NEW APPROXIMATION FOR HANDOFF RATE AND NUMBER OF HANDOFF PROBABILITY IN CELLULAR SYSTEMS UNDER GENERAL DISTRIBUTIONS OF CALL HOLDING TIME AND CELL RESIDENCE TIME

Mohammed Alwakeel

Computer Science Department, Faculty of Computers and Information Technology, Tabouk University

Saudi Arabia, Tabouk 71341, P.O. Box 1458

ABSTRACT

In cellular networks, quantities such as number of handoff probability and handoff rate are very important parameters in the cellular system performance analysis. In previous literature, several techniques were introduced to evaluate these parameters; however, there are some limitations in the introduced techniques. In this paper, we approximate number of handoff probability and handoff rate in cellular systems in which the call holding time and cell residence time follow arbitrary statistical distributions. Specifically, we derive approximate expressions to evaluate the probability mass function of handoff number and handoff rate. The technique used does not require knowledge of the distribution of the cell residence times but only their first two moments, which may be determined easily from empirical data. The analytical results are validated by computer simulation.

Keywords: *Number of Handoff probability, Handoff Counting, Handoff Rate*

1. INTRODUCTION

Recent advance in wireless communications and cellular systems make it possible for cellular networks to support a wide variety of services to the user on the move. 4G systems and future wireless networks will enable the user to make voice, data, multimedia calls, or make an internet connection to surf the web, and retrieve data. These advance services have motivated the study of network's quality-of-service (*QoS*), in cellular networks the following QoS measures are the most important measures used to specify the quality of the connections:

- New call blocking probability (P_0): defined as the probability that a new call request be denied for lack of resources.
- Premature call termination probability (CTP) : defined as the probability that an accepted on going call is terminated due to lack of recourses.
- Call dropping probability (*CDP*): defined as the probability that a call will experience either premature call termination or new call blocking.
- Handoff failure probability (P_{hf}): defined as the probability that a handoff request is denied for lack of resources.

Some of these measures may be specified in the design, for example, In second generation cellular systems, the premature call termination probability is lower than 5%, and the handoff failure probability is lower than 2% for voice calls [1]. To evaluate these QoS measures several system parameters such as call holding time, cell residence time, handoff counting, and handoff rate must be defined. The handoff number (handoff counting), defined as the number of handoff requested during a call connection, is an important parameter in such a system since it has a direct impact on handoff arrival traffic and call admission control policy design [2], and the handoff rate is defined as the average number of handoffs undertaken during the actual call connection. In order to determine these two parameters, the call holding time and the cell residence time need to be defined clearly, where as shown in Fig. 1:

- The call holding time (T_H) is defined as the period from the instant the accepted call starts to the instant the call completes.
- Cell residence time in the origination cell (r_1) is defined as the time that the mobile user travels from the point where the call is originated to the edge of the cell.
- Cell residence time in handoff cell (t_i, $i = 2, 3, 4...$) is defined as the time that the mobile user travels through a cell (edge to edge) reached after ($i-1$, i=2,3,4,...) handoff(s).

