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AN INTEGRATED REDUNDANT RELIABILITY OPTIMIZATION FRAMEWORK FOR COHERENT HYBRID SYSTEMS USING LAGRANGE MULTIPLIER, HEURISTIC, AND INTEGER PROGRAMMING APPROACHES

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ABSTRACT

The overarching purpose of reliability engineering is to guarantee, within a certain time frame and set of circumstances, the proper operation of systems and its components. Integrating redundancy strategically while taking restrictions like mass, cost, and space needs into coherent system topologies improves system robustness, according to reliability theory. The impact of these limitations volume, weight, dimensions, and space on the improvement of system reliability is the primary emphasis of this study. Parts used in a typical representative coherent hybrid system are the focus of the inquiry. These parts are really taken from coherent systems, where dimensions, mass, and cost are critical for maintaining effective operation. The motor control unit, internal combustion engine, and electric generator must all go through thorough reliability testing. To solve this problem, we need to research and construct an integrated redundant reliability model in a coherent systems structure utilizing the Lagrange multiplier approach. This approach generates continuous-valued results for significant variables, including component count, individual and stage reliabilities, and overall system dependability. The work takes it a step further by using heuristic methods and integer programming to come up with integer-based solutions that are both realistic and useful. These strategies make the dependability research much more accurate and valuable.
Keywords: *IRR Model, Coherent Systems, System Reliability, LAM Approach, HAM Approach, IP Approach*

1. INTRODUCTION

In classical view of reliability, systems and parts of it can be described in only two states, namely operational and failed. The limited scope of the analysis is explained by the fact that in this binary framework, intermediate possibilities are omitted, but they are very relevant. This approach is also improved by the multi-state systems paradigm which recognizes that the system and its elements may be found in a spectrum of states. To get to know more about reliability, it is reasonable to consider more options and keep in mind all the numerous types of performance and deterioration that occur in reality. A lot of work has been done on the Integrated Redundant Reliability Problem (IRRP) in order to find the most suitable way to distribute additional parts and maintain low prices and provide a more reliable system. This paper is an overview of the great achievements in this field and identifies methods, statement of the problems and the future solutions.

The initial study of coherent structures focused on the improvement of component reliability to deliver the best system reliability under financial constraints. Bodian (1969), brought about a solution to this problem in coherent systems and emphasized that the optimal part dependability is related to its significance to the structure. This simplistic approach preconditioned subsequent approach in the reliability engineering that aims at the optimization of the redundancy [1]. Tillman et al. (1980), have performed an in-depth examination and analysis on optimization methods of system dependability in redundancy. They classified the methods as dynamic programming, integer programming, geometric programming, heuristic methods and others. It is a

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document that serves as a beginner tutorial and at the same time a cutting-edge guide on how redundant units in parallel and core layouts can enable a system to be more reliable. This synthesis has influenced decades of research of allocating redundancy [2]. As a measure of active redundancy in coherent systems, redundancy significance has been proposed by Boland and Proschan (1988), which is the alternative of structural importance, Birnbaum reliability importance. They examined various models of redundancy, in particular of k-out-of-n systems, parallel-series systems, and series-parallel systems. The aim of this was on strategic placement of redundancies to enhance system life [3].

Boland and El-Neweihi (1993), showed that when selection of the components in redundancy in series systems with independent lives, the most appropriate method of selecting the best redundancy is to match it with the best base component. In parallel systems, the best redundancy should be equated with the weakest base component. The research discusses the challenge in the application of these results to other coherent systems and implies some open questions [4]. Hwang and Rothblum (1994), studied coherent systems which are made of series modules whereby an optimal assembly is monotonic whereby the best modules are made of the best components. They studied systems having numerous modules which consisted of a finite number of components and were arranged in series arrangement. The effective assembling of systems under coherence is explained by this monotone optimality [5]. V.R. Prasad and W. Kuo. (2000), proposed a generalized method of improving reliability in the coherent systems applicable to assembly, redundancy distribution, and multiple choices allocation. The approach involves both heuristic methods to arrive at solutions that are near the best and optimization which is used to get local optima. Examples are provided on how useful it can be in real life involving complicated systems [6].

Sasikala et al. (2013) have improved an Integrated Reliability Model (IRM) in the context of parallel-series redundant system, considering the reliability of the components, as well as redundancy among other limitations. The Lagrangean method gave them real-valued optimal solutions. The model was tested using a real-time system to prove its effectiveness [7]. Da Costa Bueno (2016), explained the concept of standby redundancy in coherent systems whose dependence is stochastic in the view of compensator transforms and signature point processes. This was to ensure that the system was as reliable as possible and with minimal additional redundancy as possible. This is a physics-based approach that examines the life of parts to provide all the details [8]. Doostparast (2017), suggested a signature-based approach to redundancy assignment in coherent systems that can be described as having heterogeneous and interdependent components. Under independence, there were active and standby redundancies that were considered, and simple manipulations occurred. Examples of it have shown how it is applicable to complex engineering systems [9].

Fang and Li (2018), consider active allocation of redundancy in coherent systems of independent, heterogeneous components. They defined parameters of determining system lifetimes in symmetric systems or those having covered minimal cut/path sets. In Coit and Zio (2019), numerical examples were used to study the theoretical results [10]. Coit and Zio (2019), compared the evolution of system reliability optimization, such as redundancy, component enhancement, selection, fault-tolerant architectures, and maintenance. The report is a critical assessment of methodological techniques and applications identifying unsolved challenges to extended studies. This high-order structure goes past structural redundancy [11]. Pavankumar Subbara et al. (2020), enhanced the reliability of the built-in redundancy systems, involving a heuristic approach and referring to the available study on redundancy optimization. Several various restrictions are considered to make the system more reliable in the model. It has been tested on real-time systems, and it demonstrates that coherent setups are even better in the real life [12].

The series-parallel and parallel-series redundant systems were extended to the Integrated Reliability Model as proposed by Sasikala et al. (2020); the optimization of real-valued solutions is performed with the help of the Lagrangean approach. The approach considers the unknowns regarding the reliability and redundancy of the components. Real-time systems validation ensures how resilient it is in the face of limitations [13]. The dependability and redundancy allocation used in a nuclear protection system by Busse and Moreira (2021) is proposed as an architecture of compact modular reactors. The allocation of reliability established the reliability of each subsystem and redundancy kept the costs low. This renders the nuclear technology development less risky [14]. Coello and Sordo (2021), discussed multi-level allocation of redundancy in multi-module coherent systems, which are coherent sub-systems. They showed that optimum allocation

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concentrates the redundancies at specific levels. In the case of series modules (k-out-of-n parallels) it is simpler to calculate upper bounds [15].

