Journal of Theoretical and Applied Information Technology

www.jatit.org

COMPARISON OF BER BETWEEN UNCODED SIGNAL AND CODED SIGNAL (USING CONVOLUTION CODE) OVER SLOW RAYLEIGH FADING CHANNEL

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ABSTRACT

Fading problem is a major impairment of the wireless communication channel. To mitigate the fading problem and to have reliable communications in wireless channel, channel coding technique is often employed. In this paper the BER (Bit Error Rate) performances is shown from analytically and by means of simulation for Rayleigh fading multipath channels. Here the convolution code is used as a channel coding technique. The performance of convolution encoded binary phase shift keying (coded BPSK) with coherent detection and Viterbi decoding is investigated. Obviously the performance of coded signal in terms of BER is better than uncoded signal and in this research; the main focus is to investigate how much improvements of BER is occurred using convolution code with BPSK for Rayleigh fading channel. The simulation results are shown here graphically.

Keywords: *BER, BPSK, Convolution code, Rayleigh fading channel, SNR, Viterbi decoding.*

1. INTRODUCTION

The fading phenomenon occurs in radio transmission channels. It is due to the presence of multipaths that varies during the transmission [1]. There are many techniques used to compensate for fading channel impairments [2], [3]. Use of error control coding is one of the important techniques. It is used to enhance the efficiency and accuracy of information transmitted. In a communication system, data is transferred from a transmitter to a receiver across a physical medium of transmission or channel. The channel is generally affected by noise or fading which introduces errors in the data being transferred. Channel coding is a technique used for correcting errors introduced in the channel. It is done by encoding the data to be transmitted and introducing redundancy in it such that the decoder can later reconstruct the data transmitted using the redundant information. If the error control coding is doing its job properly, the bit error rate at the output should be less than the bit error probability at the decoder input [5]. In this paper convolution code is used as an error control code. The viterbi algorithm was proposed in 1967 as a method of decoding convolution codes [6]. In this paper this viterbi decoding is considered and the bit error rate performance is evaluated for convolution code and it is compared with the bit error rate for uncoded signal under slow rayleigh fading channel.

2. FADING EFFECTS DUE TO DOPPLER SPREAD

Depending on how rapidly the transmitted base signal changes as compared to the rate of change of the channel, a channel may be classified either as a fast fading of slow fading channel.

2.1 Slow fading

In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal. In this case, the channel may be assumed to be static over one or several reciprocal bandwidth intervals. In the frequency domain, this implies that the Doppler spread of the channel is much less, than the bandwidth of the base band signals.

It should be clear that the velocity of the mobile (velocity of objects in the channel) and the baseband signaling determines whether a signal undergoes fast fading or slow fading.

3. CHANNEL

3.1 Additive White Gaussian Noise (AWGN) Channel

In communications, the AWGN channel model is one in which the only impairment is the linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for the phenomena of fading, frequency selectivity, interference, nonlinearity or dispersion. However, it produces simple, tractable mathematical models which are useful for gaining insight into the underlying behavior of a system before these other phenomena are considered. AWGN is commonly used to simulate background noise of the channel under study, in addition to multipath, terrain blocking, interference, ground clutter and self-interference that modern radio systems encounter in terrestrial operation.

3.2 Rayleigh Distribution

In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of an individual multipath component. It is well known that the envelope of the sum two-quadrature Gaussian noise signals obeys a Rayleigh distribution.

The narrow band, n(t) may also be represented in terms of its envelope and phase component as follows

$$n(t) = r(t)\cos[2\pi f_c t + \psi(t)]) \quad (1)$$

where

$$r(t) = [n_I^2(t) + n_Q^2(t)]^{\frac{1}{2}} \quad (2)$$

and $$\psi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right) \quad (3)$$

The function r(t) is called the envelope of n(t), and the function ψ(t) is called the phase of n(t).The sample functions n_I(t) and n_Q(t) refer to random processes N_I(t) and N_Q(t), respectively. Let X and Y denote random variables obtained by observing the random processes N_I(t) and N_Q(t) at some fixed time, respectively. These two random variables are represented by sample values x and y. We note that X and Y are statistically independent and Gaussian distributed with zero mean and variance σ_N^2. We may thus express their joint probability density function as

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_N^2}\exp\left(-\frac{x^2+y^2}{2\sigma_N^2}\right) \quad (4)$$

