receiver is considered. These transmitters communicate with the receiver using orthogonal Space Time Block Codes (OSTBC) method. Based on Rank Revealing LU (RRLU) factorization a novel algorithm to estimate multiuser MIMO channels is proposed. The algorithm estimates the null space of the subspace spanned by the user channels using only a few training blocks to extract the users CSI from this subspace. The proposed algorithm achieves an intermediate performance between Least Squares (LS) based approach and Capon approach. Computer simulations are performed to validate the proposed algorithms.

Keywords: *Channel State Information, Channel Estimation, Space Time Block Code, Rank Revealing.*

1. INTRODUCTION

Since the well known work of Alamouti [1], and the later generalization by Tarokh et. al. [2], several families of space-time block codes (STBCs) have been proposed to exploit the spatial diversity in MIMO systems. One example is the orthogonal STBCs (OSTBCs) [2]. An ordinary assumption for most of the STBCs is that perfect channel state information (CSI) is available at the receiver, which has motivated an increasing interest on blind channel estimation algorithms [3] - [6]. Blind techniques avoid the penalty in bandwidth efficiency or signal to noise ratio (SNR) associated, respectively, to training based approaches [7], [8]. The literature on blind and semi blind channel estimation under STBC transmissions is rich, most of the research efforts have considered time-invariant flat-fading MIMO channels [9], [10].

In this paper, the problem of estimating the CSI of several transmitters that communicate with a single receiver is considered. These transmitters communicate with the receiver using OSTBC. Based on Rank Revealing LU (RRLU) factorization a novel algorithm to estimate multiuser MIMO channels is proposed. The algorithm estimates the null space of the subspace spanned by the user channels using only a few training blocks to extract the users CSI from this subspace. Traditionally LU factorization has been completed using Gaussian Elimination with Partial or Complete Pivoting [11]. The RRLU is a special LU factorization that is guaranteed to reveal the numerical rank of the matrix under consideration. It is well known that the computational load of the RRLU method is significant, as it does not involve Eigen Value Decomposition (EVD) or Singular Value Decomposition (SVD) [12]. By comparing the performances of the RRLU factorization with RRQR factorization, it is worth to mention that RRLU factorization turns out to be more reliable. This algorithm allows reduce of its complexity to $O(N^3/3)$ operations from to $O(2.N^3/3)$ [11]. So the LU method is approximately two times faster than the QR method.

This paper is structured as follows: Section II presents the problem formulation. The matrices, notation are defined in Table 1. The development of the proposed algorithms is presented in Section III. In Section IV, the performance of the RRLU is illustrated and compared with the previous work [13] and [14]. Some concluding remarks follow in Section V.

2. PROBLEM FORMULATION

Consider multi-access multi-antenna environment with P transmitters and a single receiver as shown in Figure 1. Assuming all the transmitters have N antennas and the receiver has M antennas. To simplify more all the transmitters are assumed to use the same OSTBC

Table 1.
The parameters, notation used in the paper

$\boldsymbol{Y}$	The received signal matrix of size $T \times M$
$\boldsymbol{X}(\boldsymbol{s}_p)$	The matrix of transmitted signals of size $T \times N$
$\boldsymbol{s}_p$	The length of the information vector of size $K \times 1$
$\boldsymbol{H}_p$	The channel matrix for the p^{th} transmitter of size $N \times M$
$\boldsymbol{Z}$	The noise matrix of size $T \times M$
$\boldsymbol{e}_k$	unit vector with the k th element is one
$(.)^{\perp}$	The orthogonal projection operator
$\overline{(.)}$	Converting complex valued matrix into real valued vector $\overline{(.)} := \begin{bmatrix} vec\{Re(.)\} \\ vec\{Im(.)\} \end{bmatrix}$
$vec\{.\}$	The vectorization operator stacking all the column of a matrix on the top of each other.
$\boldsymbol{h}_p$	The CSI as real valued vector $\boldsymbol{h}_p := \overline{\boldsymbol{H}_p}$
$\boldsymbol{\Phi}_k$	OSTBC specific and known matrices of size $2MT \times 2MN$ and $k = 1,2, \dots, 2K$, for a rate K/T
J_t	The number of the transmitted training blocks (pilots)
J	The number of the data blocks available at receiver pilots and information
$\boldsymbol{E}$	Satisfies $\boldsymbol{A}^T(\boldsymbol{h}_p) \perp \boldsymbol{E}$ with size $2MT \times (2MT - 2KP)$

