

THE BEHAVIOUR OF THE TWO LANES OF A QUASI-ONE-DIMENSIONAL SYSTEM CONNECTED BY AN ANISOTROPIC NODE: STUDY OF A MERGING NODE

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ABSTRACT

In the light of the growing popularity of the so-called hybrid or multimethod modelling approaches, we attempted to solve a selected problem in the field of modelling transport processes using the AnyLogic general purpose-modelling tool. Using the example of the controlled selected intersection, the procedure of creating a model with a more detailed analysis of its selected parts was discussed. Later on, the same model is investigated in the light of statistical physics. Using a cellular automaton model, we studied the response of a quasi-one-dimensional system to an anisotropic merging node. The comparison of the two simulations showed non-usual behaviours through the latter. Accordingly, following the patterns of Vehicles' extraction, we found that the exits of the two lanes are correlated, and a self-organization process governs the node. A cross-correlation test proved that the flows of both lanes are interdependent. Finally, we brought to evidence the importance of cellular automata models in understanding the process of traffic breakdowns in comparison to hybrid modelling programs.

Keywords: *Traffic Flow, Anisotropy, Anylogic; Merge, Correlation, Self-Organization.*

1. INTRODUCTION

To perform capacity analysis, every traffic facility can be reduced to a group of segments. According to the Highway Capacity Manual, those segments are connected by Points, at which the traffic enters, leaves or crosses another segment.

Points hold an important role in the traffic and capacity control. They guarantee safe crossing and turning movements [39]. They have different features and nominations, depending on the type of the facilities they are connecting. They are referred to in the fields of:

- Tolls: as the entries and the exits of those facilities[25][37][40][58];
- Stand-out facilities: as in-ramps, off-ramps, intersections [10][12][19][41][52][55].
- Traffic networks: as nodes when they are the main core of those complex systems [13][15][23][27][31][59] ...

Traffic networks are composed of links that refer to roads and nodes that denote the intersections. In this field, nodes caught the interest of many scientists and were investigated according to different approaches.

Indeed, nodes do not solely represent intersections, rather, they represent locations where the roadway characteristics change[45].

Many papers adopted macroscopic models to define the distribution of the flow in downstream and upstream links(roads), connected by the node[16]. Meanwhile, other studies opted for microscopic models, for instance the Totally Asymmetric Exclusion Process (TASEP) [33][35][42][54][56][57]. They produced the phase diagrams that define the different phases that can govern the whole system, i.e. node connecting one or numerous inputs and outputs.

Nevertheless, despite being extensively investigated, nodes still present many challenges. Indeed, to exhaust the social, economic and urgent reasons behind routes choices as much as possible, scientists are faced by a multitude of priority schemes[59]. This brings more difficulty to node's modelling.

Accordingly, studies usually differentiate between merge and diverge nodes. Despite being geometrically symmetric, the dynamics of diverge/merge nodes are different[4].

On the one hand, a diverge node models an incoming road splitting into two outgoing segments[59]. The moving flow depends on the volume of the traffic coming from the upstream link, on the capacity of the diverging segments and is strongly affected by the vehicle's destinations (road choices). Then, in certain circumstances where a vehicle cannot take its chosen path, a blockage is created upstream the node. In this case, no other vehicle can leave the upstream link until the blocked vehicle does. Subsequently, the dynamics of the diverging node are insured by a FIFO (first in first out) behaviour[15].

On the other hand, a merge node connects two upcoming flows to one outgoing link[59]. Being a reverse of the diverge[28], the moving flow in a merge is majorly defined by the capacity of the downstream segment and the traffic volume from the upstream roads. Nevertheless, the dynamics here are dictated by the priority given to each road, thus by the interactions and the conflicts of the merging streams[4].

Hence, due to those major dynamics differences, each type of nodes necessitates a standout logic[4].

Furthermore, the studies mentioned above highlighted the existence of an interdependence between the upstream roads and the downstream

roads[59]. However, to our knowledge, few if not none, deeply investigated the existence of a relationship between the downstream (upstream) links connected by the diverging (merging) node.

In this respect, by the mean of the NaSch model, we previously attempted to decorticate the behaviour of a quasi-one-dimensional system. It consists of two separated identical roads, with no lane-changing, connected by a diverging node [51]. To capture the effect of routes' choice and priority, we introduced an anisotropy parameter at the node. As defined in physical systems [7][17][53], this anisotropy parameter quantifies how each vehicle chooses a lane over the other. Using numerical simulations, we investigated the global phase diagram. Instinctively, knowing that the two roads are identical, we expected their behaviours to be symmetric. However, a spontaneous symmetry breaking occurs and three phases appear: a symmetric high-density, an asymmetric low-density and an asymmetric phase of transition low-density/high-density. Indeed, through an analysis of the node temporal occupation, we found that the entries of the two lanes are strongly cross-correlated due to a self-organization process at the node. A deeper investigation displayed that the responses of the two roads are cross correlated in the symmetric phases. Consequently, we suggested that this cross-correlation can be considered as an order parameter that characterize the transitions of this type of systems.

Consequently, the fundamental differences between the nodes in addition to the lacking literature focusing on the relation between the links connected by those, urged us to carry-on another investigation of merging nodes.

Our highest aim is to answer the following questions: Will the broken symmetry persist? Will the exits of the two roads, located at the merging node, be correlated? Will the two roads be correlated? How will the differences between merge and diverge affect the global phase diagram? And more importantly, will we be able to confirm that the cross-correlation is an order parameter despite the type of the node?

The effect of the merging behaviour was investigated in the light of the lane changing: a driver decides to leave his/her lane to merge into the left one for discretionary or mandatory reasons, based on the gap acceptance and with cost consideration. This lane changing was found to have an impact on the traffic breakdown and the capacity drop [1][2].

However, the merging, subject of this paper, is defined as the fusion of two roads into one at the node (in real life, this node can be an intersection, an on-ramp segment ...). The decision of merging does not depend on the drivers but is compulsory and governed by predefined priority parameters.

Subsequently, we propose a quasi-one-dimensional system composed of one merging node connecting two roads at their exits. We consider open boundaries and forbid any lanes changing. The two links (roads) are separated and only connected at the node. We study the response of the system to different injection rates (traffic demands) and extraction rates, by adopting the NaSch cellular automaton model[38].

Nevertheless, with the recent advancements in Cloud Computing and Software-as-a-Service (SaaS), there has been a new trend in which simulation software is employed [6][20]. This innovative approach addresses the challenges associated with integrating various methods and ensuring interoperability. In this respect, many traffic researchers have leaned towards non-formal approaches and opted for so-called hybrid or multimethod modelling tools to achieve higher modelling efficiency [6][29][34]. Consequently, to make use of the latter, we recreate the same quasi-one-dimensional system, under the same traffic and simulation conditions, by the mean of a modelling tool, i.e. AnyLogic, and we study its response to compare the results to the ones obtained through more classic methods.

This study aims to explore how a specific kind of traffic system behaves on both microscopic and mesoscopic scales of a symmetric, quasi-one-dimensional system with an anisotropic merging node. There are some important limits to the study, such as assuming all lanes as the same with no lane changes, using predictable vehicle movements, and having open ends for the system. We consider one main factor that affects priorities at the merging point and assume that traffic from both directions is equal.

This paper is divided into three main sections. We start by relying on the results found through Anylogic simulation. Next, we study the response of the system to the anisotropic merging node by the mean of a cellular automata model. Finally, we close the paper by a conclusion.

2. SIMUALTION BY ANYLOGIC

2.1. AnyLogic

AnyLogic software product is a simulation-modeling product, which allows to simulate various types of processes, including the process of road traffic, to assess the traffic situation. Indeed, AnyLogic is employed across various industries such as logistics, manufacturing, healthcare, utilities, etc., to make informed decisions based on realistic simulations.

The platform provides a multimethod approach, allowing the use of different modelling techniques, including agent-based modelling, discrete event modelling, and differential equations modelling.

To perform simulation in the used software environment as input parameters are used data on the number of vehicles - traffic intensity, as well as data on geometric parameters, in particular, the length of the simulated section, width of lanes and data on priority in traffic. As a result of building the simulated section and setting the input parameters, data on changes in the main parameters of traffic flow - traffic speed, delay value and travel time were obtained [6][46][48].

2.2. Model Design

In this paper, we consider a quasi-one-dimensional system composed of two separated roads connected by a merging node. We mirror the global schema in[51]. Here, the roads are located upstream the node. We adopt the schematic representation shown in Schema A, where vehicles exit the roads through the node.

In this section, we intend to recreate the merging node linking two roads as described above

For this purpose, a section of the road was built on the AnyLogic display, the first direction is R1, the second direction is R2 (figure 1). Following, we elaborate the programming of the logical scheme to apply it to the previous layout. To do so, it is necessary to place vehicle sources at various ends of the intersection and define the possible routes these vehicles can take.

The programming has then commenced, relying on three essential categories:

- CarSource
- CarMoveTo and
- CarDispose.

Vehicles traveling upstream will encounter a merging intersection and continue straight ahead. The priority

scheme, usually depending on empirical traffic data, is translated into probabilities.

Accordingly, firstly, the logic of the movement process is constructed (figure 2)

Next, we set the probability, and the logic of the process is changed. A probabilistic anisotropic block is added (figure 3):

- For the first direction R1, the probability was 0.1
- For the second R2, the probability was 0.9

Subsequently, the simulation procedure was launched (figure 4) and we got the result for the indicators we are interested in, namely:

- Driving Time
- Speed

2.3. Simulation results

In the following subsection, we display the results of the simulations.

We plot the travel time (figure 5) and the velocity (figure 6) of both roads R1 and R2 versus different values of demand (represented by the inflow Q_{in}).

According to the figure 5, we observe that the travel time on the second road R2 do not increase as much as on the first road R1 when increasing the Q_{in} .

