NEW DETERMINISTIC CODE MATCHED INTERLEAVER FOR TURBO CODES

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ABSTRACT

Turbo codes offer extraordinary performance, especially at low signal to noise ratio, due to a low multiplicity of low weight code words. The interleaver design is critical in order to realize an apparent randomness of the code, thus further enhancing its performance especially for short block frames. This paper presents a new algorithm of obtaining a code matched interleaver leading to very high minimum distance and performances. The design method is described in depth, and the simulation results are plotted against the Long Term Evolution (LTE) standard interleaver.

Keywords: *Interleaved coding, modulation coding, code matched, turbo codes, error correcting codes*

1. INTRODUCTION

Turbo codes represent a powerful, yet flexible class of error correcting codes. It has been proven that these codes offer remarkable performances especially over low SNR domains. The low error rate of the turbo coding scheme is achieved by combining two digital IIR (Infinite Impulse Response) filters (convolutional encoders). A non-uniform interleaver scrambles the ordering of the input bits of the second digital filter.

Turbo codes behave very well at a low signal to noise ratio (SNR) because of their sparse distance spectrum, thus generating a low multiplicity of low-weight code-words and a large multiplicity of average weight code-words. This phenomenon is known as spectral thinning [12]. This kind of structure of the distance spectrum means that at low SNR, the error correction performance is influenced by the large number of medium weight code-words, whereas at medium to high SNR, the bit error rate (BER) and frame error rate (FER) curves are determined by the few low weight code-words. In this situation, turbo codes experience an error-floor limitation, because of their low minimum distance [7].

In order to lower the error floor, the increase of the free distance is mandatory. This can be accomplished either through the increase of the size of the interleaver, or either through proper interleaver design. Increasing the interleaver size leads to longer delays and larger memory requirements, fact that is intolerable in standards such as Digital Video Broadcasting (DVB) or Universal Mobile Telecommunication Standard (UMTS). Furthermore, some mobile radio systems have short frames, typically under 300, but require a high Quality of Service (Qos) [2]. For the above mentioned situations the best option is to use a code matched interleaver.

2. OVERVIEW OF CODE-MATCHED INTERLEAVER DESIGNS

The code-matched interleaver design technique attempts to adapt the interleaver to a particular turbo code. The basic idea behind this approach is to generate the interleaver in such a manner that spectral lines which produce low-weight code-words are eliminated [18]. There are two major code-matching modalities that can lead to very good code-matching interleavers.

The first one is based on estimating the free distance using some relevant error patterns of low weight (usually 2,4 or 6). Notable results have been reported in [6], [11]. The drawback of such an approach is given by the complexity of the design stages. Furthermore, only simple low-weight input patterns have been considered due to the computational complexity. This means that while trying to increase the weight of the code-words generated by low-weight input information patterns, the actual free distance is generated by medium weight input sequences.

The second approach refers to the generation of a whole family of interleavers, following a certain deterministic algorithm. The code-matching is done by defining a distance spectrum related cost-function, that selects the best interleaver from the whole family [5],[16]. In this situation, the actual limitation in the improvement of the distance spectrum is due to the algebraic structure of the interleavers, that leads to a poor degree of randomness in the structure of the interleaver.

3. SYSTEM MODEL

The turbo encoder used in the simulation is symmetrical and uses two identical convolutional encoders with the feed-forward and feed-back polynomials equal in octal to 15 and 13 respectively. The systematic output form the second encoder is punctured.

The turbo encoder used has a post-interleaver trellis termination (flushing), which means that both convolutional encoders are independent from one another reset. This is done by commuting the two switches from the on state (after a number of clock cycles equal to the size of the interleaver) to the off state (for a number of three clock cycles, which is equal to the memory of the constituent convolutional encoders)[17].

In order to increase the coding gain for the fading channel, BICM (Bit Interleaved Coded Modulation)[14]. In this situation a random interleaver is used between the encoder and the BPSK modulator.

Two sets of simulations are run, supposing that the channel is either an Additive White Gaussian Noise (AWGN) channel, or either a Rayleigh Multiplicative Fading (RMF) channel (Figure 1). The matched filter is required in order to translate the receive symbols into a Log Likelihood Ratio (LLR) form.

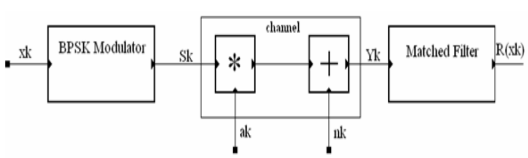

Figure 1. The Channel Model

Where a_k (channel gain) =1 if AWGN, or a Rayleigh random variable if RMF and n_k is the Gaussian noise.

The turbo decoder is based on a SW-SISO (sliding window-soft input soft output) iterative algorithm with two MAP (Maximum A Posteriori) decoders implemented in the log-domain. The LLR of the data bits is calculated by each MAP decoder and the results are passed from one decoder to the other (iterations are performed). The performance of this kind of approach depends on the number of iterations that take place between the two MAP decoders. In the simulation scenarios, the log-map approximation is used, using a look-up table and finally, a hard decision is performed on the value of the LLR [13].

