PERFORMANCE ANALYSIS OF QPSK OFDM WITH FADING, FREQUENCY OFFSET, AND CHANNEL ESTIMATION ERROR

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ABSTRACT

OFDM is a promising wireless transmission technique for high speed data transmissions. The potential benefits and obstacles are presented. The effects of fading, frequency offset, and channel estimation error are analyzed both mathematically and through computer simulations.

Keywords: *Orthogonal frequency division multiplexing (OFDM), Frequency offset, Channel estimation.*

I. INTRODUCTION

Multicarrier modulation has a lot of potential for high speed data transmissions [1]. Fundamentally, multicarrier modulation is the concurrent transmission of data over multiple parallel channels. By dividing a wideband frequency channel into subchannels with bandwidth less than the coherence bandwidth of the channel, ISI can be reduced to negligible levels [2]. This reduction is due to the multifold increase of the symbol time, leading to flat fading instead of frequency-selective fading. With flat fading, channel equalization is very simple [3,4], and requires only a one-tap equalizer to compensate for multiplicative channel distortion [4].

Multicarrier modulation can be implemented several different ways. First, frequency-division multiplexing (FDM) as described in [1,2] consists of non-overlapping subchannels each with bandwidth of at least $2*B_n$ where B_n is the baseband bandwidth of the transmitted pulse. The spectral inefficiency of this method is its main disadvantage. We can increase the spectral efficiency by 100% by using a technique known as orthogonal frequency division multiplexing (OFDM), which uses minimum subcarrier separation of B_n. Despite spectral overlap, the subchannels can be perfectly separated at the receiver as shown in [2] as long as the subchannels are orthogonal. Thus the orthogonality property is key in preventing interference from adjacent subchannels. In OFDM implementations, several issues emerge that disrupt orthogonality including:

- Carrier frequency offset
- Sampling frequency offset
- Timing jitter

Additionally, fading and channel estimation error can lead to further performance degradation. In this paper, we will analyze the effect of these phenomena on BER performance for QPSK constellations.

II. OFDM SYSTEM MODEL

Modern DSP technology has enabled a Fast-Fourier Transform (FFT) based implementation of OFDM. As described in [2-4,8,9], the structure of the OFDM transmitter is illustrated in Fig. 1. First, digital data is mapped to a signal constellation.

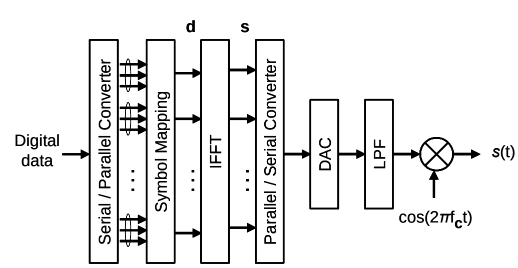

Figure 1. OFDM transmitter structure

For the purposes of reducing length and complexity of this paper, we will be focusing exclusively on QPSK constellations although OFDM implementations are not restricted to using

QPSK. These complex symbols are modulated to baseband subchannels using the FFT and hence can be expressed as follows:

$$s_k = \frac{1}{M}\sum_{i=0}^{M-1} d_i \exp\left(\frac{j2\pi ik}{M}\right),$$
$$k = 0,\ldots, M-1 \quad (1)$$

Then the signal is converted to analog and modulated up to the specified carrier frequency f_c. Conversely, at the receiver the signal comes in with channel distortion and additive white Gaussian noise (AWGN).

Figure 2, depicts the structure of the receiver.

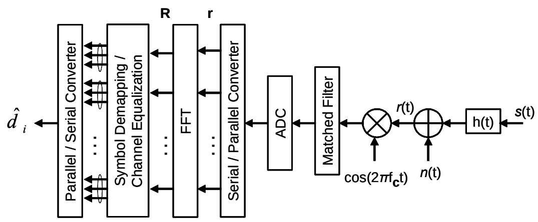

Figure 2. OFDM receiver structure

The received signal is demodulated back to baseband, sampled, and then fed into the FFT for subcarrier demodulation. With perfect symbol timing, carrier frequency, sampling frequency, and no timing jitter. The received data R can be represented as:

$$R_i = H_i d_i + W_i, \qquad i = 0, \ldots, M-1, \quad (2)$$

where H_i denotes the channel response and W_i is the frequency domain AWGN.