Intuitively, those results are to be expected since the R2 is privileged at the intersection.

The same remark holds for the velocities on both the first and the second roads (figure 6).

However, around $Q_{in}=0.45$, we observe two peaks of the velocity and the travel time of the second road (figures 5, 6). What do they reflect?

We believe that those peaks are not the subject of statistical errors but are more likely to reflect the complex dynamic of the response of the traffic flow to nodes or intersections.

Nevertheless, due to the lack of microscopic details and to the bulkiness of the data provided by the AnyLogic simulations, we are unable of understanding the process generating those peaks.

Subsequently, we think that a study based on a statistical physics approach, by the mean of Cellular automata model, may bring more clarity about the matter.

3. SIMULATION BY CA:

3.1. the Simulation Model, Injection and Extraction Strategies

3.1.1. The merging model:

In order to investigate the problematic defined above, we adopt the same schema previously defined, and we translated according to cellular automata.

In each link, a vehicle is injected with a probability α and a velocity corresponding to the gap available[8]. Vehicles circulate by the NaSch model too[38].

When vehicles arrive at the exits, they are extracted by probability β .

However, with the existence of a merge, an extra restriction is applied to the right boundaries since the node can only deserve one vehicle at a time. Thus, the choice of the prioritized road needs to be made.

Many priority schemes were previously proposed. To name a few[59]:

- Optimal merge model[16]
- Fairness model[28]
- Fractional off-merge model[30]

Few other papers estimated a priority scheme empirically [4][44].

In this paper, as in the case of the diverge, we define the anisotropy parameter P_e as the probability of choosing one road over the other to be freed. The vehicle departures are modeled according to one of the scenarios illustrated in Schema B.

3.1.2. The Choice of the injection strategy:

a. The injection strategies:

In the investigation of traffic models with open boundaries, vehicles are usually injected at the left end with a rate α .

In this respect, many injection strategies have been proposed, namely:

- The standard injection strategy where an additional site is created at the beginning of the system[14].
- The expended injection strategy where $(V_{max}+1)$ cells are created at the beginning of the system[5].
- The strategy proposed by Bouadi et al where a new vehicle is injected directly into

the lattice with a velocity V_{max} and a probability α [8].

b. The appearance of the re-entrance:

Previously, it has been found that the transitions of traffic systems are induced by the boundaries themselves.

As far as the standard and the expended injection strategies are concerned, the open traffic systems only undergo one transition from the low-density phase to the high-density phase.

However, for the latter injection strategy, in addition to the previous statements, the global density variation exhibits an unusual behaviour. For $\beta=0.7$, the system undergoes a first order transition from the low-density phase to the high-density phase. Then, a first order transition from the high density to the low-density phase is exhibited. By analogy with physical systems, this transition has been named “reentrance” [8].

What is the process responsible of this reentrance?

Actually, according to simulations and mean field approximations, it was found that due to the hindrance at the entrance of the system, satisfying the injected rate α does not mean necessary that the corresponding vehicle can enter to the system. In other words, the Inflow is lower than the actual injection rate α . Subsequently, a lower flow, in comparison to the extraction rate that defines the capacity, is travelling down the road [8].

The system transits then to the low-density phase from the high-density phase [8].

For this reason, we will adopt the injection strategy proposed by Bouadi et al [8]

3.1.3. The symmetric boundary:

In real traffic, each road can be subject to a different demand. Those demands are usually translated into an injection probability α_1 into the first road and α_2 into the second lane.

However, unequal demand ($\alpha_1 \neq \alpha_2$) engenders an extrinsic asymmetry. This may bias our investigation, knowing that our aim is to verify the occurrence of a spontaneous symmetry breaking as a response to the anisotropic merge.

Therefore, we consider a single injection rate $\alpha_1 = \alpha_2 = \alpha$ and a single extraction rate β with an anisotropic parameter Pe .

N.B.: since no lane changing is allowed, each link of the considered quasi-one-dimensional system can be referred to either by link, by lane or by road.

Moreover, as the time is discretized, probability and rate refer to the same variable.

3.2. Results And Discussion

3.2.1. Simulation conditions:

To run the simulation, we consider a cellular automaton double lattice. Each lattice represents an independent road of $L=1000$ cells.

To reach the steady state, we execute 50000 Monte Carlo iterations and average 40 initial configurations to eliminate statistical fluctuations.

We consider that the two roads are identical and have the same geometric and traffic characteristics: both roads have the same length, the same maximum velocity $V_{max}=5$. Only the deterministic case $Pr=0$ (Pr being the ratio of randomization[38]) is investigated, to only focus on the effect of the anisotropic node.

Finally, unless stated differently later in the paper, we adopt an anisotropy parameter $Pe=0.1$. This value favours a road over the other. Indeed, when two vehicles aim to leave the system through the node at the same time, the vehicle in the second road is more likely to exit with a probability $1-Pe=0.9$.

3.2.2. Recall on the main problematic:

As stated above, we have previously exhibited unusual behaviours of the flow and the density on the case of the diverge node.

Intuitively, we believe that, knowing the differences between the operating mechanisms of merging and diverging nodes, a quasi-one-dimensional system connected by a merge will also exhibit new responses.

This is also exhibited through AnyLogic simulation by the peaks observed in the plots of speed and travel time of the privileged second road.

Thus, to tackle the investigation, we study the flow Q and the density ρ of the two roads, each one aside.

3.2.3. Investigation of the flow and the density:

3.2.3.1. 1st road:

The figure 7 displays the behaviours of the flow and the density of the first road: they have three distinct behaviours with respect to β .

We recall that a one-dimensional system undergoes only one discontinuous transition from the low-density phase to the high-density at $\alpha_c \approx \beta$ (figure 9) [8].

The response of the first lane is unusual. Thus, we will investigate the latter according to the three intervals of β .

a) When $\beta < 0.7$:

i. Description of the behaviour of the flow and density:

When $\beta = 0.5$, both the flow and the density undergo **two transitions**: a discontinued transition at $\alpha = \alpha_{c1} = 0.25$ and a continued one at $\alpha = \alpha'_{c1} = 0.42$ (figure 7). Those transitions are unusual in the measure where they do not appear in the case of a one-dimensional system[8].

At $\alpha = \alpha_{c1} = \beta/2$, the first road undergoes a first order transition, i.e. the space-time diagram displays the co-existence of two distinct phases (figure 8-a). At $\alpha = \alpha'_{c1}$, it transits continuously to the High-density phase (the space-time diagram (figure 8-b) does not show any domain wall).

According to the figures 7-9-10, the flow and the density exhibit three phases:

- When $\alpha < \alpha_{c1}$: the first road is in the low-density phase:
 - The mean velocity and the gap are very important.
 - The density is very low while the flow is increasing.
- When $\alpha > \alpha'_{c1}$: the first road is in the high-density phase:
 - The density and the flow become constant.
 - The mean velocity and the gap are very low and almost zero.
- When $\alpha_{c1} < \alpha < \alpha'_{c1}$: the first road is in an unusual phase. Against the typical behaviour of the density of a one-dimensional system[8], the density continues increasing while the flow starts decreasing.

Such uncommon behaviour leads us to wonder about the identity of this phase. Is it a low-density phase or a high-density phase?

ii. The identification of the additional phase:

We recall that:

- $\rho(\alpha < \alpha_{c1}) < \rho(\alpha_{c1} < \alpha < \alpha'_{c1})$ and $\rho(\alpha > \alpha'_{c1}) > \rho(\alpha_{c1} < \alpha < \alpha'_{c1})$.
- The gap in this phase is lower than the gap of the low-density phase but higher in comparison to the high-density phase.
- The mean velocity in this phase is lower than the velocity of the low-density phase but higher in comparison to the high-density phase.

Subsequently, we suspect that this phase corresponds to another form of high-density phase.

In order to confirm our hypothesis, we plot the space-time diagrams of the two phases when $\alpha > \alpha_{c1}$ (figure 11).

Accordingly, we observe that this new phase belongs to the high-density phase (figure 11-a).

Therefore, in this range of $\beta < 0.7$, knowing the differences in gap and velocity, we can state that the first road undergoes firstly a discontinuous transition at $\alpha = \alpha_{c1}$ from the low-density phase to a high-density that we will refer to by HD1. Then, at $\alpha = \alpha'_{c1}$, it transits again, yet continuously, from the HD1 to the usual High-density phase that we will denote HD2.

However, are those two phases any different?

With the aim of responding to this question, we investigated the distribution of the dominant velocities in time in both regions of the high-density (figure 12).

According to the figure 12, we observe that in the HD1, a wider range of velocities are present on the road $\{0,1,2\}$; while in the HD2, there are only two velocities $\{0,1\}$ with a greater predominance of the stopping vehicles ($V=0$).

To push the investigation further, we plotted the velocity profile at a random instant in both phases HD1 and HD2 (figure 7).

The velocity profiles highlight a difference between the two phases. Indeed, they exhibit wider velocity fluctuations when in the HD1 (figure 13-a) in comparison to the HD2 (figure 13-b).

Nevertheless, in some literature, the high-density phase is usually referred to as congestion. This same congestion contains a region called jamming phase in which the velocity is close to none and the density is maximum [36][47]. Other works preferred to divide the high-density phase into a congestion phase where the traffic is variable and a jamming phase where the density is stable and maximal[24].

Subsequently, keeping in mind the differences of the mean gap and velocity in figure 4, and according to the last definition, we can suggest that the high-density phase HD1 corresponds to **the congestion phase** while the HD2 can be represented as **the jamming phase**.

b) When $\beta=0.7$:

When $\beta=0.7$, the first road transits three times (figure 8).

Indeed, it undergoes the same two transitions as when $\beta<0.7$, firstly discontinuously from the low-density phase to the congestion phase at $\alpha=\alpha_{c1}$ and then continuously to the jamming phase at $\alpha=\alpha'_{c1}$.