Patil et al. (2021) introduced DVE, which is a hardware-based DRAM stability replication scheme in cache-coherent NUMA systems, transmitting data between sockets. It continues to operate normally until faults occur where it causes execution. The tests reveal insignificant performance effects and increased reliability [16]. Velampudi et al. (2022) explored the history of organizational k-out-of-n systems, highlighting the concept of redundancy as a method of reliability by using heuristic programming. The approach enhances integrated models by maximizing these models with various constraints. This assists in enhancing redundant design of the system [17].

Velampudi et al. (2022) developed, and implemented a combined redundant reliability system of k-out-of-n solutions. They proposed an IRM which considers reliability as well as efficiency. The system will continue until every component of it functions. Its effectiveness is proved by practical implementation [18]. Washington and Brooks (2022), developed redundancy allocating models to ensure that systems are more robust due to the balancing of goals and boundaries. Being redundant or possessing many independent methods of doing things makes it difficult to make sure that a task is successful. This optimization comes into play to look into complex systems of resource efficiency [19]. In a case study on integrated redundant reliability models by Velampudi et al. (2023), k-out-of-n arrangements are used that state that the efficiency of the system is greater than that of the individual components. The reliabilities and efficiency are considered in the IRM. This simplifies the understanding of the impacts of configuration changes on things [20].
In the parallel-series systems, Kapu et al. (2024), performed a case study of integrated redundancy reliability models that improve dependability by using backups. The optimization and growth within constraints increases dependability. Real time testing reveals that performance is improved [21]. The model of production-inventory developed by Maji et al. (2024) relies on reliability and utilizes fuzzy logic to address uncertainty. The approach will enhance the performance of the system through optimization under fuzzy constraints. This is a combination of inventory and reliability to real world use [22]. Research by Velampudi et al. (2024), explored the use of the integrated redundant reliability models in k out of n configuration using integer programming. The process obtains the dependabilities and competence of components of actual devices, including furnaces. This contributes to optimization in cases where limits are presented [23].

Surapati et al. (2025) developed an interactive reliability model of series-parallel systems which relied on the heuristic and dynamic programming in order to identify optimal balance between redundancy and internal rate of return. Lagrange multipliers generate component reliabilities. This is a balance that helps in making things to last longer [24]. By developing a mathematical model of series-parallel systems as allocation of redundancy as well as considering weight, volume, and space constraints, Surapati et al. (2025) enhanced the reliability of the latter. Lagrangian method provides the real numbers, whereas dynamic programming offers the whole numbers. It enhances analysis when applied in oil burners [25]. You and Li (2025) showed the stochastic optimality of component-level redundancy over system-level redundancy in coherent systems with lifetimes that depend upon each other. A homogeneous component of Archimedean copulas has conditions that are used to determine hazard rate orders. There are illustrations of generalization based on literature [26].

The researchers in this paper investigated the coherent system configurations through the derivation of an Integrated Redundancy Reliability (IRR) model and applied the traditional Lagrange Multiplier Technique to derive real valued solutions both with non-rounded and rounded solutions. A heuristic algorithm, as well as an integer programming method were used to obtain practical solutions in the form of integers. These methods allowed making a comparative analysis with the Lagrangian method and provided a scientifically reliable result. The primary task was to maximize the system reliability ($S_{Co\ Systems}$) besides the optimum parts (N_elem) in each step of the system. The research problem explicitly discussed in the study is how to maximize the reliability of a system in coherent hybrid system through various constraints like cost, mass, and volume, with the IRR model. Moreover, the research design is well organized as it is done by the use of a methodological framework that combines three strategies namely, the Lagrange Multiplier Technique used in the continuous optimization, the Heuristic Algorithm Approach, and the Integer Programming Method used in discrete solutions.

2. A FEW DEFINITIONS AND NOTATIONS.:

To find out how well systems and their parts work under certain circumstances, reliability testing is necessary. Elements are assumed to have the same dependability levels within each stage, which means that performance requirements are consistent. Also, the system treats each component as if it were statistically independent, so if one part fails, it won't affect how the other parts work. Components, phases, overall system reliability, and the Integrated Redundant Reliability (IRR) Model are defined below to help with comprehension.

Definition 2.1. A Coherent System Configuration is a system configuration or architecture in which all the components, subsystems or processes work in a coherent, synchronized and unified fashion to accomplish a shared objective or to be in a stable and predictable state. The concept of coherent is that the system components are interoperable, logically aligned together, and do not have any conflict, or inconsistency in their functionality, data flow or communication. Such arrangement results in the system to be reliable and efficient with each part operating together.
Definition 2.2. The component reliability is the probability that the system component will be able to complete the intended task without failing under stated operating conditions and times. It shows the reliability of the component to add to system functionality and efficiency in regard to design, material quality, and environmental conditions.
Definition 2.3. Process Stage Reliability Probability, that a given stage in a multi-stage process will produce the desired output and neither fail under the inputs and operating conditions. When in a sequential process, a series of stages are related to each other, reliability of one stage adds to the reliability of the system. The failure at one or more stages can affect the other stages or the whole process.
Definition 2.4. System reliability is the likelihood that a complete system will operate during a certain period of time within designated operating parameters. It is an indicator of the overall functioning of all the parts and the steps of the system, which takes into consideration their composition and their relation toward one another to guarantee their overall functionality and reliability.
Definition 2.5. Integrated Redundant Reliability (IRR) Model is used to test the redundancy-based systems of continuous operation. It looks at the interrelation between primary and redundant components to avoid failures. The model takes the interaction of primary and redundant components to enhance the performance of the system and minimize the downtime to ascertain the system dependability.
Note 2.6: Each stage is assumed to consist of identical elements, meaning every element has the same reliability level.
Note 2.7: All elements are considered statistically independent, so the failure of one does not affect the performance of the others in the system.
Note 2.8: Ensure $l_s \neq 0, \; and \; m_s \neq 0.$
Note 2.9: If m_s is even, $\frac{C\_elem}{l_s}$ must be positive to ensure the expression under the root is real.
Note 2.10: The specific value of $'k'$ depends on the context (e.g., physical constraints or domain of r_{comp}).
Note 2.11: If additional context about C_elem, $l_s, m_s \; or$ the expected range of r_{comp} is available, the solution can be refined to a single value or a specific interval.
$S_{Coh\ Systems}$: System reliability in Coherent configuration
S_{phs}: Process stage reliability, $0 < S_{phs} < 1$
r_{comp}: Component reliability, $0 < r_{comp} < 1$
N_elem: Collection of elements in stage 'ab'
C_elem: Cost-element of each item in the stage 'ab'
M_elem: Mass-element of item in the stage 'ab'
V_elem: Volume-element of item in the stage 'ab'
$C\_\max c$: Maximum allowable cost of the element
$M\_\max m$: Maximum allowable mass
$V\_\max v$: Maximum allowable volume
LRA: Lagrange relaxation approach
HM: Heuristic methodology
IPA: Integer programming approach
CRAM: Comprehensive redundancy availability model
$l_s, m_s, p_s, q_s, u_s, v_s$ are constants.