Many previous literature introduced techniques and expressions to evaluate number of handoff probability and handoff rate. In early literature the following assumptions are commonly used : the call holding time and the cell residence time are assumed to be exponentially distributed, and calls arrival is a Poisson process [3,4,5,6]. However, because of technological advances and the growing interest in personal communication services, and because of new marketing service plans (e.g. flat-rate service), mobile users behavior pattern was changed such that they use there mobile devices for longer period of time and more frequently. Hence, the exponential distribution may no longer appropriately models the service time or the interarrival time of practical 3G networks [7,8,9,10]. In recent literature, several techniques were introduced to determine number of handoff probability and handoff rate where more general call holding time and cell residence time distributions were assumed [1,2,11], however, there are some limitations in the introduced techniques. For example, in [1,11], the authors introduced techniques to determine number of handoff probability and handoff rate for general call holding time and cell residence time, however, it is assumed that the Laplace Stieltjes Transform is existed for call holding time, which may not be satisfied for heavy tailed call holding time distributions such as gamma and Log-Normal distributions [2]. In [2], the authors used Transform Approximation Method (TAM) to approximate the Laplace Stieltjes Transform of the call holding time, however, this technique requires recursive procedure to determine the approximated transform which may requires more time, resources, and effort. In this paper, we introduced a new approximation that may be used to evaluate the number of handoff probability and handoff rate in systems with arbitrary call holding time and cell residence time distributions. Unlike past researches which require that the probability density function (pdf) of the cell residence time and/or the pdf of the call holding time are known and have known rational Laplace transforms, our technique only requires that the first two moments of the cell residence time, and the cumulative distribution function (cdf) of the call holding time are known. The rest of this paper is organized as follow: In section 2, the analytical formula for handoff number probability mass function (pmf) and handoff rate are derived. In section 3, the developed expressions are applied, and numerical results based on the analysis as well as computer simulation are presented. Finally, some concluding remarks are made in section 4.

2. NUMBER OF HANDOFF PROBABILITY AND HANDOFF RATE

Let (H) be the number of handoffs of a nonblocked call during the call connection, then for nonblocked call, $H = 0$ if the cell residence time in the origination cell r_1 is longer than the call holding time (T_H), $H = 1$ if the call terminated because of the first handoff failure or the call make the first successful handoff and completed successfully in the new cell, and so on. Then, for a nonblocked call the handoff number probability mass function may be found as

$$\Pr(H=k)=\begin{cases}\Pr(T_H<r_1), & k=0\\ \Pr(r_1+t_2+..+t_k<T_H\le r_1+t_2+..+t_{k+1})(1-P_{hf})^k+ & k\ge 1\\ \Pr(T_H>r_1+t_2+..+t_k)(1-P_{hf})^{k-1}P_{hf} & \end{cases} \quad (1)$$

which may be simplified as

$$\Pr(H=k)=\begin{cases}1-\Pr(T_H>r_1), & k=0\\ \Pr(T_H>r_1+t_2+...+t_k)(1-P_{hf})^{k-1}- & k\ge 1.\\ \Pr(T_H>r_1+t_2+...+t_{k+1})(1-P_{hf})^k & \end{cases} \quad (2)$$

The main key to evaluate $\Pr(H=k)$ using (2) is to find the probability density function of the random variable $\eta_m = r_1 + t_2 + ... + t_m$, assuming that $(t_2, t_3, \ldots, t_m)$ are independent and identically distributed random variables. In literature, several techniques were used to do so, such as using Laplace Stieltjes Transform or using transform approximation method, as we mentioned above. In this paper, we will start by approximating the pdf of the cell residence times $r_1, t_2, \ldots, t_m$ and then use the approximated result(s) to derive the pdf of the random variable η_m. It was shown that two processes are (approximately) equivalent if they have the same first few moments [12], hence, a moment matching technique may be used to approximate the pdf of the random variables $r_1, t_2, \ldots, t_m$. One of the most flexible standard distributions is the gamma distribution, by changing its parameters it can take many shapes that may be used to approximate other distributions as shown in Fig. 2, in addition, gamma distribution is the general case of several distributions, such as Exponential distribution and Erlang distribution , which are used in many literature to model the call holding time and the cell residence time. Based on that, to derive an approximated pdf of the random variable η_m, the residence time in the first cell r_1 is approximated by a gamma distributed random variable that has the same mean μ_{r_1}, and the residence time in all subsequent cells are approximated by a gamma distributed random variable that have the same mean μ_{t_i} and variance $\sigma_{t_i}^2$, such that

$$f_{r_1}(t) \cong \frac{\beta_1^{\alpha_1} t^{\alpha_1 - 1} e^{-\beta_1 t}}{\Gamma(\alpha_1)}, \tag{3}$$

and

$$f_{t_i}(t) \cong \frac{\beta_i^{\alpha_i} t^{\alpha_i - 1} e^{-\beta_i t}}{\Gamma(\alpha_i)}, \tag{4}$$

where $f_X(t)$ is the pdf of the random variable x, and $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$. Hence, the mean and the variance of the random variable t_i may be found from (4), respectively, as

$$\mu_{t_i} = \frac{\alpha_i}{\beta_i}, \qquad \sigma_{t_i}^2 = \frac{\alpha_i}{\beta_i^2}. \tag{5}$$