However, we perceive that at $\alpha=\alpha''_{c1}$, the flow starts increasing again while the density is decreasing (figure 7-purple squares). According to the gap and the mean velocity (figure 15), we can affirm that this phase is the congestion phase.

Consequently, in comparison with the behaviour of a one-dimensional system that exhibits a re-entrance phenomenon from the High-density phase to the low-density phase (figure 14) [8], we believe that this behaviour matches the re-entrance phenomenon from the jamming phase to the congested phase.

c) When $\beta>0.7$:

When $\beta=0.9$, the first road only transits once from the low-density phase to the high-density phase at $\alpha=\alpha_{c1}$ (figure 7-green asterisk).

If $\alpha < \alpha_{c1}$, the first road is governed by the low-density phase. At $\alpha=\alpha_{c1}$, the density transits discontinuously to the high-density phase.

If $\alpha > \alpha_{c1}$, the density slightly increases until $\alpha=\alpha'_{c1}$ when it start decreasing within the phase high-density phase. Since the flow and density are not constant in this range of α , this phase corresponds to the congestion phase.

Thus, for the high values of β (figure 7-green asterisk), the first road only transits from the low-density phase to the congested phase at $\alpha=\alpha_c$.

3.2.3.2. 2nd road:

The figure 16 exhibits the flow and the density of the second road.

Same as those of the first road, their behaviours vary with respect to $\beta=0.7$. However, this road has a similar behaviour to a one-dimensional system.

When $\beta<0.7$, i.e. $\beta=0.5$, the density undergoes a first-order transition from the low-density to the high-density phase at $\alpha=\alpha_{c2}=0.42$. According to the figure 17, the gap and the velocity vary from a maximum value directly to zero. No congested phase is observed. Subsequently, the second road transits discontinuously from the low-density phase to the jamming phase.

When $\beta=0.7$, the density undergoes two first-order transitions from the low-density to the jamming phase at $\alpha=\alpha_{c2}=0.57$ and the cross way at $\alpha=\alpha'_{c2}=0.87$. This is re-entrance phenomenon.

When $\beta>0.7$, i.e. $\beta=0.9$, the density remains in the low-density.

3.2.4. Conclusion:

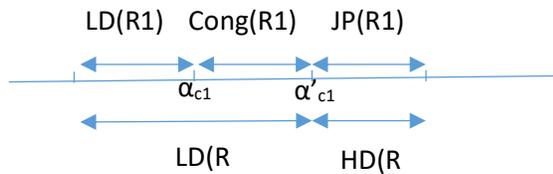
In accordance with the subsections (3.2.3.1. and 3.2.3.2), non-usual behaviours emerged.

We observed that the high-density phase of the first road contains two regions: Congestion and Jamming phase. In addition, we highlighted a new type of re-entrance from the jamming phase to the congested phase.

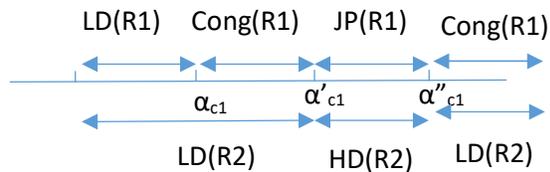
Meanwhile, the second road has conserved the well-known behaviour of a one-dimensional system.

Finally, we found that the boundaries between those phases governing the first road correspond to the transition of the second road:

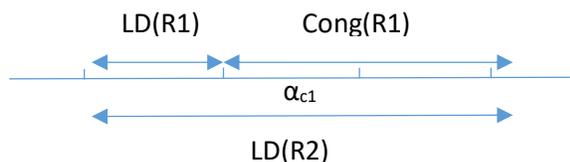
- When $\beta<0.7$:



- When $\beta=0.7$:



- When $\beta>0.7$:



Consequently, many questions rise:

- Why does the first transition of the first road occur at $\alpha = \alpha_{c1} = \beta/2$?
- Why the flow and the density of the first road are not constant in the congested phase unlike their usual behaviour in a high-density phase?
- How does the re-entrance phenomenon occur in both roads?
- Why do the two roads transit at the same $\alpha = \alpha'_{c1} = \alpha_{c2}$?
- Are the two roads interdependent?

Previously, we found that the diverging node also generates a cross-correlation between the two connected roads. This correlation guarantees the establishment of the different phases in each road and represents the response of the global system to a self-organization process located at the anisotropic node.

This confirms that understanding the behaviour of the whole system could only be achieved by the investigation of the anisotropic merging node in the light of the evolution of the second road.

3.2.5. Investigation of the effect of a merge:

In the case of a diverging node, the anisotropy is responsible of the spontaneous symmetry breaking[51]. The effect of this anisotropy was most showcased through the probability of occurrence of the different injection scenarios.

In this respect, we believe that the key to interpreting the new results is the investigation of the extraction scenarios.

3.2.5.1. The self-organization at the node:

Let us consider $\beta=0.7$ to guarantee the establishment of all the phases previously described.

In the figure 18-a, the predominant scenarios are Se_1 and Se_4 :

- When $\alpha < \alpha_{c1}$: the two scenarios are equal and are increasing with α .
- When $\alpha_{c1} < \alpha < \alpha'_{c1}$: the scenario Se_4 continues increasing while the scenario Se_1 is decreasing.
- When $\alpha'_{c1} < \alpha < \alpha''_{c1}$: the behaviours of the two scenarios are constant: Se_4 is the predominant scenario.
- When $\alpha''_{c1} < \alpha$: the behaviours of the two scenarios switch: Se_4 is decreasing while Se_1 is increasing.

In the respect of the latter, we compute the cross-correlation of the ratio of occurrence of the preponderant scenarios Se_1 and Se_4 . According to the

figure 18-b, the scenarios Se_1 and Se_4 are anti-correlated (negative cross-correlation):

- When $\alpha < \alpha_{c1}$: the cross-correlation is weak at first and increases with α until it reaches -0.45.
- When $\alpha_{c1} < \alpha < \alpha'_{c1}$: the cross-correlation is important, but it slightly decreases.
- When $\alpha'_{c1} < \alpha < \alpha''_{c1}$: the cross-correlation is constant.
- When $\alpha''_{c1} < \alpha$: the cross-correlation starts increasing again.

Knowing that only one vehicle can be extracted at a time, i.e. only one scenario occurs at each iteration, we can suspect that a scenario induces the other one [9][18].

In order to understand the mechanism of the establishment of this predominance, we follow the time occurrence of all the scenarios.

The figures 19 show that there is a pseudo-alternation of the two predominant scenarios Se_1 and Se_4 . Moreover, we observe that the smaller the demand α , the keener is this alternation.

This alternation highlights a self-organization process at the node.

However, **why are those two scenarios preponderant? How does this self-organization process at the node? And what are the consequences of this self-organization?**

3.2.5.2. Phase diagram:

In order to answer the last question, we plot the global phase diagram of the whole system: we combine the transitions of the two roads (figure 20).

According to figure 20, there are three distinct phases:

- Symmetric LD: (LD/LD)
- Asymmetric phase of transition HD/LD: (CONGESTION/LD)
- Asymmetric HD: (JAMMING/HD)

The phase diagram brought to the evidence that, for all the values of β , the transition of the privileged road corresponds to the transition of the non-privileged one (1st road) at $\alpha = \alpha_{c2} = \alpha'_{c1}$.

Moreover, the existence of those phases highlights a spontaneous symmetry breaking as for the bridge model [21][22][26][43][50][61].

a) Symmetric LD: (LD/LD)

When $\alpha < \alpha_{c1}$, both roads are in the low-density phase. In this range, the two scenarios controlling the exits are equally probable (figure 18-a).

To explain these results, we follow the temporal evolution of the flow in the roads from the injection of the first vehicles.

At $t=t_0$ (first instant at which the injection condition is verified, and a vehicle can be injected), since the two roads are identical, empty and symmetric, two vehicle are injected simultaneously, each one in a lane (figure 21-a). They travel with a velocity V_{max} . Then, at $t=t_1$, they arrive at the two exits at the same time.

By the effect of the anisotropic parameter P_e , a vehicle is more likely to be extracted from the second road by Se_4 while the other vehicle is obliged to wait for a later instant. The symmetry of the two roads is broken.

However, the very low injection rate makes it harder for another vehicle to arrive at the following instant to the exit of the second lane. Subsequently, in a later iteration (the effect of $\beta \neq 1$), the remaining vehicle will leave the first road according to Se_1 . **The symmetry is restored.**

This process is then repeated throughout the simulation (vehicles are injected simultaneously (figure 21-b)). Each time the symmetry is broken, it would be restored by the effect of the low demand and the very large gap (figure 22-a).

This symmetry is confirmed by the difference between the mean gaps. Indeed, we computed the $\Delta(GAP) = GAP_2 - GAP_1$ (figure 22-b). We observed that, in this range of α , the difference is almost none. Recalling that vehicles are injected at the same time, they leave by the same process.

An alternation of the two scenarios Se_1 and Se_4 takes place guaranteeing a self-organization (figure 19-a).

To conclude, this self-organization at the node guarantees the formation of equal flows in the two roads.

Hence, the behaviour of the system is symmetric (figure 20).

b) Asymmetric phase of transition HD/LD: (CONGESTION/LD)

- Transition of the 1st road:

At $\alpha = \alpha_{c1}$, the 1st road transits to the congested phase while the second one still is in the low-density phase.

By the mechanism of the merge, i.e. the extraction of one vehicle at a time, the extraction probability is divided by two $\beta_{eff} = \beta/2$.

As for $\alpha < \alpha_{c1}$, the process of injection and extraction from $t=t_0$ to $t=t_1$ is the same. Since the two roads are empty, the symmetry is restored quickly.

However, by the effect of the growing demand α , more vehicles are present in the roads and arrive at the ends simultaneously and rapidly. Indeed, the gap between the vehicles in both roads is smaller (figure 22-a).