3. ARCHITECTURAL ANALYSIS OF THE SUGGESTED SYSTEM FRAMEWORK

The system's dependability concerning the given value function

$$r_{comp} = \mathrm{Cot}^{-1}\left[\frac{C_{elem}}{l_s}\right]^{\frac{1}{m_s}} \quad (01)$$

System Dependability to the provided

$$S_{Sy\ Re}\,(t) = \prod_{a,b=1}^{ps}[1 - \Pi(1 - S\_phs)] \quad (02)$$

From Equation - (01),

$$C_{elem} = l_s\, Cot[r_{comp}]^{m_s} \quad (03)$$

Where C_{elem} is the Price-component dependability.
Similarly, M_{elem} is the Mass component and V_{elem} is Volume-component can be expressed as

$$M_{elem} = p_s\, Cot[r_{comp}]^{q_s} \quad (04)$$

$V_{elem} = u_s\, Cot[r_{comp}]^{v_s}$ (05)

Since Price-component is linear in C_{elem}

$\sum_{b=1}^{s} C_{elem}\ N\_elem \leq C\_\max c$ (06)

$\sum_{b=1}^{s} M_{elem}\ N\_elem \leq M\_\max m$ (07)

$\sum_{b=1}^{s} V_{elem}\ N\_elem \leq V\_\max v$ (08)

From 3, 4, 5 we get

$\sum_{b=1}^{s} l_s\, Cot[r_{comp}]^{m_s}\ N\_elem \leq\ C\_\max c$ (09)

$\sum_{b=1}^{s} l_s\, p_s\, Cot[r_{comp}]^{q_s}\ N\_elem \leq\ M\_\max m$ (10)

$\sum_{b=1}^{s} l_s\, u_s\, Cot[r_{comp}]^{v_s}\ N\_elem \leq\ V\_\max v$ (11)

The equation modified using the relation

$N\_elem = \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}}$ (12)

$\sum_{b=1}^{s} l_s\, Cot[r_{comp}]^{m_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} \leq\ C\_\max c$ (13)

$\sum_{b=1}^{s} l_s\, p_s\, Cot[r_{comp}]^{q_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} \leq M\_\max m$ (14)

$\sum_{b=1}^{s} l_s\, u_s\, Cot[r_{comp}]^{v_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} \leq\ V\_\max v$ (15)

The Lagrangian function is formulated as

$$L_{CRAM} = S\_phs + \varphi_1 \left[\sum_{b=1}^{s} l_s\, m_s\, Cot[r_{comp}]^{m_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} - C\_\max c\right] + \varphi_2 \left[\sum_{b=1}^{s} p_s\, q_s\, Cot[r_{comp}]^{q_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} - M\_\max m\right] + \varphi_3 \left[\sum_{b=1}^{s} u_s\, v_s\, Cot[r_{comp}]^{v_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} - V\_\max v\right] \quad (16)$$

Where $S\_phs, r_{comp}, \varphi_1$, φ_2 and φ_3 are represent ideal points.

$$\frac{\partial L_{CRAM}}{\partial S\_phs} = 1 + \varphi_1 \left[\sum_{b=1}^{s} l_s\, m_s\, Cot[r_{comp}]^{m_s}\ \frac{1}{S\_phs\ \text{Log}\, r_{comp}}\right] + \varphi_2 \left[\sum_{b=1}^{s} p_s\, q_s\, Cot[r_{comp}]^{q_s}\ \frac{1}{S\_phs\ \text{Log}\, r_{comp}}\right] + \varphi_3 \left[\sum_{b=1}^{s} u_s\, v_s\, Cot[r_{comp}]^{v_s}\ \frac{1}{S\_phs\ \text{Log}\, r_{comp}}\right] \quad (17)$$

$$\frac{\partial L_{CRAM}}{\partial r_{comp}} = -\varphi_1 \left[\sum_{b=1}^{s} l_s\, m_s\, Cot[r_{comp}]^{m_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}}\ [\,Tan\,(r_{comp}) + Cot\,(r_{comp}) + \frac{1}{r_{comp}\,.Log\, r_{comp}}]\right] - \varphi_2 \left[\sum_{b=1}^{s} p_s\, q_s\, Cot[r_{comp}]^{q_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}}\ [\,Tan\,(r_{comp}) + Cot\,(r_{comp}) + \frac{1}{r_{comp}\,.Log\, r_{comp}}]\right] - \varphi_3 \left[\sum_{b=1}^{s} u_s\, v_s\, Cot[r_{comp}]^{v_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}}\ [\,Tan\,(r_{comp}) + Cot\,(r_{comp}) + \frac{1}{r_{comp}\,.Log\, r_{comp}}]\right] \quad (18)$$

$$\frac{\partial L_{CRAM}}{\partial \varphi_1} = \sum_{b=1}^{s} l_s\, Cot[r_{comp}]^{m_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} - C\_\max c \quad (19)$$

$$\frac{\partial L_{CRAM}}{\partial \varphi_2} = \sum_{b=1}^{s} p_s\, Cot[r_{comp}]^{q_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} - M\_\max m \quad (20)$$

$$\frac{\partial L_{CRAM}}{\partial \varphi_3} = \sum_{b=1}^{s} u_s\, Cot[r_{comp}]^{v_s}\ \frac{\text{Log}\, S\_phs}{\text{Log}\, r_{comp}} - V\_\max v \quad (21)$$

Where φ_1, φ_2 and φ_3 are Lagrangean multipliers.