Solving (5) for α_i and β_i we have

$$\alpha_i = \mu_{t_i} \beta_i, \qquad \beta_i = \frac{\mu_{t_i}}{\sigma_{t_i}^2}. \tag{6}$$

Similarly, for the residence time in the first cell we have $\mu_{r_1} = \frac{\alpha_1}{\beta_1}$, and letting $\beta_1 = \beta_i$ we have

$$\alpha_1 = \mu_{r_1} \beta_1 = \frac{\mu_{r_1} \mu_{t_i}}{\sigma_{t_i}^2}. \tag{7}$$

Based on the above, the random variable η_m is a summation of (m) gamma distributed random variables that have the same shape parameter β_i, then η_m is also a gamma distributed random variable with parameters

$$\alpha_m = \alpha_1 + (m-1)\alpha_i, \quad \text{and} \quad \beta = \beta_1 = \beta_i, \tag{8}$$

hence, the pdf of η_m may be found as

$$f_{\eta_m}(t) = \frac{\beta^{\alpha_m} t^{\alpha_m - 1} e^{-\beta t}}{\Gamma(\alpha_m)}. \tag{9}$$

From (2) we have

$$\Pr(H = k) = \begin{cases} 1 - \Pr(T_H > r_1), & k = 0 \\ \Pr(T_H > \eta_k)(1 - P_{hf})^{k-1} - \\ \Pr(T_H > \eta_{k-1})(1 - P_{hf})^{k} & k \geq 1, \end{cases} \tag{10}$$

then

$$\Pr(H=k)=\begin{cases} 1-\int_0^\infty f_{\eta_1}(t)\left(1-F_{T_H}(t)\right)dt, & k=0 \\ \left(1-P_{hf}\right)^{k-1}\int_0^\infty f_{\eta_k}(t)\left(1-F_{T_H}(t)\right)dt- & \\ \left(1-P_{hf}\right)^{k}\int_0^\infty f_{\eta_{k-1}}(t)\left(1-F_{T_H}(t)\right)dt & k\geq 1, \end{cases} \tag{11}$$

where $F_{T_H}(t)$ is the cumulative distribution function of the call holding time. Hence, the mean of H may be found as

$$E[H]=\sum_{k=0}^{\infty} k \ \Pr(H=k). \tag{12}$$

3. Illustration and Discussion

To Illustrate the use of the proposed method, we evaluate the probability mass function of the number of handoff and the mean of handoff in a cellular network, in which the cell residence times are assumed to be a generalized gamma random variables, with parameters a_1, b_1, and c_1, for the origination cell, and a_i, b_i, and c_i for all other subsequent cells, meanwhile, the call holding time is assumed to be exponentially distributed random variable with mean $\frac{1}{\mu_{T_H}}$. Hence, we have [13]

$$F_{T_H}(t)=1-e^{-\mu_{T_H} t}, \tag{13}$$

and

$$\mu_1=\frac{\Gamma\left(a_1+\frac{1}{c_1}\right)}{\Gamma(a_1)b_1},\ \mu_i=\frac{\Gamma\left(a_i+\frac{1}{c_i}\right)}{\Gamma(a_i)b_i}, \tag{14}$$

$$\text{and } \sigma_i^2=\left\{\Gamma\left(a_i+\frac{2}{c_i}\right)\Big/\Gamma(a_i)-\left(\Gamma\left(a_i+\frac{1}{c_i}\right)\Big/\Gamma(a_i)\right)^2\right\}\Big/b_i^2$$