Consequently, due to the parameter anisotropy P_e , when a vehicle is extracted from the privileged road according to Se_4 , the probability that the vehicle remaining in the first lane is extracted in the next iteration decreases ($P(Se_1)$ is decreasing (figure 18-a)). The difference between the gaps in both lanes is more important (figure 22-b). The reestablishment of the symmetry becomes harder.

Thus, the reduced extraction rate strikes the non-favoured lane first. The latter transits then to the high-density phase, generating the transition to the phase (CONGESTION/LD) (figure 20).

- Continuous evolution of the first road in the High-density phase:

As for the low-density phase, because of the self-organization process that favors the second road (figure 19-b), a hindrance is growing at the end of the first road. The flow of the first road decreases.

This phase is considered as a phase of transition of the global quasi-one-dimensional system to the High-density phase.

c) Asymmetric HD: (JAMMING/HD)

When $\alpha'_{c1} < \alpha < \alpha''_{c1}$, the hindrance generated by β also affects the second road. The shockwave reaches the entry of the second lane, and vehicles cannot be injected. The second road is also in the high-density phase. At this stage, both lanes are in the high-density phase and only β controls the system. The flows are constant.

However, by the pattern of the self-organization (figure 19-c), i.e. the pseudo-alternation of the two scenarios Se_1 and Se_4 , vehicles are more likely to leave the second road due to the anisotropy. The ratio $P(Se_4)$ is greater than $P(Se_1)$.

Subsequently, the symmetry is broken, which generates the asymmetric HD.

d) Re-entrance: (CONGESTION/LD)

When $\alpha''_{c1} < \alpha$, due to the injection strategy, the second road transits to the low-density phase through the re-entrance phenomenon [8]. In fact, by the effect of the frustration at the entry, less vehicles enter this road. Then, vehicles arrive sparsely to the end of the second road and do not pile up at its exit [8] (wider gap figure 22-a).

Here again, a pseudo-alternation of the two scenarios occurs (figure 19-d), but with a wider presence of the Se_1 (the ratio $P(Se_4)$ decreases while the $P(Se_1)$ is increasing).

As a response to the self-organization, more vehicles leave the first road (figure 18-a, 19-d). The first road also transits to the congested phase.

The asymmetric phase of transition (CONGESTION/LD) reappears.

e) Summary:

The analysis of the temporal occurrence of the different extraction scenarios have confirmed the establishment of a self-organization process.

Indeed, at the node, only two scenarios are preponderant and anti-correlated. Their alternation generates a self-organization at the node. This process generates a spontaneous symmetry breaking and controls the transitions of the two roads.

3.2.5.3. The correlation of the roads:

In the case of the diverging node, this punctual cross-correlation guarantees the cross-correlation of the two roads. Would this cross-correlation persist considering a merging node?

We plot the cross-correlation of the flows of the two roads, as a function of the injection rate α .

For each value of (α, β) , this cross-correlation is a mean value that was calculated, over the whole time simulation and averaged, according to the following formula:

$$C_c = \frac{\sum_t [(q_1(t) - \langle q_1 \rangle) * (q_2(t) - \langle q_2 \rangle)]}{\sqrt{\sum_t [(q_1(t) - \langle q_1 \rangle)]^2} \sqrt{\sum_t [(q_2(t) - \langle q_2 \rangle)]^2}}$$

The random variables are q_1 and q_2 , the flows in the first road and in the second road successively.

With regards to the figure 23, the flows of the two roads are highly correlated ($C_c \approx 1$) when they both are in the low-density phase. When the first road undergoes a discontinuous transition to the high-density phase, the cross-correlation follows the same path. Afterword, the cross-correlation continues

decreasing until the two roads transit to the asymmetric high-density.

Hence, the cross-correlation persists for the merging node.

Moreover, order parameter is a physical quantity that varies according to phases. It distinguishes the symmetry-broken states in the ordered phases.

Accordingly, this same cross-correlation can be considered as an order parameter [3][11][47] that defines the transitions of a quasi-one-dimensional system composed of two lanes connected by a node despite its type.

4. CONCLUSION

The complexity of most real-world systems often necessitates the use of simulation for in-depth study. This holds true for the domain of road transportation and traffic operations as well [6].

Moreover, the different types of nodes have a major impact on the performance, the levels of service and the safety of connected facilities.

In this article, we attempted to understand microscopically, and through various forms of simulations, the impact of the differences between a merging node and a diverging one on the results mentioned in [51].

In this respect, we opposed the results of the simulation of a merging node connecting two roads by the mean of multimethod modelling tool AnyLogic and by the mean of microscopic model-based method.

For this purpose, we considered a quasi-one-dimensional system composed of a single merging node connecting two separated roads at the left boundaries. Then, we implemented an anisotropy parameter Pe that favours one lane over the other when both exits are occupied. We were able to point out the relationship between the connected roads, in the response of the existence of the anisotropic merge.

On the one hand, by running the simulation developed in AnyLogic, we have highlighted the differences in the response of the two roads to the anisotropy at the intersection. Indeed, by favouring a road over the other, the travel time and speed are extremely compromised in comparison to the second road. This result may be expected. Moreover, we also pointed out an extreme change in the behaviour

of the privileged road. However, due to the fact that these types of simulation are more results focused the process focused, we could not understand the process or the reason behind those peaks.

Subsequently, we continued our study by adopting the so-called NaSch model so as to re-explore the results cited above.

Firstly, we described the new response of the flow and the density of each road. While the privileged (second) road behaves as one-dimensional system, the first road undergoes two transitions from the low-density phase to the high-density phase. Indeed, the latter road transits discontinuously from the low-density phase to the high-density phase and continues evolving continuously within the same phase until both the density and the flow become constant. According to a deep investigation of the velocity, we suggested that this road transits from the low-density phase to a congested phase then to a jamming phase.

Then, to understand the mechanism behind these behaviours, we investigated the merge, i.e. the ratio of occurrence of the different extraction scenarios. We found that only two scenarios control the node. Those scenarios were found to be alternating in such a way that they generate a self-organization process. The latter is confirmed by a cross-correlation of the exits guaranteeing the establishment of the different phases.

Furthermore, we found a spontaneous symmetry breaking by plotting the global phase diagram. Indeed, this diagram exhibited the existence of a symmetric low-density phase, an asymmetric high-density phase and an asymmetric phase of transition. This broken symmetry is proposed to be the organization of the system as a response to the anisotropy at the node. Finally, the flows of the two roads are strongly correlated in the symmetric low-density phase.

By comparing those results to the findings of the case of the diverge, we can state that despite the differences in the behaviours of the two roads, both the merging and the diverging nodes with an anisotropic parameter, create a cross-correlation between the two in the symmetric phases, through a self-organization process.

The primary novelty of this work is the identification and validation of lane cross-correlation as a universal order parameter for phase transitions in quasi-one-dimensional systems with anisotropic nodes, regardless of the node being a merge or

diverge. This contributes a new metric for characterizing traffic system behavior at microscopic junctions. Furthermore, our methodological contribution lies in the deliberate parallel use of a multi-method simulation tool (AnyLogic) and a fundamental cellular automaton model, demonstrating how each can be used to interrogate different aspects—operational performance versus mechanistic cause—of the same traffic phenomenon.

Still, by recalling the results of the Anylogic simulation, we believe that the changes observed in the behaviour of the privileged road are to be correlated to the behaviour of the other road.

In brief, in this work, we were able to confirm that the bare existence of an anisotropic node creates an important connection between the two connected segments regardless of it being a merge or a diverge. This connection is a cross-correlation that we suggest as an order parameter quantifying the transition of quasi-one-dimensional systems with nodes.

It is worth to mention that this paper is a humble effort aiming to shell the mechanism and the outcome of a different type of nodes. The bigger challenge would then be investigating the assembling of numerous types of nodes while keeping in mind that previously observed phenomena would not necessarily be present due to the complexity and the nonlinearity of the traffic.

Another aspect that would be challenging is confronting all the previously mentioned results to real traffic data. This would be interesting as far as it is important to verify the validity of the phase diagram and to complete the fundamental diagram. An empirical investigation by the mean of AnyLogic, or similar simulation programs, would also enlarge the spectra of the types of the facilities studied. Indeed, in addition to nodes connecting symmetric roads, we would be capable of treating the effect of merging or diverging behaviour on on-ramps or off-ramps, arterial and principal roads, joined roads with lane changing... We would reproduce fundamental diagram by the mean of different CA models or multimethod modelling tools, reinvestigate the establishment of different traffic phases... Finally, we would be able to follow the evolution of the shock waves generated by the effect of the merging behaviour, calculate their velocity and define their effects and boundaries.