Employing the LMT, we determine the number of elements in each phase (N_elem), discover the optimal reliability of components (r_{comp}), compute the reliability of each stage (S_phs), and evaluate the overall structural reliability ($S_{Coh\ Systems}$). The procedure produces an accurate numerical answer for the component's volume, weight, and cost.

4. CASE STUDY PARAMETERS:

When it comes to coherent hybrid systems, where factors like weight, volume, and cost are paramount to operational efficiency, this study is all about optimizing system parameters. A power-split mechanism allows the engine and electric motor to run independently or together in a coherent hybrid system, which incorporates the benefits of both types of hybrids. This setup allows the engine to operate at its most economical operating point while the motor supplies electric-only or combined power, thanks to a planetary gearset or other power-split mechanism that dynamically routes power for best efficiency and performance. You may switch between all-electric, regenerative braking, engine-only, and hybrid power assist modes with this setup, Figure 1 shows a schematic of a typical coherent hybrid system.

With a focus on redundant reliability in industrial systems, this work presents a coherent system based on the analysis of coherent topologies in Integrated Redundant Reliability (IRR) Models. A case study centered around a standard coherent hybrid system's electric generator, motor control unit, and internal combustion engine. These crucial components, which range in price from $70 to $530, are stacked in series-parallel topologies to increase reliability, and have a weight of 120 lbs to 250 lbs and a volume of 1,500 cm3 to 5,500 cm3. While the system can

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meet all in coherent system properties in a series flow to preserve process continuity, it is common practice to install the internal combustion, electric generator, and motor control unit in parallel for backup. Because of its importance, the internal combustion system may have redundant components. In order to model and optimize system reliability, which aims to balance cost, performance, and uptime, mathematical methods including Lagrange multipliers, heuristics, and integer programming are utilized.

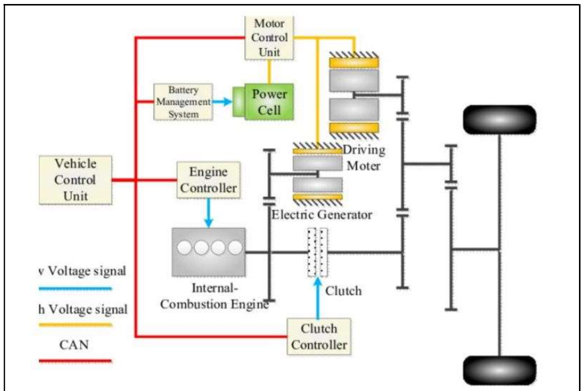


Figure 1: Coherent Hybrid System Configuration

4.1 Input Design Criteria for the Investigation:

Table 1 contains the necessary constants for the case study.

Table 1: *Establish Fixed Parameters for Cost, Load, and Size in Series-Parallel Configuration Systems*

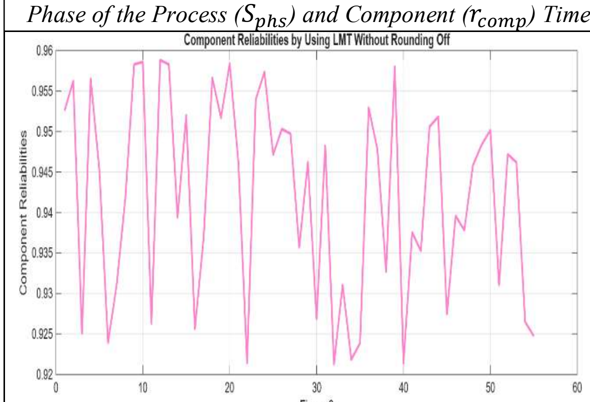
Level	Cost Element Values		Mass Element Values		Volume Element Values	
	l_s	m_s	p_s	q_s	u_s	v_s
I	70	0.9243	120	0.9021	1500	0.9157
II	330	0.9364	185	0.9292	3500	0.9256
III	530	0.9421	250	0.9346	5500	0.9339

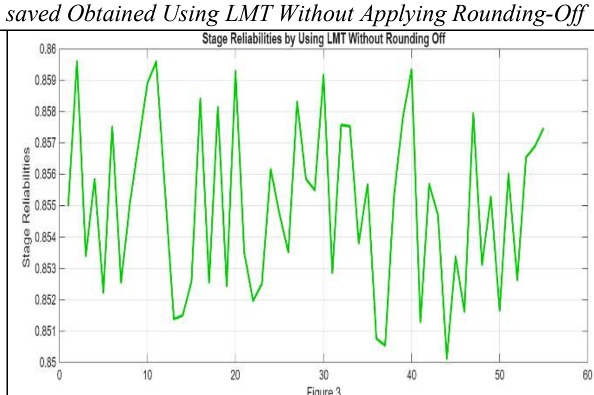
You can see the structural efficiency, along with the efficiency values for each component, phase, and quantity of elements at each step, in Tables 2, 3, and 4 below.

4.2 Evaluation of Component and Stage Reliabilities Considering Cost, Load, and Size Constraints Through the LMT Without Rounding in Coherent Systems

Figures 2 and 3 show the results of component and process stage performance under price, load, and volume constraints. These outcomes were produced following around forty iterations of an experimental procedure based on MATLAB based trial-and-error. Incorporating constraints such as budget, capacity, and dimensions, the program was used to create a thorough redundant reliability framework in a coherent configuration.