4. THE LTE STANDARD INTERLEAVER

The LTE (Long Term Evolution) standard is a 4G communication standard developed by of the 3gpp (3^{rd} Generation Partnership Project) organization, that is due to replace the current 3G UMTS (Universal Mobile Telecommunication Standard) [15]. In terms of the error correcting coding scheme involved, the most important breakthrough of this standard was the replacement of the UMTS standard interleaver with a new LTE interleaver, derived from the QPP (Quadratic Permutation Polynomial) interleavers. This type of interleavers is among the best one known for turbo codes [16].

5. THE PROPOSED CODE-MATCHED INTERLEAVER

The main idea behind the proposed code matched interleaver design is to improve the last three spectral lines. In order to generate a high performance code-matched permutation, a method of computing the distance spectrum of the specific code is necessary. Furthermore, because in the simulations, the post-interleaver trellis termination is considered, the distance spectrum calculation algorithm, has to take this aspect into account as well. There are several distance measurements methods, such as the true distance measurement method [8], the error-impulse method [1], the all-iterative decoding method [9] or the double impulse iterative decoding method [3]. From all of these, the most reliable is the true distance measurement method, which is able to reliably compute the first three terms of the distance spectrum. The disadvantage of this approach is that the complexity increases severely with the free distance (which in its turn is dependent on the interleaver's length).

The design algorithm for the code matched interleaver can be synthesized as follows:

1). Start from a given interleaver. The initial interleaver used in the simulations is a high spread random interleaver, which generates an interleaver with a certain S_{new} spreading factor [4], but the algorithm can be applied to any kind of interleaver, thus providing increased flexibility.

2). Calculate the S_{new}-spread of the interleaver and the first three terms of the distance spectrum {d(1),n(1),w(1)},{d(2),n(2),w(2)} and {d(3),n(3),w(3)} taking into consideration the post-interleaver flushing termination. Furthermore, the normalized dispersion γ and the S spreading factor are computed [10]. A cost function I is defined, where $I=(S+S_{new})^* \gamma$

3). For the desired number of iterations perform the following operations:

4). With two indexes i and j that are incrementally built, the interleaver can be completely scanned, and the positions given by these indexes are swapped

5). The swap is kept only if a series of conditions in the following order are met:

a). If the new interleaver doesn't have a spreading factor at least equal to the initial S_{new} spread value, the swap is discarded and the algorithm returns to step 4, otherwise jump to 5.b

b). The first term of the distance spectrum is computed. If there is an improvement in the sequence d(1)-n(1)-w(1) (if FER optimization is desired) or in the sequence d(1)-w(1)-n(1) (if BER optimization is desired) the swap is kept and the algorithm returns to step 4. In case there is no change in the distance spectrum, the algorithm computes the second term of the distance spectrum and makes the same evaluation, keeping only the swap that improves the second term. If still there is no improvement in the distance spectrum the third term is computed and the same procedure is applied. The swap is kept if there is an improvement and discarded if the distance spectrum is damaged. In case there is no change in the distance spectrum after computing the first three terms, then the algorithm jumps to step 5.c

c). The normalized dispersion γ, the S spreading factor are computed and the cost function $I=(S+S_{new})^* \gamma$ are computed. The swap is kept if the cost function suffers an improvement, otherwise the algorithm jumps to step 4.

The algorithm has several advantages over other code matched interleavers. First of all, there is the possibility to start from any kind of interleaver. The choice of the start interleaver influences the final performance of the code matched interleaver. Provided a very good interleaver is chosen as the start-up structure, then the performance would be improved, but the generation time will be less, because a lower number of iterations would have to be performed. Secondly, the design offers flexibility, not only in terms of the number of iterations that are user definable, but also from the point of view of FER or BER optimization. Third of all, a real distance spectrum calculation algorithm is used, instead of a distance spectrum estimation using various error patterns. Finally, in case a non-random interleaver is used as the starting interleaver, the design is fully deterministic, thus having the advantage of being easy to implement in VLSI structures, with a low memory capability.

6. SIMULATIONS AND RESULTS

The simulations were run for two code matched interleavers, deriving from the high-spread random interleavers of lengths equal to 64 and 152, in case of both Additive White Gaussian Noise (AWGN) and Rayleigh Multiplicative Fading (RMF). The number of iterations for generating the interleavers was set to 6, the number of decoder iterations was set to 12, the modulation used was Binary Phase Shift Keying (BPSK) with Bit Interleaved Coded Modulation (BICM). The constituent convolutional encoders chosen were identical and equal to (15/13) in octal with post interleaver trellis termination and the decoding algorithm was log-MAP. The the BER and FER curves are shown in figures 2,3,4 and 5 which depict the proposed code matched interleaver against the Long Term Evolution (LTE) standard interleaver. Additionally, tables 1, 2, 3, 4, 5 and 6 illustrate the first terms of the distance spectrum computed for post-interleaver trellis termination and the most important parameters as computed for the high-spread random, matched high-spread random and LTE interleavers for the simulated lengths. The results of the simulations yield to a clear improvement of the performance, due to the better distance spectrum and the proper choice of the cost function I, which maximizes both the S spread and the dispersion γ.