The impact of these findings is twofold for current traffic science and engineering. They provide a

theoretical foundation for understanding how mandatory merge points (e.g., on-ramps, lane drops) can induce correlated congestion patterns in seemingly separate lanes, which is critical for predictive traffic management and control algorithms

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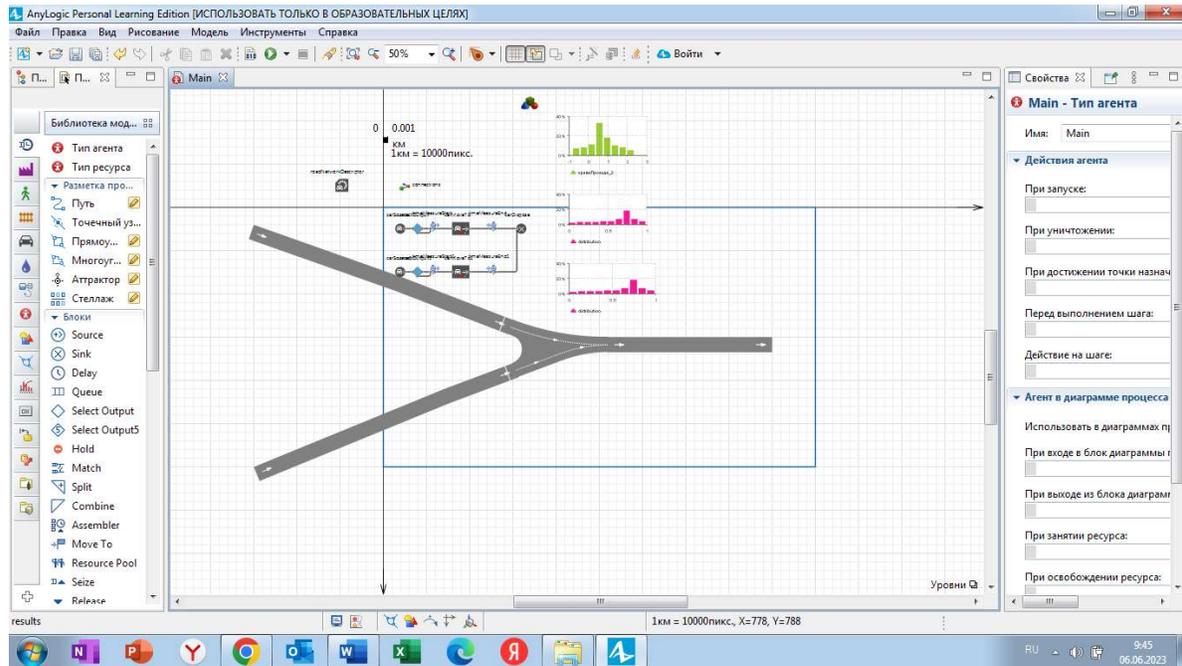


Figure 1: Roads Mapping On The Anylogic Display

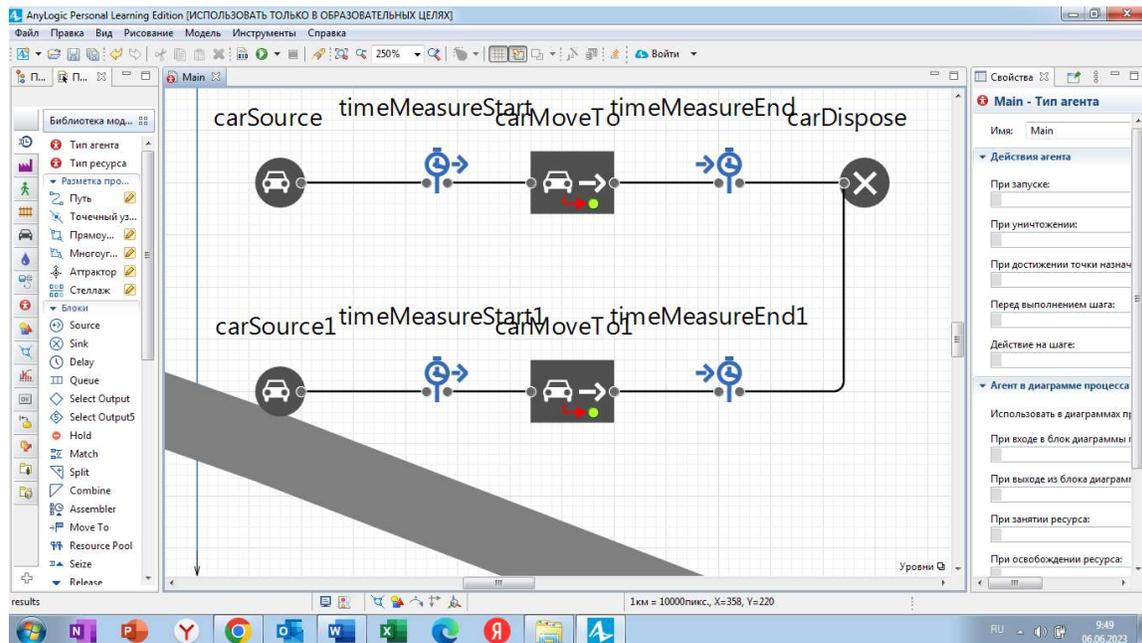


Figure 2: Programming Of The Movement Process

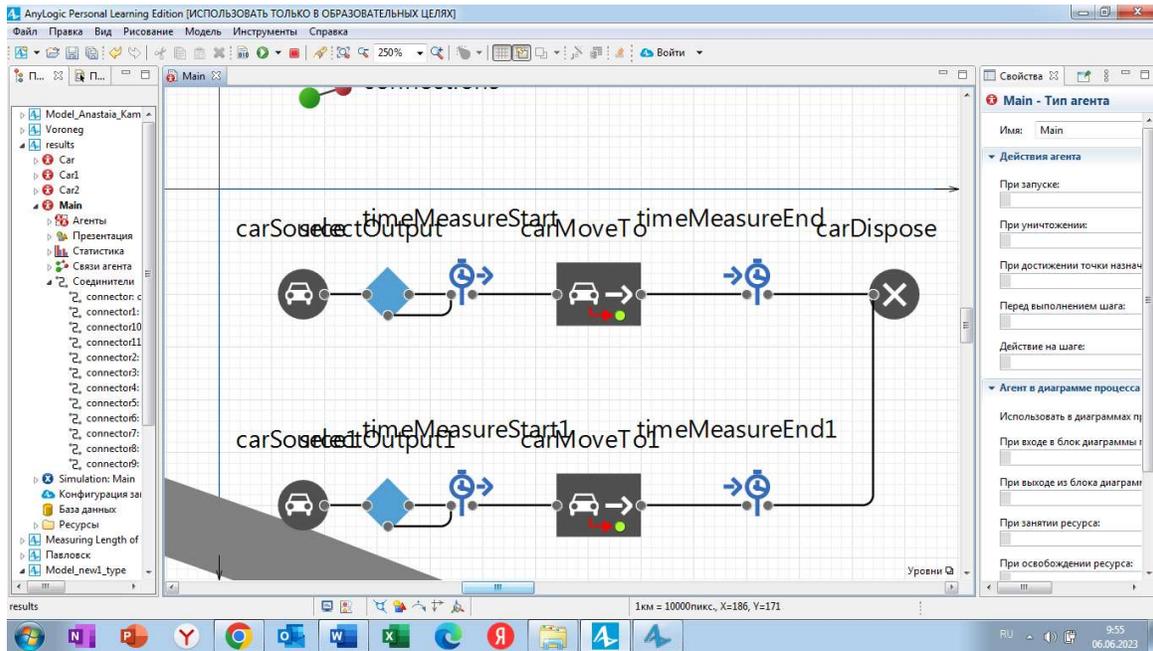


Figure 3: Reprogramming Of The Movement Process By Including The Probability Scheme