Phase of the Process (S_{phs}) and Component (r_{comp}) Time saved Obtained Using LMT Without Applying Rounding-Off





4.3 Comprehensive Assessment of Component Cost Limitations Utilizing the LMT Without Rounding in Coherent Configuration Systems

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Table 2 The Lagrange Multiplier Method for Cost Limitations in Coherent Systems Configuration

Level	l_s	m_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	C_elem	C_cost	$C_{elem\ total}$
I	70	0.9243	0.9389	-0.0274	0.8525	-0.0693	2.53	51	128
II	330	0.9364	0.9414	-0.0262	0.8645	-0.0632	2.41	238	575
III	530	0.9421	0.9519	-0.0214	0.8515	-0.0698	3.26	375	1,224
Ultimate Value									1,927
Efficiency of System ($S_{Coh\ Systems}$)									0.9349

4.4 Comprehensive Assessment of Component Mass Cost Limitations Utilizing the LMT Without Rounding in Coherent Configuration Systems

Table 3: The Lagrange Multiplier Method for Mass Limitations in Coherent Systems Configuration

Level	p_s	q_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	M_elem	M_mass	$M_{elem\ total}$
I	120	0.9021	0.9389	-0.0274	0.8525	-0.0693	2.53	87	220
II	185	0.9292	0.9414	-0.0262	0.8645	-0.0632	2.41	134	322
III	250	0.9346	0.9519	-0.0214	0.8515	-0.0698	3.26	177	577
Ultimate Mass									1,119
Efficiency of System ($S_{Coh\ Systems}$)									0.9349

4.5 Comprehensive Assessment of Component Size Limitations Utilizing the LMT Without Rounding in Coherent Configuration Systems

Table 4: The Lagrange Multiplier Method for Volume Limitations in Coherent Systems Configuration

Level	u_s	v_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	V_elem	V_Vol	$V_{elem\ total}$
I	1,500	0.9157	0.9389	-0.0274	0.8525	-0.0693	2.53	1,087	2,750
II	3,500	0.9256	0.9414	-0.0262	0.8645	-0.0632	2.41	2,526	6,091
III	5,500	0.9339	0.9519	-0.0214	0.8515	-0.0698	3.26	3,892	12,691
Ultimate Dimension									21,532
Efficiency of System ($S_{Coh\ Systems}$)									0.9349

5. LAGRANGE MULTIPLIER-BASED OPTIMIZATION OF SYSTEM EFFICIENCY

The system's efficiency incorporates ′pq′ values rounded ′pq′ to the nearest whole number, treating them as integers. The tables offer a full explanation of the acceptable outcomes for cost, load, and size. To collect the required information, you must determine the difference between the values before and after rounding ′pq′ to the nearest integer, as well as the variations caused by load, dimensions, value, and construction capacity.

5.1 Analysis of Efficiency Design Using the LMT with Consideration of Cost, Mass, and Volume Parameters, Incorporating Rounding-Off Techniques in Coherent Configuration System

Table 5: *Lagrange Multiplier Method with Rounding for Cost, Mass, and Volume Limitations in Coherent Systems Configuration*

Level	r_{comp}	S_{phs}	N_elem	C_cost	$C_{elem\ total}$	M_mass	$M_{elem\ total}$	V_vol	$V_{elem\ total}$
I	0.9593	0.8827	3	94	282	161	483	2,013	6,043
II	0.9614	0.9244	2	442	883	248	495	4,692	9,370

III	0.9588	0.8814	3	712	2,135	336	1,007	7,385	22,159
Total Cost, Mass and Volume				3,301		1,985		37,571	
Efficiency of System ($S_{Coh\ Systems}$)								0.9523	

5.1.1 Examining the Impact of LMT on the Price-Component Variation = 41.62%
5.1.2 Examining the Impact of LMT on the Mass-Component Variation = 43.63%
5.1.3 Examining the Impact of LMT on the Size-Component Variation = 42.68%

5.1.4Variations in System Efficiency with WR-LAM = 01.82%

6. HEURISTIC LOGIC IN RELIABILITY OPTIMIZATION:

Heuristic problem-solving entails locating a previously established collection of procedures or rules that perform admirably in a certain setting. When conventional methods fail to provide a satisfactory classification for the situation at hand, a heuristic approach is typically used in lieu of optimization. Problems may not fit neatly into any one category, however optimization solutions may still be out of the question due to constraints on available resources like computing power or data. Problems that may necessitate heuristics are described in the following list. Unstructured problems are those for which there are no known algorithmic solutions.

6.1 Common Heuristic Techniques:

Step 1. Greedy Algorithm: At each step, it makes the best decision locally in the hopes of discovering the best choice globally.
Step 2. Simulated Annealing: This method uses probability to explore the solution space, simulating the process of heating and cooling material to obtain a minimal energy state.
Step 3. Genetic algorithms: These algorithms take their cues from natural selection and use processes like mutation, selection, and crossover to build solutions.
Step 4. Tabu Search: Discovers a solution using local search methods, avoiding solutions that have been studied before.
Step 5. Ant Colony Optimization: This method takes its cues from ant behavior when solving complex issues; it employs pheromone trails to locate good solutions.
Author used Tabu Search to examine critical parameters in present IRR model, including component reliabilities, stage reliabilities, total system reliabilities, number of components, and overall model. This method helped discover a nearly ideal configuration for the system by exploring and optimizing its performance while avoiding previously investigated solutions.

6.2 Heuristic Algorithm

An approach to problem-solving in computer science known as a heuristic algorithm (or just heuristic) can be applied in many real-world contexts. It borrows heavily from heuristics in principle but provides no evidence of its validity. While heuristics can often yield correct results, they don't always ensure efficiency or optimality. When finding the best solution to a problem, given its restrictions or otherwise, is not an obvious task, heuristics are often used.
Novel heuristic procedures were created to enhance the system's dependability in redundant reliability systems with a coherent system design and different limitations in the author's proposed work. Detailed below is an explanation of the heuristic algorithm's methodology.

6.3 Improving Process Phase Efficacy and Component Reliability with the Use of Heuristic Algorithms

The component and process phase efficiencies (r_{comp} and S_{phs} respectively) under the size constraint are shown in Figures 5 and 6, respectively. These efficiencies were obtained using around 40 iterations of a trial-and-error procedure utilizing the Heuristic Algorithm Approach in the MATLAB software. Taking size, weight, and cost into account, this program's designers built an integrated redundant reliability model for use in a series-parallel architecture.