Define the bivariate function transformation

$$x = r\cos\psi \quad (5)$$

and

$$y = r\sin\psi \quad (6)$$

The Jacobian of the transformation is defined by

$$J(r,\psi) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \psi} & \frac{\partial y}{\partial \psi} \end{vmatrix} = \begin{vmatrix} \cos\psi & \sin\psi \\ -r\sin\psi & r\cos\psi \end{vmatrix} = r\cos^2\psi + r\sin^2\psi = r \quad (7)$$

The R and ψ denote the random variables resulting from the transformation of X and Y. The joint probability density function of R and ψ is given by

$$f_{R,\Psi}(r,\psi) = f_{X,Y}(x,y)\,J(r,\psi) = \frac{r}{2\pi\sigma_N^2}\exp\left(-\frac{r^2}{2\sigma_N^2}\right) \quad (8)$$

Where it has made use of Eqs. 4 - 6. This probability density function is not dependent on the angle ψ, which means that the random variables R and ψ are statistically independent. We may thus express $f_{R,\Psi}(r,\psi)$ as the product of $f_R(r)$ and $f_\psi(\psi)$. In particular, the random variable ψ is uniformly distributed inside the range 0 to 2π, as shown by

$$f_\Psi(\psi) = \begin{cases} \frac{1}{2\pi} & ;\ 0 \le \psi \le 2\pi \\ 0 & ;\ \text{elsewhere} \end{cases} \quad (9)$$

This leaves the probability density function of R as

$$f_R(r) = \begin{cases} \frac{r}{\sigma_N^2}\exp\left(-\frac{r^2}{2\sigma_N^2}\right); & r \ge 0 \\ 0 & ;\ \text{elsewhere} \end{cases} \quad (10)$$

Where σ_N^2 is the variance of the original narrow-band noise process N(t). A random variable having the probability density function is said to be Rayleigh distributed [4].

4. MODULATION AND DEMODULATION USING CONVOLUTION CODE

4.1 Coherent Binary PSK

In a coherent binary PSK system, the pair of signals, $S_1(t)$ and $S_2(t)$, used to represent binary symbols 1 and 0 respectively, are defined by

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \quad (11)$$

$$s_2(t) = \sqrt{2\frac{E_b}{T_b}}\cos(2\pi f_c t + \pi)$$
$$= -\sqrt{2\frac{E_b}{T_b}}\cos(2\pi f_c t) \quad (12)$$

Where 0 < t < T_b, and E_b is the transmitted signal energy per bit. In order to ensure that each transmitted bit contains an integral number of cycles of the carrier wave, the carrier frequency f_c is chosen equal to n_c/T_b for some fixed integer n_c.

A pair of sinusoidal waves that differ only in a relative phase-shift of 180 degrees, as defined above, are referred to as antipodal signals.

From equation 11 and 12, it is clear that there is only one basis function of unit energy, namely

$$\Phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t); \quad 0 \le t < T_b \quad (13)$$

Then we may expand the transmitted signals $s_1(t)$ and $s_1(t)$ in terms of $\Phi_1(t)$ as follows,

$$s_1(t) = \sqrt{E_b}\Phi_1(t); \quad 0 \le t < T_b \quad (14)$$

and

$$s_2(t) = -\sqrt{E_b}\Phi_1(t) \quad 0 \le t < T_b \quad (15)$$

The coordinates of the message points equal

$$s_{11} = \int_0^{T_b} s_1(t)\,\Phi_1(t)dt = +\sqrt{E_b} \quad (16)$$

And

$$s_{21} = \int_0^{T_b} s_2(t)\,\Phi_1(t)\,dt = -\sqrt{E_b} \quad (17)$$

The message point corresponding to s_1(t) is located at $s_{11} = +\sqrt{E_b}$, and the message point corresponding to $s_2(t)$ is located at $s_{21} = -\sqrt{E_b}$.

To realize a rule for making a decision in favor of symbol 1 or symbol 0, specifically, we must partition the signal space into two regions:

1. The set of points closest to the message point at $+\sqrt{E_b}$
2. The set of points closest to the message point at $-\sqrt{E_b}$ [4].

Without loss of generality, consider the detection of a symbol transmitted over the time interval [0,T_b]. The transmitted signal is

$$x(t) = \begin{cases} \sqrt{E_b}\Phi_1(t), & \text{For symbol '1'} \\ -\sqrt{E_b}\Phi_1(t), & \text{For symbol '2'} \end{cases} \quad (18)$$

Where $\Phi_1(t)$ is the basis function.