In [2], it is shown that the probability of bit error (P_b) for a QPSK constellation with no channel distortion in AWGN is:

$$P_b = Q(\sqrt{2\gamma_b}) \quad (3)$$

A. FADING

Considering the time varying nature of wireless multipath channels, we characterize the channels statistically. Flat fading is the term used to describe the narrowband models of these time-varying channels. As shown in [2], the average probability of bit error in the presence of fading can be expressed as:

$$\overline{P_b} = \int P_b(\gamma_b)p(\gamma_b)d\gamma_b \quad (4)$$

In OFDM, the time-varying channel has effects on the amplitude and phase of the received symbol R_i, increasing the BER of the system. In this paper, we will be primarily concerned with Rayleigh fading. However, Ricean and Nakagami-m=0.5 (worse than Rayleigh) will also be considered.

III. CHANNEL ESTIMATION ERROR

Based on equation (2), if H_i is known perfectly, the maximum-likelihood (ML) receiver would decode the symbol as the following:

$$\hat{d}_i = \frac{R_i}{H_i} \quad (5)$$

However, in OFDM systems (just as many other systems) the channel is not perfectly known. The channel estimate can be obtained using pilot symbols as described in [1, 4, 6]. Channel equalization is performed using this estimate, $\hat{H}_i$. The resulting $\hat{d}_i$ can then be shown to be [4]:

$$\hat{d} = \frac{Hd + W}{\hat{H}} = \frac{1}{|\hat{H}|}\left(|H|e^{i(\angle H - \angle\hat{H})}d + |W|e^{i(\angle W - \angle\hat{H})}\right) = \frac{1}{|\hat{H}|}\left(|H|e^{i\varphi}d + W'\right) \quad (6)$$

Where W' is a zero-mean complex GRV with the variance equal to that of W. By examining equation 6, it is clear that constellation decisions are independent of $|\hat{H}|$. Therefore, the QPSK bit error probability can be expressed conditioned upon the fading SNR (γ_b) and a fixed phase error (φ) as follows:

$$P_b(E|\varphi,\gamma_b) = \frac{1}{2}Q(\sqrt{[\cos\varphi + \sin\varphi]^2\gamma_b}) + \frac{1}{2}Q(\sqrt{[\cos\varphi - \sin\varphi]^2\gamma_b}) \quad (7)$$

Based on equation (4), the unconditional BEP for a QPSK OFDM system with channel estimation error can be shown to be:

$$\overline{P}_b = \int_0^{\infty}\int_0^{\infty}\int_{-\pi}^{\pi} P_b(E|\varphi,\gamma_b)\,p(\gamma_b,\hat{\gamma}_b,\varphi)\,d\varphi\, d\gamma_b\, d\hat{\gamma}_b \quad (8)$$

The joint probability $p(\gamma_b,\hat{\gamma}_b,\varphi)$ is derived in [4] and the resulting $\overline{P}_b$ is shown in [4] to be:

$$\overline{P}_b = \frac{1}{2}\left[1-\frac{1}{2}\frac{\frac{(\rho_1+\rho_2)}{\sqrt{2}}}{\sqrt{1+\frac{1}{2\overline{\gamma}_b}-\frac{(\rho_1-\rho_2)^2}{2}}}-\frac{1}{2}\frac{\frac{(\rho_1-\rho_2)}{\sqrt{2}}}{\sqrt{1+\frac{1}{2\overline{\gamma}_b}-\frac{(\rho_1+\rho_2)^2}{2}}}\right] \quad (9)$$

where ρ_1 is the correlation coefficient for H with respect to $\hat{H}$ and ρ_2 is the correlation coefficient for $\hat{H}$ with respect to H. For a reasonably good estimate, $\hat{H} \approx H$, which leads to $\rho_2 \approx 0$ [4-6].

Analysis for channel estimation error on constellations other than QPSK is provided in [4-6].