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Phase of the Process (S_{phs}) and Component (r_{comp}) Time saved Obtained Using HAA

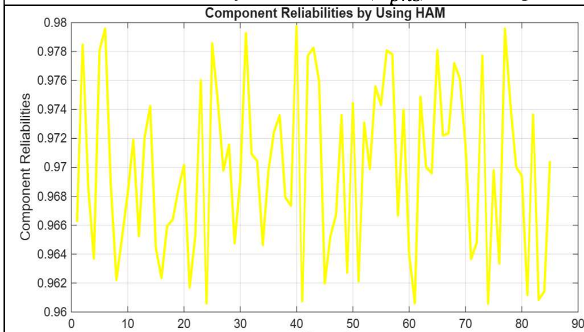

Figure 4

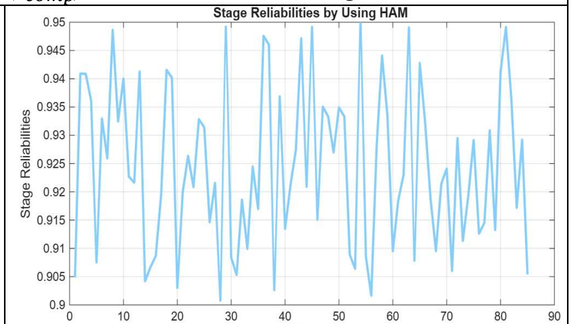

Figure 5

6.4 Thorough Assessment of Coherent System Configuration about Cost Constraints through the Use of Heuristic Algorithms

The authors determined the ideal component (r_{comp}) and process phase (S_{phs}) efficiencies according to price limitations generated from the iterative approach; Table 6 displays the value-related efficiency design.

Table 6: *The Heuristic Algorithm Approach for Cost Limitations in Coherent Systems Configuration*

Level	l_s	m_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	C_elem	C_cost	$C_{elem\ total}$
I	70	0.9243	0.9686	-0.0139	0.9088	-0.0415	3	79	237
II	330	0.9364	0.9745	-0.0112	0.9497	-0.0224	2	376	751
III	530	0.9421	0.9657	-0.0152	0.9005	-0.0455	3	597	1,792
Ultimate Value									2,780
Efficiency of System ($S_{Coh\ Systems}$)									0.9864

6.5 Thorough Assessment of Coherent System Configuration about Mass Constraints through the Use of Heuristic Algorithms

The authors determined the ideal component (r_{comp}) and process phase (S_{phs}) efficiencies according to mass limitations generated from the iterative approach; Table 7 displays the load-related efficiency design.

Table 7: *The Heuristic Algorithm Approach for Mass Limitations in Coherent Systems Configuration*

Level	p_s	q_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	M_elem	M_mass	$M_{elem\ total}$
I	120	0.9021	0.9686	-0.0139	0.9088	-0.0415	3	136	407
II	185	0.9292	0.9745	-0.0112	0.9497	-0.0224	2	211	421
III	250	0.9346	0.9657	-0.0152	0.9005	-0.0455	3	282	846
Ultimate Load									1674
Efficiency of System ($S_{Coh\ Systems}$)									0.9864

6.6 Thorough Assessment of Coherent System Configuration about Size Constraints through the Use of Heuristic Algorithms

Based on the size constraints imposed by the iterative method, the authors calculated the optimal component (r_{comp}) and process phase (S_{phs}) efficiencies; the design of these efficiencies is shown in Table 8.

Table 8: *The Heuristic Algorithm Approach for Volume Limitations in Coherent Systems Configuration*

Level	u_s	v_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	V_elem	V_vol	$V_{elem\ total}$
I	1,500	0.9243	0.9686	-0.0139	0.9088	-0.0415	3	1,696	5,085
II	3,500	0.9364	0.9745	-0.0112	0.9497	-0.0224	2	3,985	7,963

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III	5,500	0.9421	0.9657	-0.0152	0.9005	-0.0455	3	6,192	18,595
Ultimate Dimension									31,642
Efficiency of System ($S_{Coh\ Systems}$)									0.9864

6.6.1.1 Examining the Impact of HAA on the Price-Component Variation = 30.68%
6.6.1.2 Examining the Impact of HAA on the Load-Component Variation = 33.15%
6.6.1.3 Examining the Impact of HAA on the Size-Component Variation = 31.95%
6.6.1.4 Variations in System Efficiency with HAA = 05.22%

Reliability models for redundant systems subject to multiple limitations can be created by integrating heuristics into the recommended mathematical function. Using the Lagrangean algorithm, the heuristic strategy produces inputs for the current case problem. Because it produces integers for the number of elements in each step (N_elem), the heuristic method is well-suited for practical use. Classical approaches, such integer or dynamic programming, can yield integer solutions, but their calculation time grows exponentially with issue complexity. Heuristics make complex systems practical and scalable by quickly finding integer solutions.

A solution is provided by the heuristic approach, but its main drawback is that it is simply an approximation. It may not be the best answer, but it is still possible and within the scope of possibility.

When you have a lot of variables to deal with in an industrial challenge, this strategy can help you get answers that are close to perfect. The author subsequently employed the integer programming method to resolve the specified mathematical models utilizing solely integer values. This part of the study goes into great detail about the results of this strategy.

7. INTEGER PROGRAMMING TECHNIQUE:

Using the Lagrangian approach could be hard because it needs exact numbers for the amount of components ("ab") at each stage. Truncating result values often alters attributes such as cost, mass, and size, which in turn impacts system dependability and model efficiency. The author proposes an empirical

technique utilizing integer programming to identify integer solutions. This method takes the outputs of the Lagrangian method and uses them as input parameters for integer programming, which gives a more useful and accurate solution.

7.1 Improving System Dependability and Efficient Use of Resources in a Series-Parallel Architecture through the Application of Integer Programming

Integer programming can figure out the overall dependability of a system, the number of parts in each stage, and the dependability of each stage by taking into consideration the dependability of each part. Integer programming is useful for making integrated reliability models, but it has a big flaw: you can't use it directly without entering the reliabilities of the parts. Integer programming solves this problem by using the component reliabilities from the Lagrangian method and giving the stage reliabilities, system reliabilities, and component numbers for each stage as output. Integer programming lets you choose the amount of components at each phase, the reliability of each stage, and the overall reliability of the system, as long as you follow the rules set by the challenge. This allows for a more customized and effective solution within the set boundaries.

7.2 Integer Programming Method

Integer Programming (IP) solves optimization problems with integer decision variables. This method is appropriate for discrete decisions like system component or stage counts. General steps for tackling integer programming problems:

Step 1: Define decision variables that indicate problem decisions. The number of components and stages will be integers.

Step 2: Create the objective function and set the optimization goal, such as system dependability or cost reduction. A mathematical statement involving decision variables is the objective function.