with a rate K/T. The flat block fading channel scenario is assumed. The received signal is given by:

$$\boldsymbol{Y} = \sum_{p=1}^{P} \boldsymbol{X}(\boldsymbol{s}_p)\boldsymbol{H}_p + \boldsymbol{Z} \qquad (1)$$

The elements of the Channel matrices are assumed to be independent complex Gaussian random variables. The matrix $\boldsymbol{X}(\boldsymbol{s}_p)$ is OSTBC matrix where the elements in this matrix are function of $s_1, s_2 \dots s_K$ information symbols and its complex conjugates. $\boldsymbol{X}(\boldsymbol{s}_p)$ is given by:

$$\boldsymbol{X}(\boldsymbol{s}_p) = \sum_{k=1}^{K} (\boldsymbol{C}_k Re\{s_k\} + \boldsymbol{D}_k Im\{s_k\}) \qquad (2)$$

Where

$$\boldsymbol{C}_k := \boldsymbol{X}(\boldsymbol{e}_k) \ \& \ \boldsymbol{D}_k := \boldsymbol{X}(i\boldsymbol{e}_k)$$

Using (2) one can write (1) as:

$$\overline{\boldsymbol{Y}} = \sum_{p=1}^{P} \boldsymbol{A}(\boldsymbol{h}_p)\overline{\boldsymbol{s}_p} + \overline{\boldsymbol{Z}} \qquad (3)$$

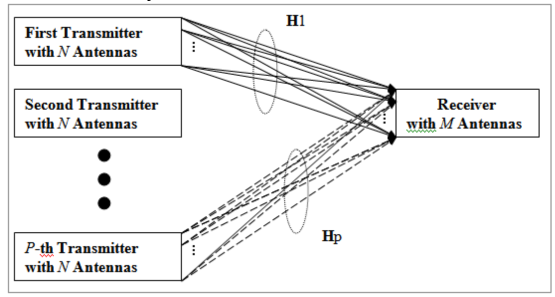

Figure 1. Multiuser MIMO environment of P transmitters and a single receiver.

The vectorized form of the matrix $\boldsymbol{A}(\boldsymbol{h}_p)$ is:

$$\boldsymbol{A}(\boldsymbol{h}_p) := [\overline{\mathbf{C}_1\mathbf{H}_p}, \dots, \overline{\mathbf{C}_K\mathbf{H}_p}, \quad \overline{\mathbf{D}_1\mathbf{H}_p}, \dots, \overline{\mathbf{D}_K\mathbf{H}_p}] = [\boldsymbol{a}_1(\boldsymbol{h}_p) \ \boldsymbol{a}_2(\boldsymbol{h}_p) \dots \dots \boldsymbol{a}_{2K}(\boldsymbol{h}_p)] \qquad (4)$$

The matrix $\boldsymbol{A}(\boldsymbol{h}_p)$ is linear in $\boldsymbol{h}_p$, and it is orthogonal with column norm of $\|\boldsymbol{h}_p\|^2$. There exists $2K$ real matrices such that

$$\boldsymbol{a}_k(\boldsymbol{h}_p) = \boldsymbol{\Phi}_k\boldsymbol{h}_p \quad for\ k = 1,2, \dots, 2K \qquad (5)$$

Thus, using (5) one can rewrite (4) as:

$$\boldsymbol{A}(\boldsymbol{h}_p) := [\boldsymbol{\Phi}_1\boldsymbol{h}_p \ \boldsymbol{\Phi}_2\boldsymbol{h}_p \dots \dots \boldsymbol{\Phi}_{2K}\boldsymbol{h}_p] \qquad (6)$$

and

$$vec\{\boldsymbol{A}(\boldsymbol{h}_p)\} = \boldsymbol{\Phi}\boldsymbol{h}_p \qquad (7)$$

where

$$\boldsymbol{\Phi} := [\boldsymbol{\Phi}_1^T \ \ \boldsymbol{\Phi}_2^T \ \dots \dots \boldsymbol{\Phi}_{2K}^T]^T$$