IV. CARRIER FREQUENCY OFFSET

Carrier frequency offset (CFO) is the result of a mismatch in carrier frequencies at the transmitter and receiver. This mismatch results in loss of subcarrier orthogonality and introduces intercarrier interference (ICI). The effects of this offset first appear at the receiver where given a CFO ε_c and a phase offset θ_o, the received signal can be represented by:

$$r_n = e^{j\theta_O}\sum_{k\in K} H_k d_k e^{j2\pi n(k+\varepsilon_c)/M} + w_n \quad (10)$$

As shown in [6-8] the result of the FFT with CFO results in:

$$R_i = H_i^{CFO} d_i + I_i^{CFO} + W_i \qquad i = 0, ..., M-1, \quad (11)$$

where H_i^{CFO} represents the distorted channel response, which can be written as [6-8]:

$$H_i^{CFO} = \frac{H_i \sin \pi\varepsilon_c}{M \sin(\pi\varepsilon_c/M)} e^{j(\pi\varepsilon_c(M-1)/M+\theta_O)} \quad (12)$$

and leads to amplitude reduction and phase shift of the symbol d_m. The ICI due to CFO can be expressed as [6-8]:

$$I_i^{CFO} = \sum_{\substack{k=0\\k\neq i}}^{M-1} H_k d_k \frac{\sin \pi\varepsilon_c e^{j(\pi\varepsilon_c(M-1)/M+\theta_O)}}{M \sin(\pi(k-i+\varepsilon_c)/M)} e^{-j(k-i)/M}$$

i = 0,1, …, M-1 (13)

If M is sufficiently large, by the Central Limit Theorem the ICI can be approximated by a zero-mean GRV[6]. Using the technique of establishing an upper bound and using a Taylor series approximation as in [9,6], the normalized ICI variance can be approximated as the following for small ε_c:

$$\sigma_{ICI}^2 \approx \frac{(\pi\varepsilon_c)^2}{3} \quad (14)$$

Using the technique in [6], the average effective SNR for OFDM with channel estimation error, η, and CFO can be represented as:

$$\overline{\gamma}_{eff} = \left[\frac{\sin^2 \pi\varepsilon_c}{M^2 \sin^2(\pi\varepsilon_c/M)} - \sigma_{ICI}^2 - \frac{1}{\overline{\gamma}}\right] \cdot I(\sigma_{ICI}^2 + \frac{1}{\overline{\gamma}}, \sigma_\eta^2) + 1 \quad (15)$$

where σ_η^2 is the variance of η, and the $I(x,y)$ is the definite exponential integral defined as:

$$I(x,y) = \begin{cases} \frac{1}{x}, & y=0 \\ \int_0^{\infty} \frac{1}{y(t+x)} e^{-t/y} dt = e^{x/y}\Gamma\left(0,\frac{x}{y}\right), & y>0 \end{cases} \quad (16)$$

where Γ(a,b) is the incomplete gamma function.

The corresponding average BEP for a QPSK OFDM system with with CFO and channel estimation error in fading can be expressed by combining equations (3, and 15) as:

$$\overline{P}_b = \int P_b(\overline{\gamma}_{eff})\,p(\gamma_b)\,d\gamma_b \quad (17)$$

Note that the phase rotation due to the residual frequency offset is compensated for and included in channel estimation.

V. SAMPLING FREQUENCY OFFSET

Sampling frequency offset (SFO) is the mismatch in frequencies of the A/D and D/A converters at the transmitter and the receiver. Similar to CFO, SFO results in amplitude reduction, phase shift, and the loss of orthogonality between subcarriers. Using a derivation equivalent to CFO the effects of SFO can be expressed as [8,9]:

$$R_i = H_i^{SFO} d_i + I_i^{SFO} + W_i \qquad i = 0, \ldots, \text{M-1}, \tag{17}$$

where

$$H_i^{SFO} = \frac{H_i \sin \pi i \varepsilon_s}{M \sin(\pi i \varepsilon_s / M)} e^{j(\pi \varepsilon_s i(M-1)/M + \theta_O)} \tag{18}$$

and

$$I_i^{SFO} = \sum_{\substack{k=0 \\ k \neq i}}^{M-1} H_k d_k \frac{\sin \pi k \varepsilon_s e^{j(\pi \varepsilon_s k(M-1)/M + \theta_O)}}{M \sin(\pi[k(1+\varepsilon_s) - i]/M)} e^{-j(k-i)/M}$$

i = 0,1, …, M-1 (19)

Note that the effects of SFO are very similar to that of CFO except that the amplitude reduction, and phase offset are not equal for all subcarriers. The effects of SFO get larger as the number of subcarriers increase.