Step 3: Establish Limits and define decision variable constraints. These constraints may limit resources, component capacity, system reliability, etc. Most constraints are linear, but complex issues may include nonlinear constraints.

Step 4: Enter Component Reliabilities or Other parameters provide values from past methods (e.g., the Lagrangian method) if the problem concerns dependability or other parameters. These values will feed the Integer Programming model.

Step 5: Solve the integer programming problem and find the best solution using branch-and-bound, cutting planes, or simplex approaches. The solver will choose choice variables (e.g., number of components,

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stage reliabilities) that maximize or reduce the objective function while fulfilling all constraints.
Step 6: Interpret Solution after optimisation, analyse the outcomes. This includes interpreting component counts, stage reliabilities, and system performance. Based on objectives and restrictions, the solution should deliver the best configuration.

7.3 Results of Integer Programming Method

We used the Lagrange multiplier approach to continuously solve the series-parallel Integrated Redundant Reliability (IRR) systems that were recommended. To enhance system reliability, reliability engineers use these topologies, which integrate components in series and parallel. For use in the tested models, the method yielded integer and real-valued (continuous) solutions.
The Integer Programming Approach was used to study and comprehend these models' major results. This method optimized judgments at each stage, ensuring optimal management of series and parallel components in the IRR system to maximize reliability. Tables 9, 10 and 11 shows the mathematical function's performance and reliability results. This table shows how the Lagrange multiplier, Heuristic Algorithm, and Integer Programming methods solved the IRR model's series-parallel reliability problems.

7.4 Complete IP Evaluation of Component Cost Constraints in Coherent System Configuration

Table 9: *The Integer Programming Method for Cost Limitations in Coherent Systems Configuration*

Level	l_s	m_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	C_elem	C_cost	$C_{elem\ total}$
I	70	0.9243	0.9726	-0.0121	0.9201	-0.0362	3	80	238
II	330	0.9364	0.9867	-0.0058	0.9736	-0.0116	2	382	762
III	530	0.9421	0.9734	-0.0117	0.9222	-0.0352	3	603	1,810
Ultimate Value									2,811
Efficiency of System ($S_{Coh\ Systems}$)									0.9901

7.5 Complete IP Evaluation of Component Mass Constraints in Coherent System Configuration

Table 10: *The Integer Programming Method for Mass Limitations in Coherent Systems Configuration*

Level	p_s	q_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	M_elem	M_mass	$M_{elem\ total}$
I	120	0.9021	0.9726	-0.0121	0.9201	-0.0362	3	136	409
II	185	0.9292	0.9867	-0.0058	0.9736	-0.0116	2	214	427
III	250	0.9346	0.9734	-0.0117	0.9222	-0.0352	3	284	854
Ultimate Mass									1,691
Efficiency of System ($S_{Coh\ Systems}$)									0.9901

7.6 Complete IP Evaluation of Component Volume Constraints in Coherent System Configuration

Table 11: *The Integer Programming Method for Volume Limitations in Coherent Systems Configuration*

Level	u_s	v_s	r_{comp}	Log r_{comp}	S_{phs}	$Log\ S_{phs}$	V_elem	V_vol	$V_{elem\ total}$
I	1,500	0.9243	0.9726	-0.0121	0.9201	-0.0362	3	1,705	5,109
II	3,500	0.9364	0.9867	-0.0058	0.9736	-0.0116	2	4,046	8,085
III	5,500	0.9421	0.9734	-0.0117	0.9222	-0.0352	3	6,253	18,784
Ultimate Size									31,979
Efficiency of System ($S_{Coh\ Systems}$)									0.9901

subsequently compare the results obtained from the integer programming approach to those obtained from the initial Lagrangean multipliers method, which does not use rounding, to determine the initial sensitivity levels.

7.6.1 IPM's Effect on Price-Component Variation = 31.45%
7.6.2 IPM's Effect on Mass-Component Variation Assessing = 33.83%
7.6.3 IPM's Effect on Size-Component Variation = 32.67%
7.6.4 IPM-related System Efficiency Variations = 05.58%

Specifically, it was noted that minor changes in component constraints (cost, mass, and volume) had a strong impact on the reliability of the stages and the system overall. Also, now a comparative discussion with the works that have been published in the field of reliability optimization and redundancy allocation is introduced more in detail in the next section. This comparison indicates that the combination of the proposed IRR model with heuristic and integer programming models has a high reliability in its system as opposed to the previous models that have been used in the literature. This supports the legitimization of the suggested framework.

The results of four different approaches are compared in this study: integer programming, rounding-off, heuristic technique, and Lagrangean multiplier. The study looks at the whole system reliability, component reliability, stage reliability, and the number of components. The model takes the statistically independent components.

- The case study is founded on a representative coherent hybrid system.
- The large-scale industrial systems can be associated with increasing computational complexity.
- Heuristic solutions deliver almost optimal instead of optimum solutions.

8. ANALYTICAL COMPARISON:

.8.1 LMT with Rounding, HAM, and IPM Improve Cost IRR in Coherent Configuration Systems

Table 12: *Comparison of LMT, Rounding Methods, HAM, and IPM in Cost-Coherent System Configuration*

Maximum-Cost		LMM Without Rounding-Off			Heuristic Algorithm			Integer Programming		
Level	C_elem	r_{comp}	S_{phs}	$C_{elem\ total}$	r_{comp}	S_{phs}	$C_{elem\ total}$	r_{comp}	S_{phs}	$C_{elem\ total}$
I	3	0.9389	0.8525	128	0.9686	0.9088	237	0.9726	0.9201	238
II	2	0.9414	0.8645	575	0.9745	0.9497	751	0.9867	0.9736	762
III	3	0.9519	0.8515	1,224	0.9657	0.9005	1,792	0.9734	0.9222	1,810
Efficiency of System		1,927			2,780			2,811		
$(S_{Coh\ Systems})$		LMT $S_{Coh\ Systems} = 0.9349$			HAA $S_{Coh\ Systems} = 0.9864$			IPM $S_{Coh\ Systems} = 0.9901$		

8.2 LMT with Rounding, HAM, and IPM Improve Mass IRR in Coherent Configuration Systems

Table 13*: Comparison of LMT, Rounding Methods, HAM, and IPM in Mass Coherent System Configuration*