Based on the received training blocks (3) can be reformulated as:

$$\overline{\boldsymbol{y}(n)} = \sum_{p=1}^{P} \overline{\overline{\boldsymbol{A}}}(\overline{\boldsymbol{s}_p(n)})\boldsymbol{h}_p + \overline{\boldsymbol{zz}}(n) \qquad (8)$$

Here, $\overline{\boldsymbol{s}_p(n)}$ and $\overline{\boldsymbol{zz}(n)}$ is vectorized signal and noise components, respectively. The matrix $\overline{\overline{\boldsymbol{A}}}(\overline{\boldsymbol{s}_p})$ is of size $(2MT \times 2MN)$ with k^{th} column is given by:

$$\overline{\overline{\boldsymbol{A}}}(\overline{\boldsymbol{s}_p}) = \boldsymbol{A}(\boldsymbol{e}_k)\overline{\boldsymbol{s}_p} \qquad (9)$$

Rewrite (8) as:

$$\boldsymbol{y}(n) = \boldsymbol{\Gamma}(n)\boldsymbol{v} + \boldsymbol{zz}(n) \qquad (10)$$

where,

$$\boldsymbol{\nu} := [\boldsymbol{h}_1^T \;\; \boldsymbol{h}_2^T \;\; \ldots\ldots \boldsymbol{h}_P^T]^T$$

and

$$\boldsymbol{\Gamma}(n) := [\bar{\bar{\boldsymbol{A}}}(\overline{\boldsymbol{s}_1(n)})\; \bar{\bar{\boldsymbol{A}}}(\overline{\boldsymbol{s}_2(n)}) \ldots\ldots \bar{\bar{\boldsymbol{A}}}(\overline{\boldsymbol{s}_P(n)})].$$

Arranging $\boldsymbol{y}(n)$, and $\boldsymbol{zz}(n)$ as:

$$\boldsymbol{\chi} := [\mathbf{y}^{\mathrm{T}}(1) \;\; \mathbf{y}^{\mathrm{T}}(2) \ldots\ldots \mathbf{y}^{\mathrm{T}}(U_t)]^T$$

and

$$\boldsymbol{\eta} := [\mathbf{zz}^{\mathrm{T}}(1) \;\; \mathbf{zz}^{\mathrm{T}}(2) \ldots\ldots \mathbf{zz}^{\mathrm{T}}(U_t)]^T$$

Also, redefining matrix $\boldsymbol{\Gamma}(n)$ of size $2MTU_t \times 2PMN$

$$\bar{\bar{\boldsymbol{\Gamma}}} = [\boldsymbol{\Gamma}^{\mathrm{T}}(1) \;\; \boldsymbol{\Gamma}^{\mathrm{T}}(2) \ldots\ldots \boldsymbol{\Gamma}^{\mathrm{T}}(U_t)]$$

$$\boldsymbol{\chi} = \bar{\bar{\boldsymbol{\Gamma}}}\boldsymbol{\nu} + \boldsymbol{\eta} \qquad (11)$$

Therefore training based LS estimation is given by:

$$\hat{\boldsymbol{\nu}} = (\bar{\bar{\boldsymbol{\Gamma}}}^{\mathrm{H}}\bar{\bar{\boldsymbol{\Gamma}}})^{-1}\bar{\bar{\boldsymbol{\Gamma}}}\boldsymbol{\chi} \qquad (12)$$

To be more specific, the matrix $\bar{\bar{\boldsymbol{\Gamma}}}$ should be full rank. This can be accomplished by considering appropriate size of J_t.