The normalized ICI variance can be shown to be:

$$\sigma_{ICI,i}^2 \approx \frac{(\pi i \varepsilon_s)^2}{3} \tag{20}$$

Thus the average effective SNR per subchannel, i, can be represented as:

$$\bar{\gamma}_{eff,i} = \left[\frac{\sin^2 \pi i \varepsilon_s}{M^2 \sin^2(\pi i \varepsilon_s / M)} - \sigma_{ICI,i}^2 - \frac{1}{\bar{\gamma}} \right] \cdot I(\sigma_{ICI,i}^2 + \frac{1}{\bar{\gamma}}, \sigma_\eta^2) + 1 \tag{21}$$

and the corresponding average BEP can be expressed as

$$\bar{P}_b = \frac{1}{M} \sum_{i=0}^{M-1} \int P_b(\bar{\gamma}_{eff,i}) p(\gamma_b) d\gamma_b \tag{22}$$

Note that the phase offset is incremental and is not corrected by channel estimation, but can be easily tracked and compensated for since it is a linear phase increase. Therefore the effects of phase rotation due to SFO will be neglected in BEP analysis.

VI. TIMING JITTER

Timing jitter is the result of noisy sampling. The A/D and D/A may experience slight errors due to thermal noise. This phenomenon is different than sampling frequency offset because the difference in sampling frequencies is not constant. However, timing jitter also disrupts the orthogonality of subcarriers in a way similar to maligned sampling in SFO. In [3], timing jitter is mathematically characterized and the effect on BEP is demonstrated. These results are not recreated in this paper and are only mentioned for completeness in OFDM characterization.

VII. NUMERICAL EXAMPLES

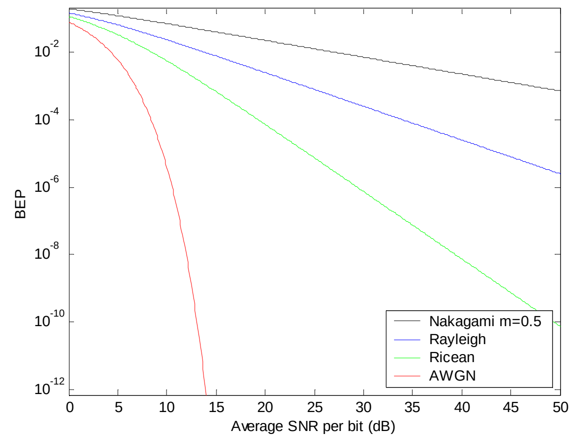

Figure 3. Average BEP in a variety of fading channels (QPSK).

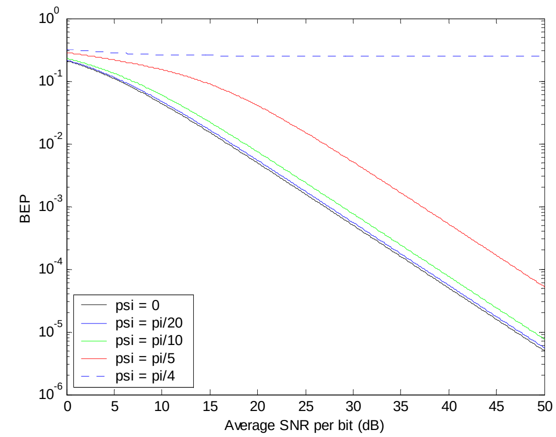

Figure 4. Average BEP conditioned on a fixed channel estimation phase error , φ, in Rayleigh fading channels (QPSK).

As shown in [2], using the Craig's alternate form of the Q-function and expressing the Rayleigh distribution as a moment generating function, we can express all of the P_b formulas presented in this work as closed form integrals which are easily calculated. The results of these calculations are presented in the graphs above.