Maximum-Mass		LMM Without Rounding-Off			Heuristic Algorithm			Integer Programming		
Level	M_elem	r_{comp}	S_{phs}	$M_{elem\ total}$	r_{comp}	S_{phs}	$M_{elem\ total}$	r_{comp}	S_{phs}	$M_{elem\ total}$
I	3	0.9389	0.8525	220	0.9686	0.9088	407	0.9726	0.9201	409
II	2	0.9414	0.8645	322	0.9745	0.9497	421	0.9867	0.9736	427
III	3	0.9519	0.8515	577	0.9657	0.9005	846	0.9734	0.9222	854
Efficiency of System		1,119			1,674			1,691		
$(S_{Coh\ Systems})$		LMT $S_{Coh\ Systems} = 0.9674$			HAA $S_{Coh\ Systems} = 0.9799$			IPM $S_{Coh\ Systems} = 0.9823$		

8.3 LMT with Rounding, HAM, and IPM Improve Volume IRR in Coherent Configuration Systems

Table 14: *Comparison of LMT, Rounding Methods, HAM, and IPM in Volume Coherent System Configuration*

Maximum-Size		LMM Without Rounding-Off			Heuristic Algorithm			Integer Programming		
Level	V_elem	r_{comp}	S_{phs}	$V_{elem\ total}$	r_{comp}	S_{phs}	$V_{elem\ total}$	r_{comp}	S_{phs}	$V_{elem\ total}$
I	3	0.9389	0.8525	2,750	0.9686	0.9088	5,085	0.9726	0.9201	5,109
II	2	0.9414	0.8645	6,091	0.9745	0.9497	7,963	0.9867	0.9736	8,085
III	3	0.9519	0.8515	12,691	0.9657	0.9005	18,595	0.9734	0.9222	18,784
Efficiency of System		21,532			31,642			31,979		
$(S_{Coh\ Systems})$		LMT $S_{Coh\ Systems} = 0.9349$			HAA $S_{Coh\ Systems} = 0.9864$			IPM $S_{Coh\ Systems} = 0.9901$		

The present section describes ways on how the proposed reliability optimization model could be implemented in real-life industrial systems like hybrid power systems, manufacturing equipment, and reliability-sensitive engineering infrastructures. The discussion now points out how the proposed approach can be used by system designers to establish the optimal allocation of redundancy and balancing operation constraints like cost, space and weight which are very critical in the modern industry use.

9. CONCLUSION:

In a coherent hybrid system, it is very important to optimize weight, volume, and cost for the system to work well. The power-split mechanism, which is usually a planetary gearset, lets the internal combustion engine (ICE) and electric motor work together or separately, combining the best parts of series and parallel hybrids. The ICE can run at its most fuel-efficient point while the electric motor can give power solely or in combination with the ICE. The motor control unit (MCU) handles smooth transitions between all-electric, regenerative braking, engine-only, and hybrid power assist modes. Figure 1 depicts a diagram of a typical power-split hybrid system. The system's capacity to dynamically route power improves performance and efficiency. To improve system reliability, the study presents an integrated redundant reliability model for a cohesive hybrid architecture, emphasizing interdependent components such as electric generators, electric motors, and combustion engines configured in parallel for redundancy. The model uses the Lagrange multiplier approach to look at performance at three different stages of operation. It tries to find the best quantity and dependability of parts while keeping costs, weight, and volume low. We changed the initial real-valued solutions to practical integer values to make sure they could be used in the real world. This method made components and stages more reliable, giving useful information on how to make strong hybrid systems that work in real-world situations.

This research formulates an enhanced reliability framework intended for a unified system architecture, using several performance measures. Using the Lagrange multiplier method, we can get important parameters like the number of components (N_elem), the efficiency of each component (r_{comp}), the dependability of each phase (S_{phs}), and the overall reliability of the system ($S_{Co\quad Systems}$). This is done after making sure the data is real-valued. The computed efficiencies for cost, mass, and volume are r_{comp} = 0.9389, 0.9414 & 0.9519, with corresponding phase reliabilities of S_{phs} = 0.8525, 0.8645 & 0.8515, yielding a system reliability of $S_{Co\quad Systems}$ = 0.9349 along with the final component values are cost \$1,927, mass 1,119 lbs & volume 21,532 cm^3 including all the three stage. Applying Lagrange multiplier methods with rounding, the efficiencies remain r_{comp} = 0.9593, 0.9614, & 0.9588, with adjusted phase reliabilities of S_{phs} = 0.8827, 0.9244 & 0.8814, resulting in an improved system reliability of $S_{Coh\ Systems}$ = 0.9523 along with the final component values are cost \$3,301, mass 1,985 lbs & volume 37,571 cm^3 including all three stages.

Due to inaccuracies in rounded results from the initial approach, an alternative Heuristic Algorithm was employed to derive integer-based solutions. Using inputs from the Lagrange multiplier analysis, this method produces revised component efficiencies for cost, mass and volume are r_{comp} = 0.9686, 0.9745 & 0.9657, with corresponding phase reliabilities for cost, mass and volume are S_{phs} = 0.9088, 0.9497 & 0.9005. The resulting system reliability is $S_{Co\quad Systems}$ = 0.9864 along with the final component values are cost \$2,780, mass 1,674 lbs & volume 31,642 cm^3 including three stages.

In practical use, an integer programming approach uses Lagrange inputs to further improve these values. The component efficiencies for cost, mass, and volume are r_{comp} = 0.9726, 0.9867 & 0.9734, and the phase reliabilities for cost, mass, and volume are S_{phs} = 0.9201, 0.9736 & 0.9222. This gives the system a reliability of $S_{Co\quad Systems}$ = 0.9901, and the final component values are cost \$2,811, mass 1,691 lbs, and volume 31,979 cm3, which includes all three stages. Notably, little changes to the cost, weight, and volume of components made a big difference in phase dependability, which improved the overall performance of the system. The study finds that integer programming gives more accurate integer answers than the Lagrange multiplier with rounding or heuristic approaches.

This method creates a useful reliability model, especially for coherent setup with reliability redundancy. It helps design engineers pick materials that provide them the best performance for the least amount of money, which is very helpful for system dependability engineers when the system isn't worth much. Future study should look into a new method that establishes minimum and maximum reliability limits for each component while improving the overall reliability of the system. Modern heuristic

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approaches can create similar integrated reliability models with redundancy, which makes them more effective in reliability engineering.

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