3. THE PROPOSED METHOD

Consider the signal subspace of a multiuser channel estimation problem is spanned by $\{\boldsymbol{A}(\boldsymbol{h}_p)\}$ which is orthogonal to the noise subspace $\boldsymbol{E}$ [15]. That implies:

$$\boldsymbol{A}^T(\boldsymbol{h}_p)\boldsymbol{E} = \mathbf{0} \qquad p = 1,2,\ldots\ldots P \qquad (13)$$

The generalized MUSIC spectrum is defined as:

$$P_{MUSIC} = \frac{1}{\left(\left\|\boldsymbol{A}^T(\boldsymbol{h}_p)\boldsymbol{E}\right\|^2\right)} \qquad (14)$$

The noise subspace $\boldsymbol{E}$ was extracted by SVD of the covariance matrix in [13]. The problem in hand now is to estimate the null subspace $\boldsymbol{E}$. Follow similar steps as in [13], and simplify MUSIC search using (6) and (14)

$$P_{MUSIC} = \frac{1}{tr\{\boldsymbol{A}^T(\boldsymbol{h}_p)\boldsymbol{E}\boldsymbol{E}^T\boldsymbol{A}(\boldsymbol{h}_p)\}} = \frac{1}{\boldsymbol{h}_p^{\mathrm{T}}\boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{I}_{2\mathrm{K}}\otimes\boldsymbol{E}\boldsymbol{E}^T)\boldsymbol{\Phi}\boldsymbol{h}_p} \qquad (15)$$

From (15) the channel vectors are belongs to the subspace spanned by the LP minor eigenvectors of the matrix $\boldsymbol{\Omega}$ which is given by:

$$\boldsymbol{\Omega} = \boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{I}_{2\mathrm{K}}\otimes\boldsymbol{E}\boldsymbol{E}^T)\boldsymbol{\Phi}$$

Now, one can estimate the channel vector $\widehat{\boldsymbol{h}}_p$as:

$$\widehat{\boldsymbol{h}}_p = \sum_{k=1}^{LP}\beta_{pk}\widehat{\boldsymbol{v}}_k \qquad (16)$$

where $\widehat{\boldsymbol{v}}_k, k = 1,2,\ldots LP$ are the eigenvectors of $\boldsymbol{\Omega}$. Here, L is the OSTBC specific parameter. Using LS based procedure one can easily extract parameter β as:

$$\beta_k := [\beta_{1,k} \;\; \beta_{1,k} \;\; \ldots\ldots \beta_{LP,k}]$$

where

$$\boldsymbol{\beta} = [\boldsymbol{\beta}_1^T \;\; \boldsymbol{\beta}_2^T \ldots\ldots \boldsymbol{\beta}_P^T]^T$$

One can estimate $\boldsymbol{\beta}$ as:

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{\Lambda}^{\mathrm{H}}\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}^{\mathrm{H}}\boldsymbol{\psi} \qquad (17)$$

where

$$\boldsymbol{\Lambda} := [\boldsymbol{F}^T(1)\; \boldsymbol{F}^T(2) \ldots \boldsymbol{F}^T(U_t)]^T$$

$$\boldsymbol{F}(n) := [\boldsymbol{F}_1(n) \ldots \boldsymbol{F}_{LP}(n)]$$

$$\boldsymbol{F}_k(n) := [\boldsymbol{A}(\widehat{\boldsymbol{v}}_k)\overline{\boldsymbol{s}_1(n)} \;\; \boldsymbol{A}(\widehat{\boldsymbol{v}}_k)\overline{\boldsymbol{s}_2(n)} \;\; \ldots \;\; \boldsymbol{A}(\widehat{\boldsymbol{v}}_k)\overline{\boldsymbol{s}_P(n)}],$$

and

$$\boldsymbol{\psi} = \boldsymbol{\Lambda}\boldsymbol{\beta} + \boldsymbol{\eta} \qquad (18)$$

where $\boldsymbol{\eta}$ is a similar to noise matrix given by (11). The covariance matrix formulated from all the received data is:

$$\boldsymbol{R}_{cov} := \boldsymbol{E}\{\overline{\boldsymbol{Y}}.\overline{\boldsymbol{Y}}^T\} \qquad (19)$$