In an AWGN channel, the received signal is *r(t) = x(t) + n(t)*; where n(t) represents the white Gaussian noise process with zero mean and two-sided psd(power spectral density) $N_0/2$. Coherent BPSK reception over an AWGN channel, as shown in Figure 3.4

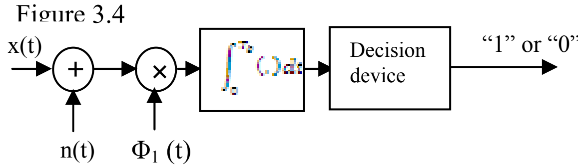

Figure 1: Coherent reception of BPSK in an AWGN channel

The output of the correlator is a Gaussian random variable with $r_1 = \int_0^{T_b} r(t)\Phi_1(t)dt$ is a Gaussian random variance $\sigma_N^2 = N_0/2$ and conditional means $\mu_1 = \sqrt{E_b}$ given that "1" was sent and $\mu_0 = -\sqrt{E_b}$ given that "0" was sent. That is, the conditional probability density function (pdf) of the decision variable r_1 is given by

$$f_{r_1}(x) = \frac{1}{\sqrt{2\pi}\sigma_N}\exp\left[-\frac{(x-\mu_i)^2}{2\sigma_N^2}\right], \; i = 0,1; \; -\infty < x < \infty \quad (19)$$

Corresponding to the decision regions shown in Figureure 3.4, the decision rule is as follows:

$$\begin{cases} \text{If } r \ge 0, \text{ symbol "1" was sent;} \\ \text{If } r < 0, \text{ symbol "0" was sent.} \end{cases}$$

With equally likely symbols "1" and "0", the probability of symbol (bit) error, or bit error rate (BER), is

P_b = P(r$\geq 0|symbol$ "0" was sent) P (symbol "0" was sent) + P(r$<$0|symbol "1" was sent) P (symbol "1" was sent)

$$= \frac{1}{2}\left\{\int_0^{\infty} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma N} exp\left[\frac{(x-\mu_0)^2}{2\sigma_N^2}\right] dx + \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma N} exp\left[-\frac{(x-\mu_1)^2}{2\sigma_N^2}\right] dx\right\}$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right). \qquad (20)$$

Where $Q(t) = \frac{1}{\sqrt{2\pi}}\int_t^{\infty} exp\,(-z^2/2)dz$ is called the Q-function, which defines the area under the standard Gaussian (with zero mean and unit variance) tail.

We consider a stationary and slow fading channel where
(a) the delay spread introduced by the multipath propagation environment is negligible compared with the symbol interval(hence, the channel does not introduce inter symbol interference) and
(b) channel fading status does not change much over a number of symbol intervals. The first condition means that the effect of the channel can be represented by a complex gain α(t)exp$[j\theta(t)]$, where α(t) is the amplitude fading and Θ(t) is the phase distortion. Given that a signal x(t), with symbol interval T_s, is transmitted, the received signal is

$$r(t) = \alpha(t)\, exp\,[j\theta(t)]x(t) + n(t) \qquad (21)$$

Where n(t) is white Gaussian noise with zero mean and two-sided power spectral density $N_0/2$. The second condition means that it is possible for the receiver to estimate Θ (t) and remove it. As a result, in the following BER performance analysis, we assume Θ (t) =0, without loss of generality.
Consider a cellular system where the effect of propagation path loss and shadowing on the received signals is compensated by power control and the received signals experienced only a multipath Rayleigh fading. In other words, we consider that the transmitted signal has a constant bit energy E_b and the received signals has instantaneous bit energy equal to $\alpha^2 E_b$, where α is the amplitude fading in the symbol interval. Based on the transmission performance analysis for an AWGN channel, for a given modulation scheme, its probability of bit error can be represented as a function, P_b(.), of the received bit energy to the one-sided noise power spectral density, E_b/N_0, (or SNR/bit), denoted by γ^b. In the following, it is extended the analysis to a fading channel in two steps: (a) to find the conditional probability $P_{b|\alpha}(\gamma^b|\alpha)$, given the amplitude fading α; and (b) to average the conditional probability $P_{b|\alpha}(\gamma^b|x)$ with the respect to the pdf of α at α=x, in order to take into account the effect of all possible amplitude fading values on the transmission performance.
Therefore,

$$P_b(\overline{\gamma_b}) = \int_{-\infty}^{\infty} P_{b|\alpha}(\gamma^b|x)\, f_\alpha(x)\, dx \qquad (22)$$