Substituting (14) in (6), (7), (8), and (13) in (11) we have

$$\Pr(H=k)=\begin{cases} 1-\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_1}, & k=0 \\ \left(1-P_{hf}\right)^{k-1}\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_k}\left[1-\left(1-P_{hf}\right)\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_k}\right], & k\geq 1. \end{cases} \tag{15}$$

From (12) and (15), the handoff rate may be found as

$$E[H]=\sum_{k=1}^{\infty} k\left\{\left(1-P_{hf}\right)^{k-1}\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_k}\left[1-\left(1-P_{hf}\right)\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_k}\right]\right\} \tag{16}$$

$$=\left[\frac{1}{\left(1-P_{hf}\right)}\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_1-\alpha_i}-\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_1}\right]\sum_{k=1}^{\infty} k\left[\left(1-P_{hf}\right)\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_i}\right]^k$$

then

$$E[H]=\left[\frac{1}{\left(1-P_{hf}\right)}\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_1-\alpha_i}-\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_1}\right]\left\{\frac{\left[\left(1-P_{hf}\right)\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_i}\right]}{\left[1-\left(1-P_{hf}\right)\left(\frac{\beta_i}{\beta_i+\mu_{T_H}}\right)^{\alpha_i}\right]^2}\right\}. \tag{17}$$

Fig. 3 shows the probability mass function of the number of handoff as a function of the number of handoff, as evaluated using (11), and computer simulation. From the figure, we can see that the analytical results derived using the introduced technique and the computer simulation results are in very good agreement for different system parameters. The handoff rate derived using computer simulation and analysis, for different values of the call holding time and cell residence time means is shown in table 1, the results show that the approximation that was used in the introduced analytical method leads to a highly acceptable accuracy.

TABLE I. *HANDOFF RATE FOR DIFFERENT VALUES OF CALL HOLDING TIME AND CELL RESIDENCE TIME MEANS*

Call holding time mean	Origination cell residence time mean	Handoff cell residence time mean	Simulation handoff rate	Analysis handoff rate
4	1	1.5	2.5758	2.5932
4	1.5	2	1.8269	1.8388
6	1	1.5	3.8324	3.8479
6	1.5	2	2.7921	2.7966
8	2	4	1.9974	2.0096
8	3	4	1.768	1.7801

4. CONCLUSION

In this paper we introduced a new approximation technique that may be used to evaluate the probability mass function of handoff number and handoff rate in cellular system. The advantages of this technique are, firstly, its flexibility such that it may be used with any assumptions for call holding time and cell residence time distributions. Secondly, unlike other techniques, which require the distribution of the cell residence time which may not be available in practical applications, this technique requires only the first moment of the origination cell residence time, and the first two moments of subsequent cells residence time to evaluate the handoff probability mass function and the handoff rate. The results obtained from computer simulation and the analysis show that the introduced approximation technique produces a highly accurate results.

5. REFERENCES

[1] Y. Fang, "Modeling and performance analysis for wireless mobile networks: A new analytical approach", *IEEE/ACM Trans. on Networking*, Vol. 13, No. 5, Oct. 2005, pp. 989-1002.

[2] Z. Yan and S. Boon-Hee, "Handoff counting in hierarchical cellular system with overflow scheme", *Computer Networks journal*, Vol. 46, 2004, pp. 541-554.

[3] R. A. Guerin, "Channel occupancy time distribution in a cellular radio system", *IEEE Trans. Veh. Technol.*, vol. 35, no. 3, Mar. 1987, pp. 89–99.

[4] D. E. Everitt, "Traffic engineering of the radio interface for cellular mobile networks", *Proc. IEEE*, vol. 82, no. 9, Sep. 1994, pp. 1371–1382.