Applying RRLU factorization to (19), the covariance matrix can be expressed as a product of a unitary matrix and a rank-revealing upper triangular matrix as:

$$\boldsymbol{R}_{cov} = \boldsymbol{L}\boldsymbol{U} = \begin{bmatrix}\boldsymbol{L}_{11} & \mathbf{0}\\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22}\end{bmatrix}.\begin{bmatrix}\boldsymbol{U}_{11} & \boldsymbol{U}_{12}\\ \mathbf{0} & \boldsymbol{U}_{22}\end{bmatrix} \qquad (20)$$

The LURR implies the separation of the eigenvalues of $\boldsymbol{R}_{cov}$ into groups of large and small eigenvalues. The RRLU reveals the numerical rank of $\boldsymbol{R}_{cov}$. Since $\boldsymbol{U}_{22}$ has small norm $\boldsymbol{R}_{cov}$ can be approximated by $\widetilde{\boldsymbol{R}}_{cov}$ as:

$$\tilde{\boldsymbol{R}}_{cov} = \begin{bmatrix} \boldsymbol{L}_{11} \\ \boldsymbol{L}_{21} \end{bmatrix} . [\boldsymbol{U}_{11} \quad \boldsymbol{U}_{12}] = \tilde{\boldsymbol{L}}\tilde{\boldsymbol{U}}$$

The none zero noise (null) space $\boldsymbol{G}$ can be extracted from $\tilde{\boldsymbol{U}}$ as:

$$\tilde{\boldsymbol{X}}.\boldsymbol{G} = \tilde{\boldsymbol{L}}\tilde{\boldsymbol{U}}\boldsymbol{G} = \boldsymbol{0}$$

simply

$$[\boldsymbol{U}_{11} \quad \boldsymbol{U}_{12}] . \begin{bmatrix} \boldsymbol{g_1} \\ \boldsymbol{g_2} \end{bmatrix} = \boldsymbol{0} \qquad (21)$$

Since $\boldsymbol{U}_{11}$ is an invertible matrix

$$\boldsymbol{g}_1 = -\boldsymbol{U}_{11}^{-1}\boldsymbol{U}_{12}\boldsymbol{g}_2$$

Then $\boldsymbol{G}$ can be written as:

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g_1} \\ \boldsymbol{g_2} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{U}_{11}^{-1}\boldsymbol{U}_{12} \\ \boldsymbol{I}_{(L-P)} \end{bmatrix} \boldsymbol{g_2} = \boldsymbol{H}\boldsymbol{g_2} \qquad (22)$$

Using the orthogonal projection, the basis of the noise space $\boldsymbol{H}$ can be derived as:

$$\boldsymbol{H}^{\perp} = \boldsymbol{H}(\boldsymbol{H}^{H}\boldsymbol{H})^{-1}\boldsymbol{H}^{H} \qquad (23)$$

Substitute (23) in (15), one can formulate MUSIC like search pseudo-spectrum. Similar to (17), the channel vectors can be estimated using subspace spanned by LP minor eigenvectors of the inner matrix of pseudo-spectrum. The unknown real multiplier coefficients can be calculated using LS based approach to estimate unknown channel parameters $\boldsymbol{h}_p,\ p = 1,2,\dots,P$.

4. SIMULATION RESULTS

Many computer simulations have been performed to validate the proposed method. The OSTBC of rate ¾ is considered [16]. The RRLU is compared with LS, Capon [13] and RRQR [14]. A multi-access scenario considered with ($P=2$, $N=M=4$, $J=300$, $J_t=5$. The SNR of one the user is assumed to be stronger than the other by 2.5 dB. A slow fading scenario has been assumed where channel is considered roughly constant over a period of time to transmit J data blocks. Each scenario simulated for MC=1000 independent channel realizations. The normalized root mean square error (NRMSE) for p^{th} users is:

$$RMSE^{p} = \frac{1}{MN_t}\sum_{n=1}^{MC}\left(\sqrt{\sum_{i=1}^{2MN}\left|\boldsymbol{h}_p(i) - \hat{\boldsymbol{h}}_p(i)\right|^2}\right)$$