Where $\overline{\gamma_b}$ is the average received SNR/bit with respect to α^2,

$$\overline{\gamma_h} = \int_{-\infty}^{\infty} x^2\, \gamma_b f_a(x)dx$$
$$=\gamma_h E(\alpha^2) \qquad (23)$$

And $f_\alpha(.)$ is the pdf of the amplitude fading α. For a Rayleigh fading channel α follows a Rayleigh distribution with pdf

$$f_\alpha(x) = \begin{cases} \frac{x}{\sigma_\alpha^2}\exp\left(-\frac{x^2}{2\sigma_\alpha^2}\right); & x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

It can be derived that E(α^2) = $2\sigma_\alpha^2$. In this case, we have $\overline{\gamma_b} = 2\sigma_\alpha^2\gamma_b$. For coherent BPSK, QPSK, and MSK, from Eq. 20, we have

$$P_{b|\alpha}((\gamma^b|x)) = Q\left(\sqrt{2\sigma_\alpha^2\gamma_h}\right) \qquad (24)$$

Substituting Eqs. 22 and Eqs. 21 into Eqs. 20, we have

$$P_b(\overline{\gamma_b}) = \int_0^{\infty} Q(\sqrt{2x^2\gamma_b})\,\frac{x}{\sigma_\alpha^2}\exp(-\frac{x^2}{2\sigma_\alpha^2})\, dx$$
$$= \frac{1}{2\sigma_\alpha^2}\int_0^{\infty} Q\,(\sqrt{2y\gamma_b})\,\exp\,(-\frac{y}{2\sigma_\alpha^2})dy$$
(where $y=x^2$)
$$= \frac{1}{2\sigma_\alpha^2}\int_0^{\infty}\left[\frac{1}{\sqrt{2\pi}}\int_{\sqrt{2y\gamma_b}}^{\infty}\exp(-u^2/2)du\right]\exp\,(-\frac{y}{2\sigma_\alpha^2})dy$$
$$= \frac{1}{2\sigma_\alpha^2}\frac{1}{\sqrt{2\pi}}\int_{\sqrt{2y\gamma_b}}^{\infty}\exp(-u^2/2)\left[\int_0^{u^2/2\gamma_b}\exp\,(-\frac{y}{2\sigma_\alpha^2})dy\right]du$$
$$= \frac{1}{2}\left[1 - \sqrt{\frac{\overline{\gamma_b}}{1+\overline{\gamma_b}}}\right] \qquad (25)$$

By using equation 25, the probability of error in a slow flat fading channel can be evaluated. It can be that for coherent binary PSK [1].

4.2. Convolution Encoding and Decoding

Convolution codes are usually described using two parameters: the code rate and the constraint length. The code rate, k/n, is expressed as a ratio of the number of bits into the convolutional encoder (k) to the number of channel symbols output by the convolutional encoder (n) in a given encoder cycle. The constraint length parameter, K, denotes the "length" of the convolutional encoder, i.e. how many k-bit stages are available to feed the combinatorial logic that produces the output symbols. Closely related to K is the parameter m, which indicates how many encoder cycles an input bit is retained and used for encoding after it first appears at the input to the convolutional encoder. The m parameter can be thought of as the memory length of the encoder. In this thesis, and in the example source code, ½ convolutional code is used.

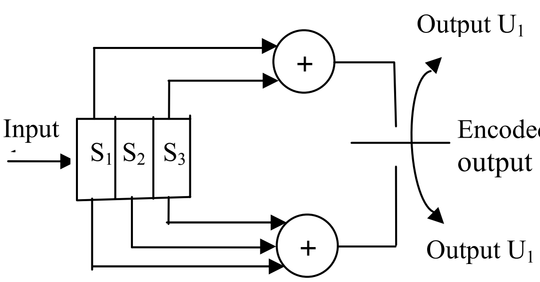

Figure 2: Example of convolution encoder

Convolution encoding works on a continuous stream of data. The source data appear as a continuous stream of bits at a given bit rate (bps), which is passed though a shift register. As the bits are temporarily stored in the shift register, they are combined in a known manner using modulo 2 adders to form the encoded output.