[5] T.-S. P. Yum and K. L. Yeung, "Blocking and handoff performance analysis of directed retry in cellular mobile systems", *IEEE Trans. Veh. Technol.*, vol. 44, no. 3, Jun. 1995, pp. 645–650.

[6] D. Hong and S. S. Rappaport, "Traffic model and performance analysis for cellular mobile radio telephone systems with prioritized and nonprioritized handoff procedures", *IEEE Trans. Veh. Technol.*, vol. VT-34, no. 3, 1986, pp. 77–92.

[7] M. Rajaratnam and F. Takawira, "Nonclassical traffic modeling and performance analysis of cellular mobile networks with and without channel reservation", *IEEE Trans. Veh. Technol.*, vol. 49, no. 3, May 2000, pp. 817–834.

[8] C. Jedrzycki and V. C. M. Leung, "Probability distributions of channel holding time in cellular telephony systems", *Proc. IEEE Vehicular Technology Conf.*, Atlanta, GA, May 1996, pp. 247–251.

[9] J. Jordan and F. Barcelo, "Statistical modeling of channel occupancy in trunked PAMR systems", *Proc. 15th Int. Teletraffic Conf.*, 1997, pp. 1169–1178.

[10] V. A. Bolotin, "Modeling call holding time distributions for CCS network design and performance analysis", *IEEE J. Sel. Areas Commun.*, vol. 12, no. 3, Mar. 1994, pp. 433–438.

[11] R. Rodriguez-Dagnino and H. Takagi, "Counting handovers in a cellular mobile communication network: equilibrium renewal process approach", *Performance Evaluation Journal*, No. 52, April 2003, pp. 153-174.

[12] S. Subramaniam, A. Somani, M. Azizoglu, and R. Barry, "The benefits of wavelength conversion in WDW networks with non-Poisson traffic", *IEEE Comm. Letters*, vol. 3, March 1999, pp. 81-83.

[13] I. Gradsheyn, M. Ryzhik, and A. Jeffrey, *Table of Integral, Series, and Products.* New York: Academic, 1994.