In Figure 4, the effect of a fixed channel estimation phase error is investigated. Note that no error floor exists until the phase error gets above the adjacent symbol boundary (pi/4 for QPSK).

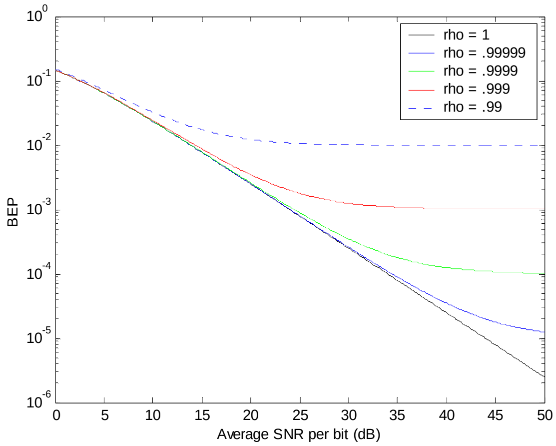

Figure 5. Average (unconditioned) BEP in Rayleigh fading channels (QPSK) with varying degrees of channel estimation error as measured by the correlation coefficient ρ_1.

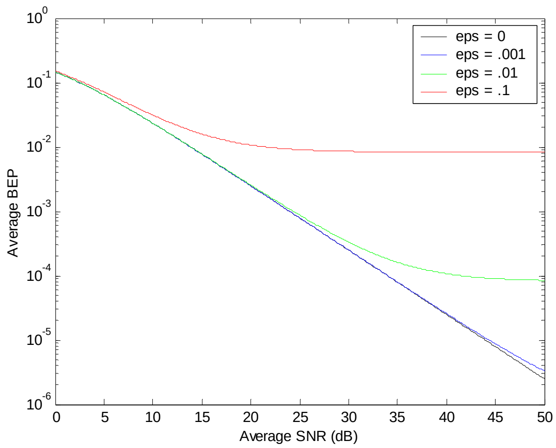

Figure 6. Average BEP with a carrier frequency offset, ε_c, in Rayleigh fading channels (QPSK).

However, a fixed channel estimation phase error is not a realistic phenomenon. To understand the effect of channel estimation error in a real system we must examine the BER in terms of the correlation coefficient between the channel and the channel estimate as in Figure 5. Note that error floors begin to form even with the slightest channel estimation error.

In Figure 6 and 7, we show the effect of carrier frequency offset and sampling frequency offset in OFDM systems. The ICI due to the deterioration of orthogonality creates an error floor in both types of frequency offset. The OFDM system is shown to be much more sensitive to sampling frequency offset due to the escalation of the ICI as the number of subchannels increases.

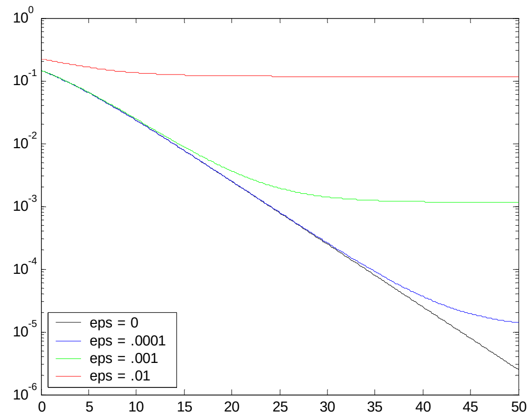

Figure 7. Average BEP with a sampling frequency offset, ε_s, in Rayleigh fading channels (QPSK) M=64.

VIII. CONCLUSIONS

The bandwidth efficiency of OFDM comes with a complexity tradeoff. System designers must account for and mimimize channel-estimation error, CFO, SFO, and timing jitter to preserve system performance in the presence of fading.
The examples provided are for OFDM-QPSK and are intended to serve as a tool for OFDM system design.

FUTURE WORK

I hope to examine the BEP of OFDM in the presence of multiple simultaneous impairments, including combinations of channel-estimation error, CFO, and SFO. This is intended to aid system designers by providing a robust system design in the face of all three impairments.

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