Here the S_1 and S_3 bits are modulo 2 added to form the U_1 output, and the S_1, S_2 and S_3 bits are similarly added to form the U_2 output. These are converted to a serial output through the switch, which is synchronized to the clocking signal for the shift register. U_1 and U_2 must be read out during one input bit period, when the shift register is in a steady-state condition. This means that if the input bit rate is r bps, the output bit rate is 2r bps [5].

4.3 The Viterbi Algorithm

Let the trellis node corresponding to state at time I be denoted. Each node in the trellis is to be assigned a value based on a metric. The node values are computed in the following manner.

1. Set = 0 and I = 1
2. At time I, compute the partial path metrics for all paths entering each node.
3. Set equal to the smallest partial path metric entering the node corresponding to state at time i. Ties can be broken by previous node choosing a path randomly. The nonsurviving branches are deleted from the trellis. In this manner, a group of minimum paths is created from.
4. If I , where L is the number of input code segments (k bits for each segment) and m is the length of longest shift register in the encoder, let I = I + 1 and go back to step 2 .

Once all node values have been computed, start at state, time I = L + m, and follow the surviving branches through the trellis. The path thus defined is unique and corresponds to the decoded output. When hard decision decoding is performed, the metric used is the Hamming distance, while the Euclidean distance is used for soft decision decoding [6].

5. SIMULATIONS

The simulations in this research are carried out using MATLAB software. The performance is simulated and evaluated for BPSK systems. Based on data generated by computer simulation of BPSK modulation techniques for BER calculation the following results are obtained. They are:

1. Bit Error Rate (BER) versus Signal-to-Noise ratio (SNR) over Rayleigh fading channel for BPSK modulation scheme without convolutional coding technique.

2. BER versus SNR over Rayleigh fading channel for BPSK modulation scheme with convolutional coding technique. This scheme that we considered in our simulations is presented in Figure 3.

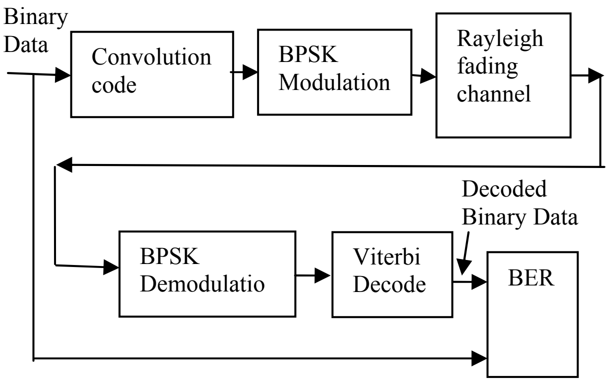

Figure 3: The convolutional coded transmission system.

3. Comparison simulated curve with respect to BER versus SNR over Rayleigh fading channel between the coded and uncoded signal.

5.1 Performance Evaluation

The simulation result of uncoded signal is evaluated on BER vs. SNR for Rayleigh fading channel when the number of data is 5000 bits and the BERs are obtained by varying the values of SNR in the range of 0 to 30 dB Figure 4(a).

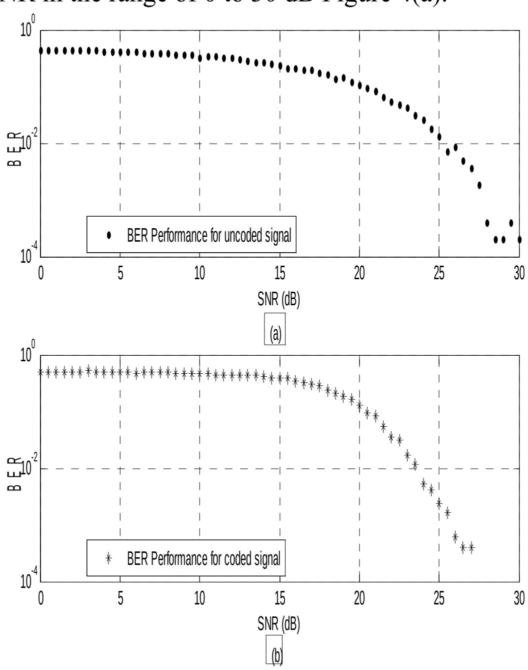

Figure 4: BER performance over slow Rayleigh fading channel for a) uncoded signal b) coded signal.