Table 1.LTE interleaver with length=64

Parameter	S	S_{NEW}	γ	d_{FREE}	n_{FREE}	ω_{FREE}
Value	4	8	0.088	12	2	4

Table 2.LTE interleaver with length=152

Parameter	S	S_{NEW}	γ	d_{FREE}	n_{FREE}	ω_{FREE}
Value	6	12	0.038	15	1	1

Table 3.High Spread Random interleaver with length=64

Parameter	S	S_{NEW}	γ	d_{FREE}	n_{FREE}	ω_{FREE}
Value	4	9	0.712	14	2	6

Table 4.High Spread Random interleaver with length=152

Parameter	S	S_{NEW}	γ	d_{FREE}	n_{FREE}	ω_{FREE}
Value	7	14	0.756	18	4	8

Table 5.Code matched interleaver with length=64

Parameter	S	S_{NEW}	γ	d_{FREE}	n_{FREE}	ω_{FREE}
Value	4	9	0.812	19	1	1

Table 6.Code matched interleaver with length=152

Parameter	S	S_{NEW}	γ	d_{FREE}	n_{FREE}	ω_{FREE}
Value	7	14	0.799	23	2	2

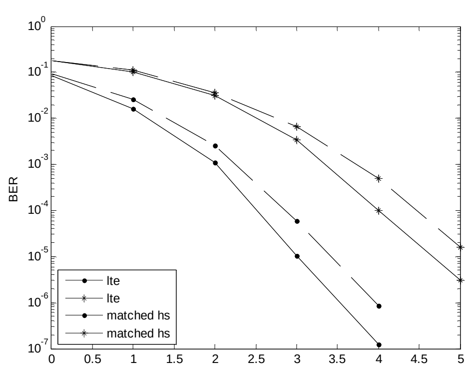

Figure 2. BER for L1=64 for AWGN (.) and RMF (*)

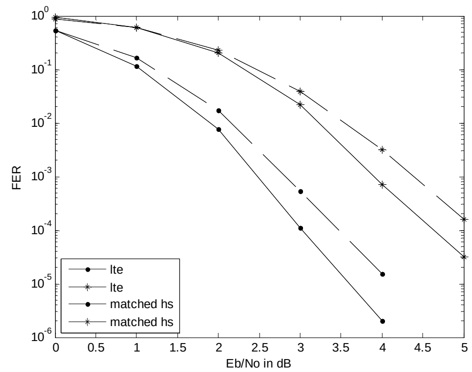

Figure 3. FER for L1=64 for AWGN (.) and RMF (*)

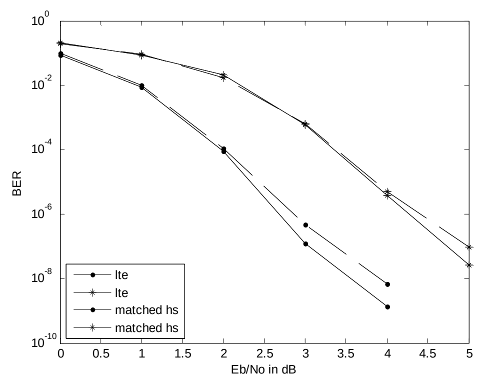

Figure 4. BER for L2=152 for AWGN (.) and RMF (*)

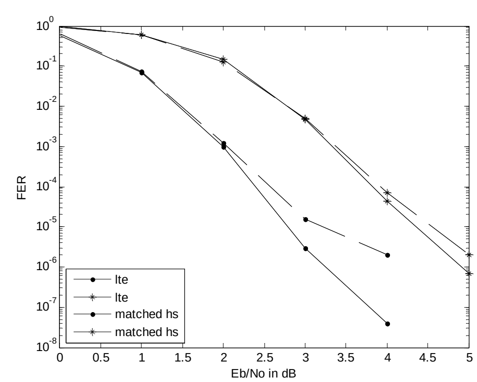

Figure 5. FER for L2=152 for AWGN (.) and RMF (*)

7. CONCLUSIONS AND FUTURE WORK

This paper presents a new code matched interleaver design. It's performances are evaluated against the LTE standard interleaver, which is one of the best known interleaver types for short frames. The results of the simulations show a clear improvement in each of the considered scenario. Among the advantages of this new design algorithm are flexibility, ease of implementation and the performance.

The main drawback of the proposed design refers to the non-algebric structure of the interleaver, which makes the algorithm difficult to implement using logical circuitry. Another limitation is given by the non-liniar variation of the generation time with the interleaver length. This feature can be surpassed by increasing the spreading values of the starting interleaver, before performing the code-matching operation.

Future work should address to the study of this code matched interleaver for longer frame sizes and the way its performance improves, as referred to the number of iterations performed to generate the permutation.

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