Figure 2 indicates the performance of the proposed estimator for the stronger transmitter. The RRLU based channel estimator is showing approximately 6.0 dB better performance with respect to LS for SNR greater than 5.0dB. Also RRLU algorithm is showing intermediate performance between LS and Capon. A similar behavior of the proposed algorithm with respect to reference methods can be observed in Figure 3 for the weak transmitter case. It is worth to notice that, the performance of the LS method weakens as per increase in SNR. For SNR greater than 5.0 dB Both RRQR and RRLU are showing the same performance while RRLU of half the complexity [12]. Figure 4 shows the performance of the proposed method with respect to number of training blocks. The SNR is considered as 10.0 dB for the stronger transmitter. Again the similar scenario depicted in Figure 5 for relatively weaker transmitter case. The proposed method is performing well even in decreased SNR while LS [13] is quite sensitive with SNR degradation.

5. CONCLUSION

A new technique is proposed for estimating channel parameters in multi-access multi-antenna system by applying the RRLU. This is a non-eigenvector based method used to explore noise subspace. This inherently save amount of arithmetic complexity compared with EVD, SVD or RRQR. The RRLU based subspace estimation methods is used to construct the null-space of signal covariance matrix while the structural behavior of OSTBC is used to explore the null-space for CSI. The CSI estimation with the proposed methods is showing an excellent performance compared with LS based method [13], and a reasonable performance compared with RRQR based method [14] and Capon method [13].

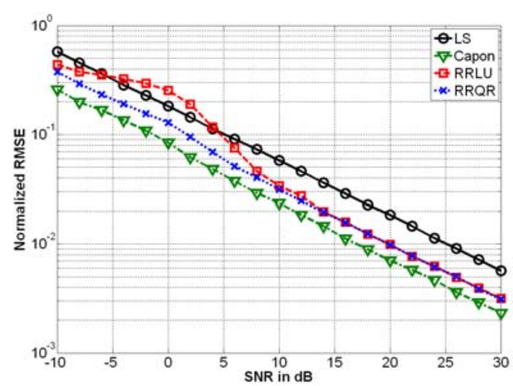

Figure 2. NRMSE versus SNR for the stronger user.

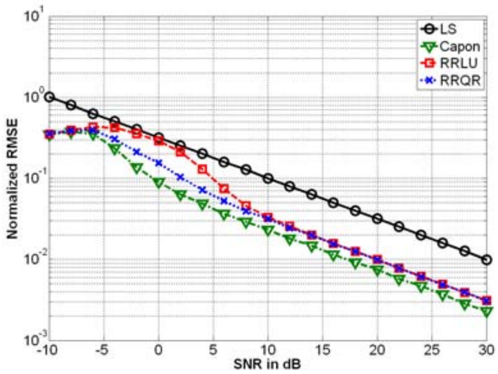

Figure 3. NRMSE versus SNR for the weaker user.

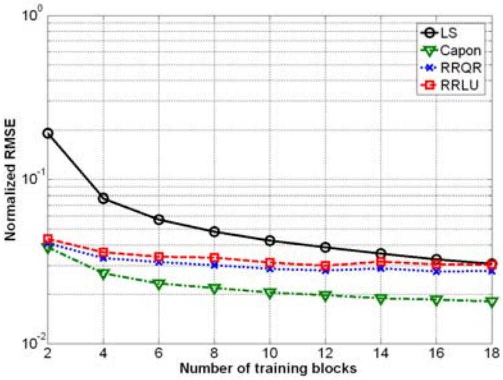

Figure 4. NRMSE versus training blocks for the stronger user.

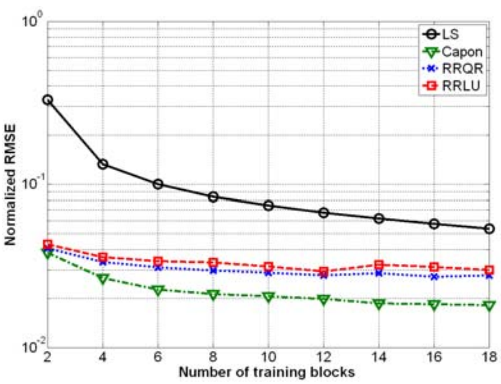

Figure 5. NRMSE versus training blocks for the weaker user.

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