This simulation result of coded signal is evaluated on BER vs. SNR for Rayleigh fading channel using convolution code with constraint length 3, generator sequences for modulo 2 adders are $(g_0^{(1)}, g_1^{(1)}, g_2^{(1)}) = (1,0,1)$ and $(g_0^{(2)}, g_1^{(2)}, g_2^{(2)}) = (1,1,1)$ and code rate is 1/2 . The Viterbi algorithm is used to decode the convolutional code when the number of data is 5000 bits and the BERs are obtained by varying the values of SNR in the range of 0 to 30 dB.The output of the simulation curve is clearly indentified; lower BER for coded signal is achieved with convolutional coded communication system Figure 4(b).

5.2 Comparison Of Ber For Rayleigh Fading Channel Between Coded And Uncoded Signal

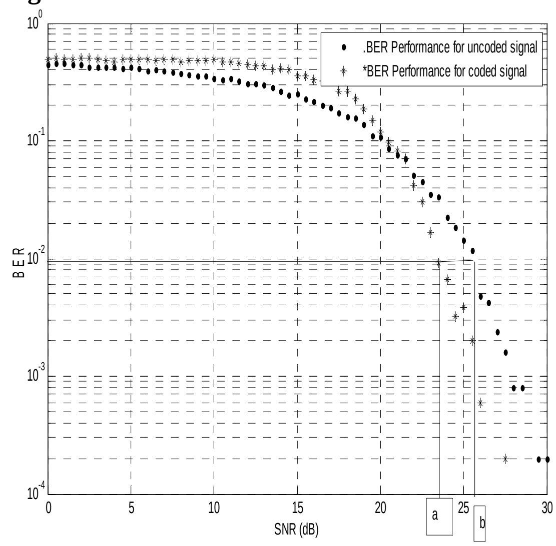

Figure 5: BER performance over slow Rayleigh fading channel for coded signal and uncoded signal.

5.3 The Significance of This Comparison Curve

The downward slope of BER curve of coded signal is sharper than uncoded signal after 21 dB in the simulated curve. Consequently, the above Figure for a specific BER 10^{-2}, the SNRs of coded and uncoded system are a and b dB respectively. So coding gain is (b-a) dB. From the cross-sectional point, the coded signal performance is better than uncoded signal. From this simulation it proves that if the data signal is transmitted using convolutional code, the system performance is clearly improved when the SNR is greater than 21 dB.

6. CONCLUSION

Convolution code is widely used in digital wireless communication system for detecting and correcting the errors in received signal message bits. Furthermore, it seems that it is one of the most important channel coding systems. In this paper, the

BER (Bit Error Rate) performances are obtained from the simulation of the convolutional coded transmission systems in the slow Rayleigh fading multi-paths channels. The Viterbi algorithm is used to decode the convolutional code. By simulation, it is shown that the improvement of BER using the convolutional code in the presence of the fading provides a better performance gain than that of the uncoded signal. Here it is noticeable that, to achieve the benefit of convolutional coded signal, it is required to cross the minimum SNR which is 21 dB

REFERENCES

[1] John G. Prokis, "Digital Communication", McGraw-Hill Series in Electrical and Computer Engineering Stephen W, 2001.

[2] Theodore S. Rappaport, "Wireless Communication: Principle and Practice", Pearson Educational International, 2nd edition, 2002.

[3] Jon W. Mark and Weihua Zhuang, "Wireless Communications and Networking," New delhi: Prentice-Hall India, 2005.

[4] Simon Haykin,"Digital Communication", Wiley – India, 1988.

[5] Dennis Roddy and John Coolen, "Electronic Communications", New Delhi: Prentice-Hall of India, 4th edition, 2003.

[6] Andrew J. Viterbi, "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm", IEEE Transactions on Information Theory, Volume IT-13, pages 260-269, in April, 1967.

[7] P. J. Mclane, P. H. Wittke, P. K. –MO, and C. Loo, "PSK and DPSK Trellis Codes for Fast Fading, Shadowed Mobile Satellite Communication Channels," IEEE Transactions on Communications, vol. 36, no. 11, November 1988.

[8] Bernard Sklar, "Rayleigh Fading Channel in Mobile Digital Communication System Part I: Characterization", IEEE Communication Magazine, pp. 90-100, July 1997.