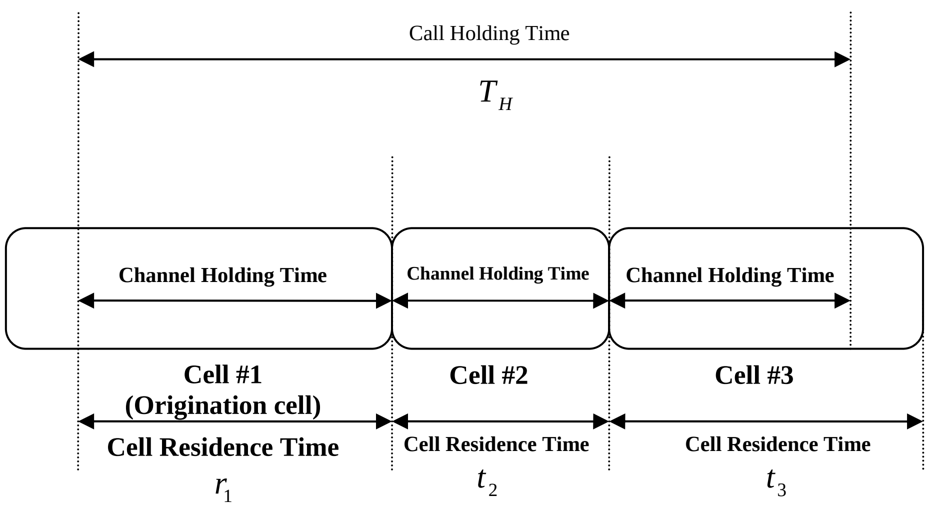

Figure 1. *Timing Diagram for Cell Residence Time and Call Holding Time.*

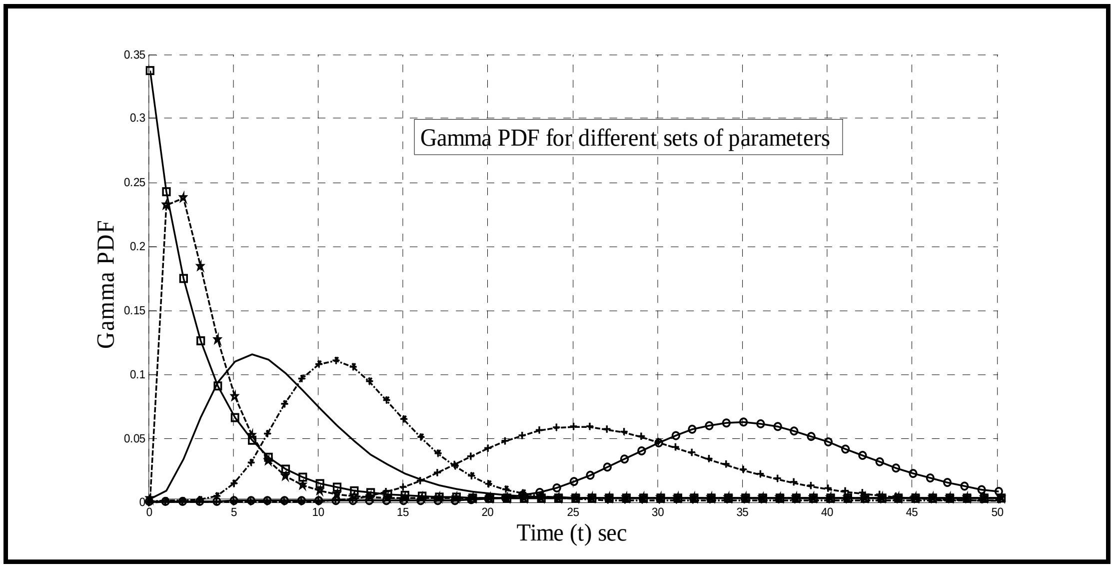

Figure 2. *Gamma Probability Density Function*

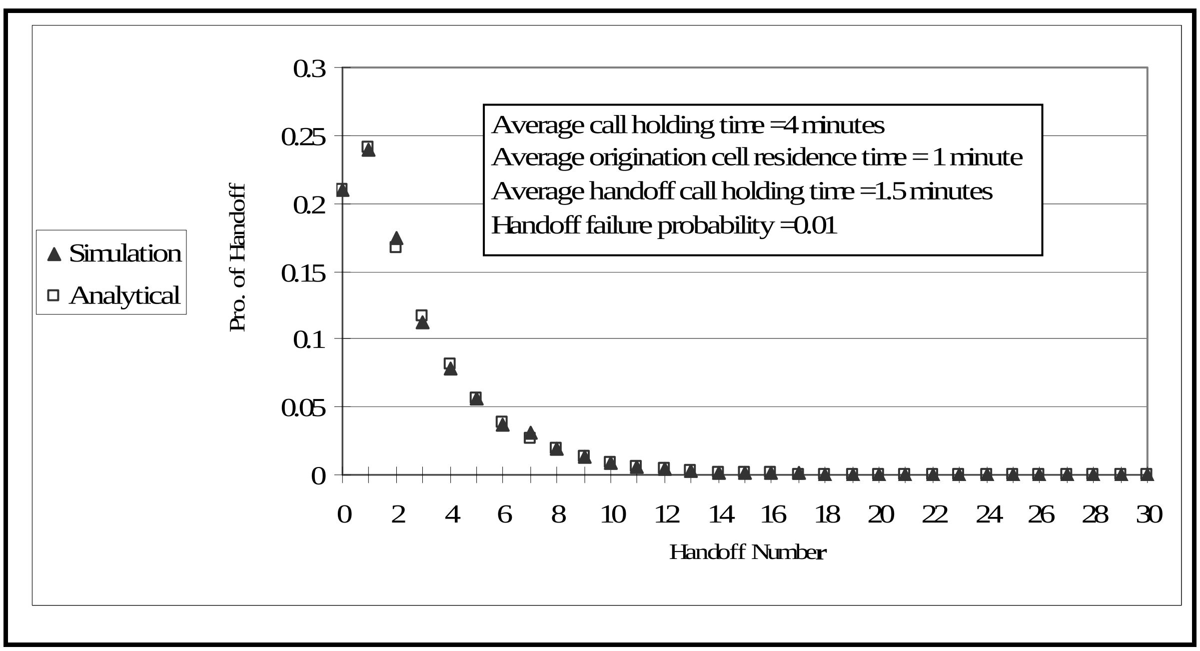

Figure 3.a *Handoff Probability Mass Function.*

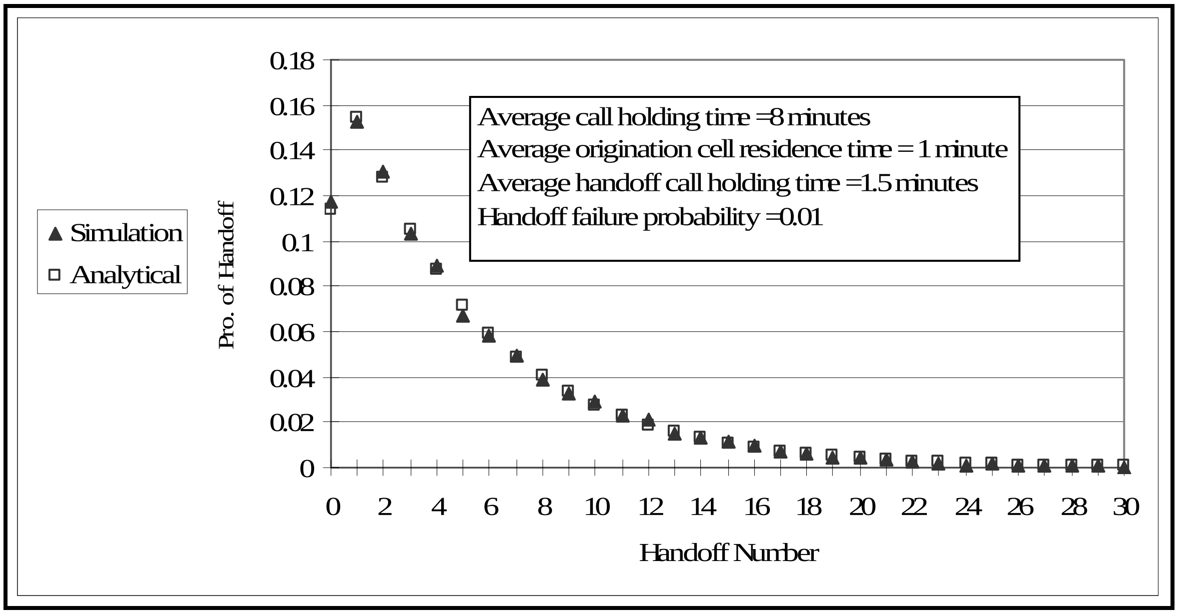

Figure 3.b *Handoff Probability Mass Function.*

ABOUT AUTHOR:

Dr. Mohammed M. Alwakeel received the B.S. degree in Computer Engineering and the M.S. degree in Electrical Engineering in 1993 and 1998, respectively, both from King Saud University, and the Ph.D. degree in Electrical Engineering in 2005 from Florida Atlantic University. From 1994 to 1998, he was employed as Communications Network Manager at The National Information Center in Saudi Arabia. From 1999 to 2001, he was employed by King Adulaziz University as a lecturer and as vice dean of Tabuk Community College. He is now the dean of Computers and Information Technology college at University of Tabuk. His current research interests include teletraffic analysis, mobile satellite communications, and cellular systems.