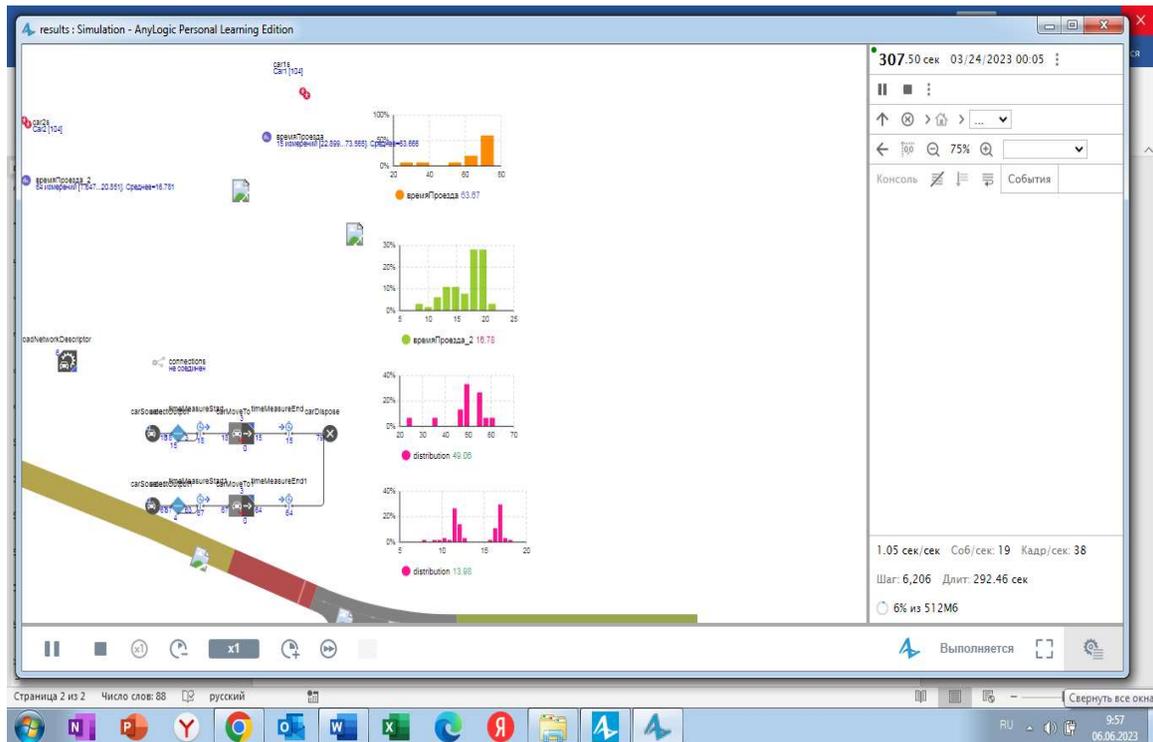


Figure 4: Simulation Of The Traffic Under The Defined Conditions

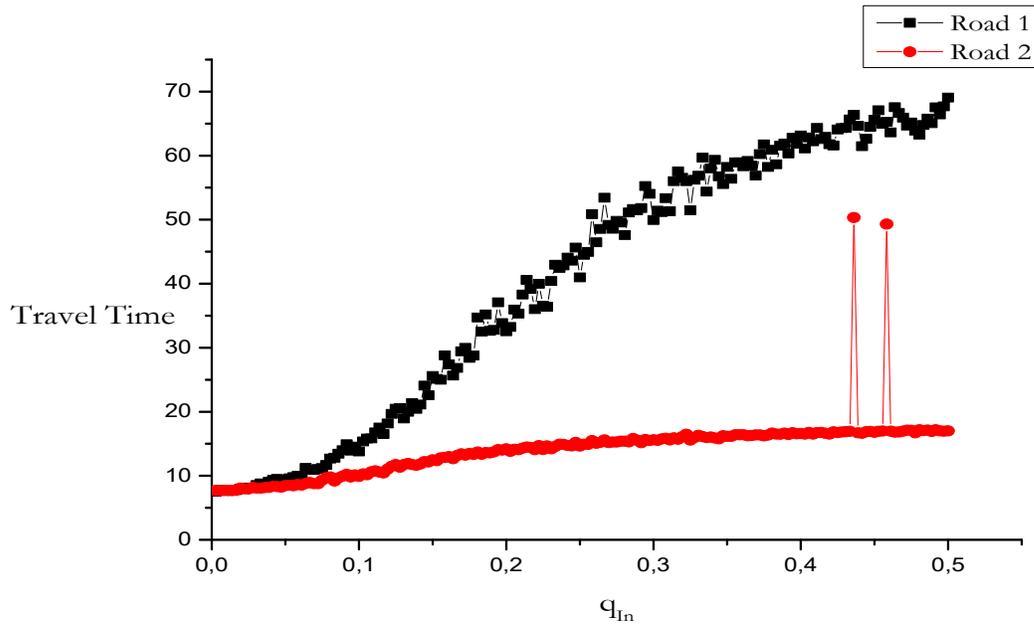


Figure 5: Mean Travel Time Of Both Roads Versus The Inflow Rate Q_{in}

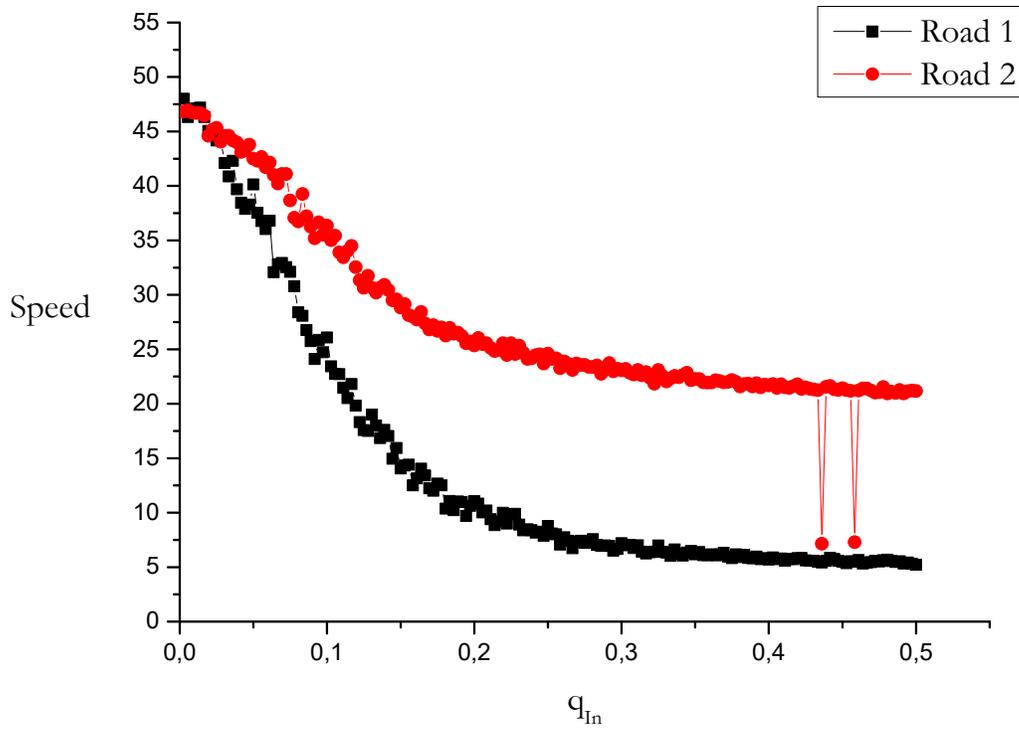


Figure 6: Mean Velocity Of Both Roads Versus The Inflow Rate Q_{in}

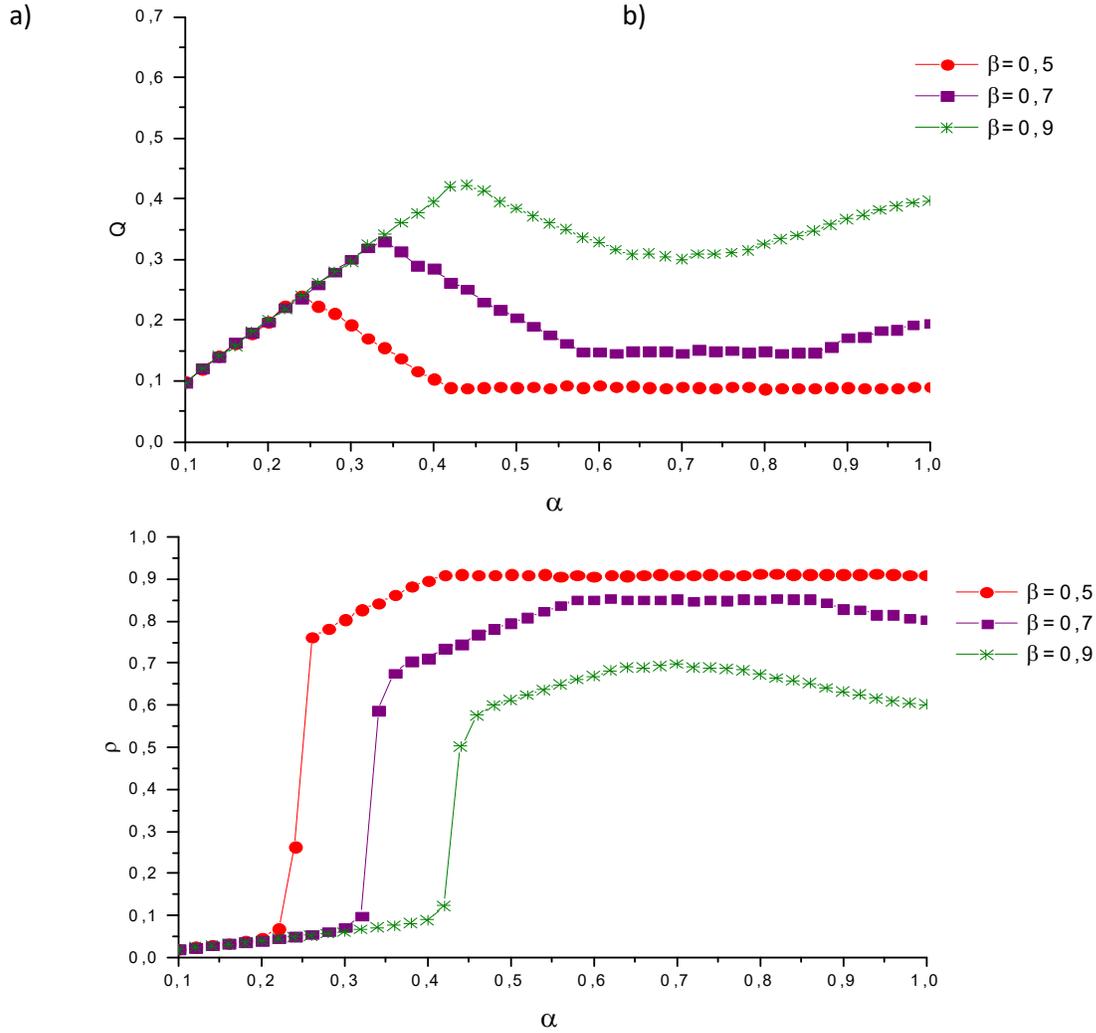


Figure 7: A) Flow And B) Density Of The First Road Versus The Injection Rate α For Different Values Of The Extraction Rate β .

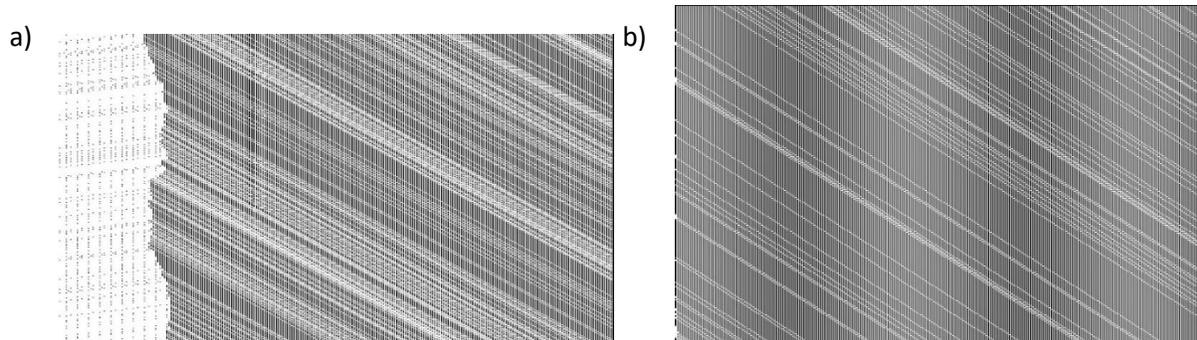


Figure 8: Space-Time Diagram For $\beta = 0.5$ And A) $\alpha = 0.25$, B) $\alpha = 0.42$: The Black Cells Are Occupied (Vehicle) And The Whites Are Empty

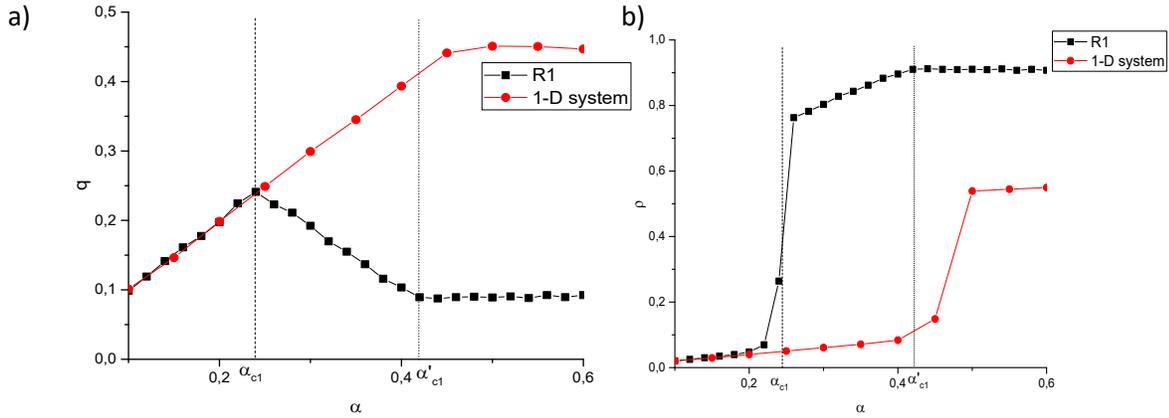


Figure 9: Comparison Between The Observables Of The First Road And Of The One-Dimensional System For $B=0.5$: A) The Flow B) The Density

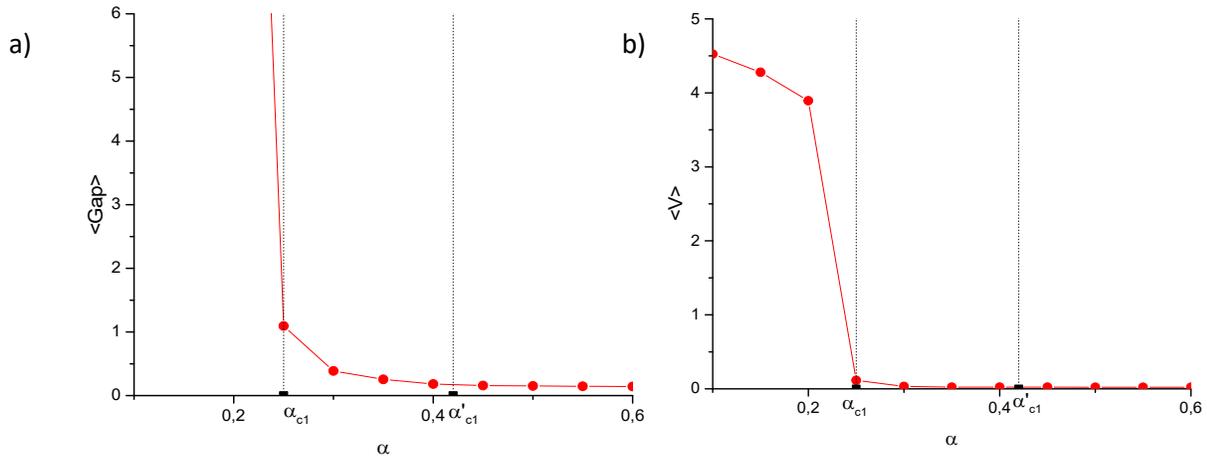


Figure 10: a) Mean gap and b) Mean velocity of the first road versus the injection rate α for the extraction rate $\beta=0.5$.

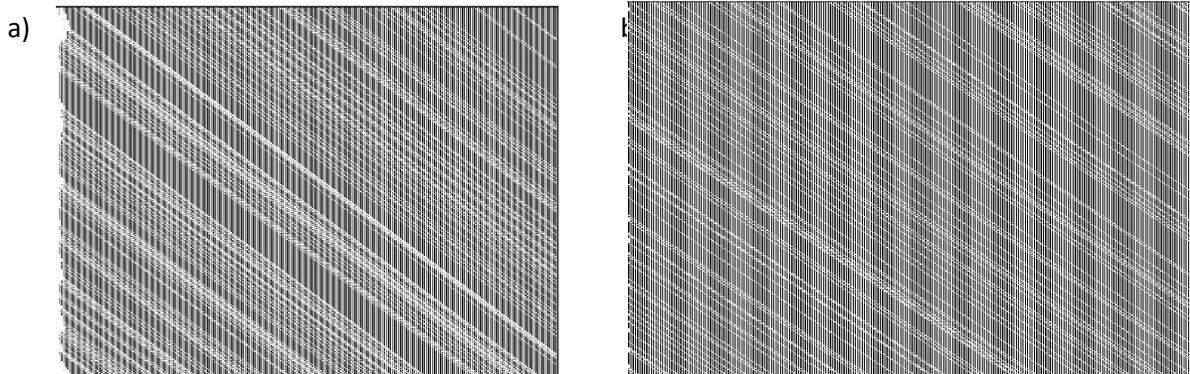


Figure 11: Space-time diagram for $\beta=0.5$ and a) $\alpha=0.3$, b) $\alpha=0.5$: the black cells are occupied (vehicle) and the whites are empty

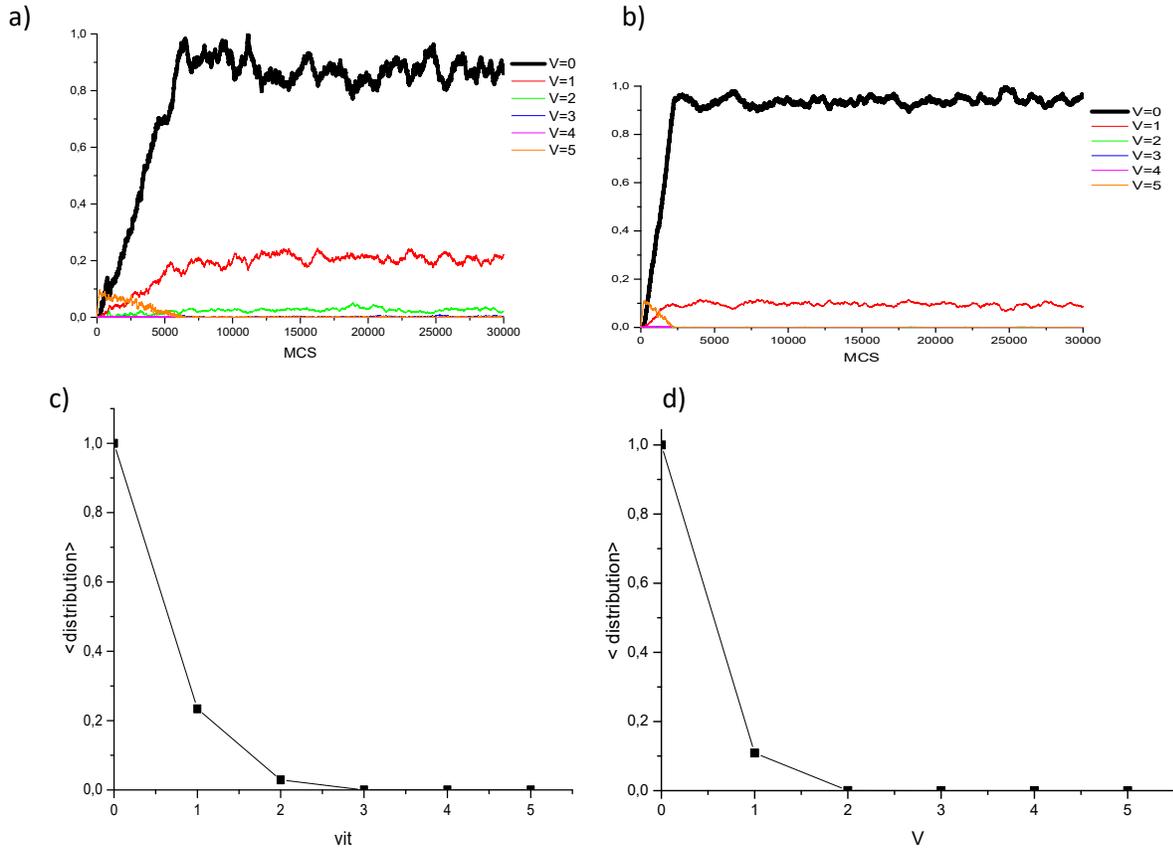


Figure 12 : Distribution Of The Velocities (V) In Time (MCS Being The Time Step) For $B=0.5$ In The HD1 A) $A=0.3$, And In The HD2 B) $A=0.5$. Mean Distribution $\langle \text{Distribution} \rangle$ Of The Velocities For $B=0.5$ In The HD1 C) $A=0.3$, And In The HD2 D) $A=0.5$

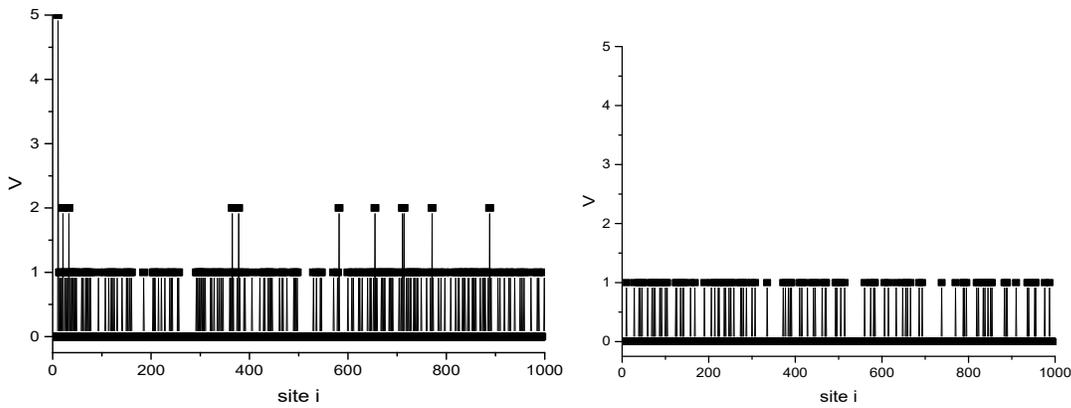


Figure 13: Velocity Profile At A Random Instant For $B=0.5$ And A) $A=0.3$ (HD1) B) $A=0.5$ (HD2), V Being The Velocity At The Random Instant For Each Vehicle Present On The Road

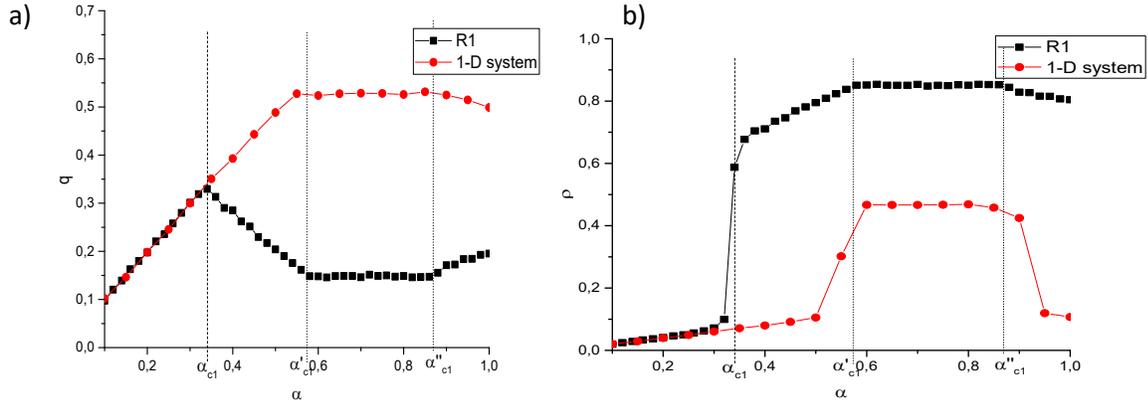


Figure 14: Comparison between the observables of the first road and of the one-dimensional system for $\beta=0.7$: a) the flow b) the density

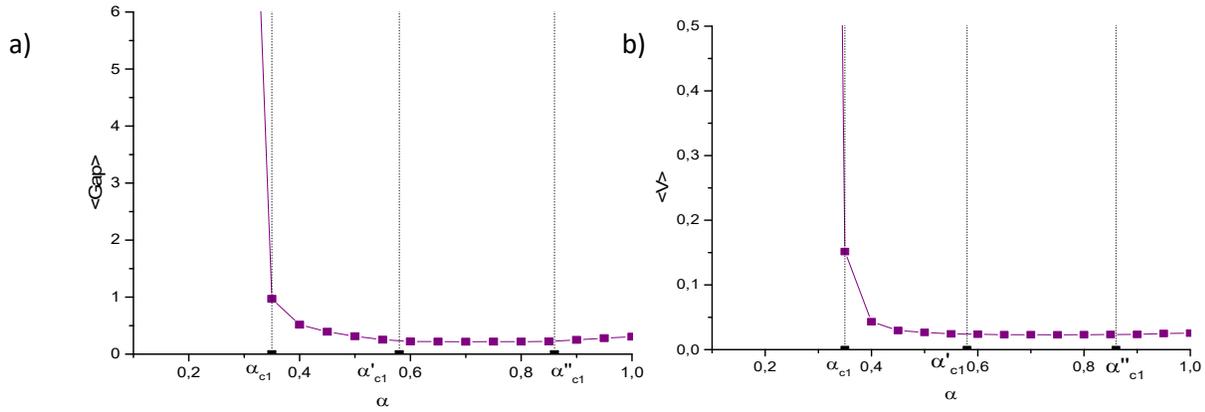


Figure 15: a) Mean gap and b) Mean velocity of the first road versus the injection rate α for the extraction rate $\beta=0.7$.

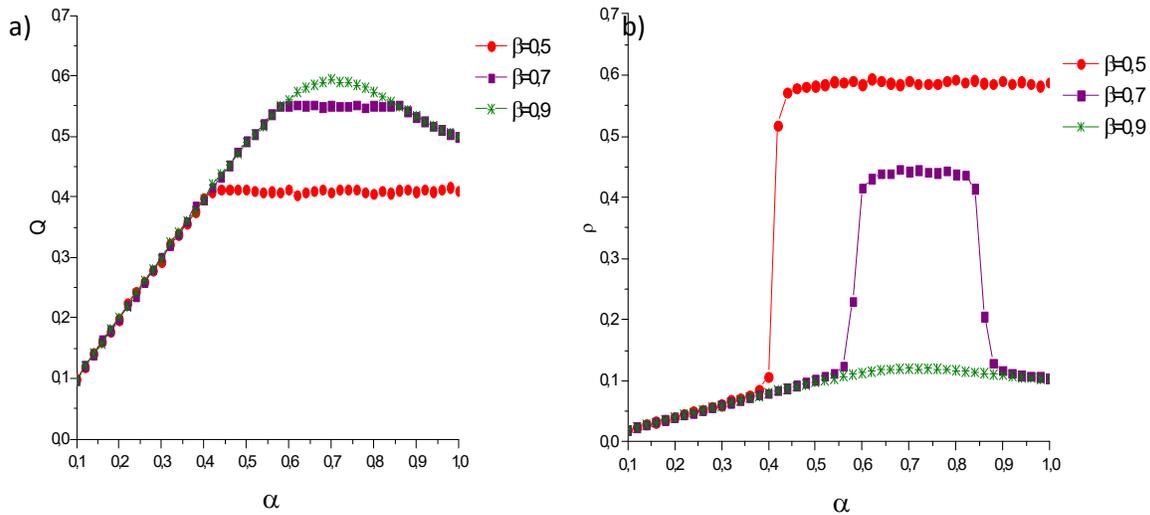


Figure 16: a) Flow and b) Density of the second road versus the injection rate α for different values of the extraction rate β .

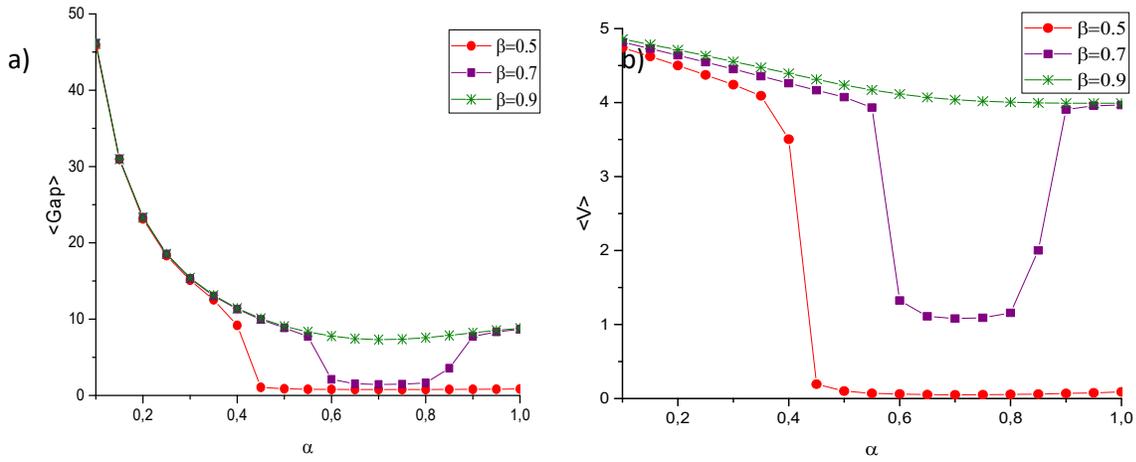


Figure 17: a) Gap and b) Mean velocity of the second road versus the injection rate α for different values of the extraction rate β .

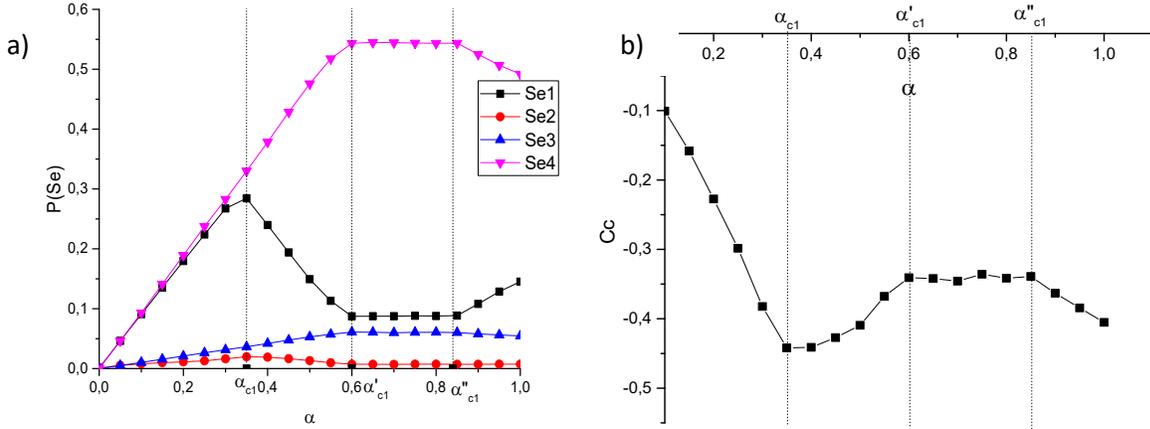
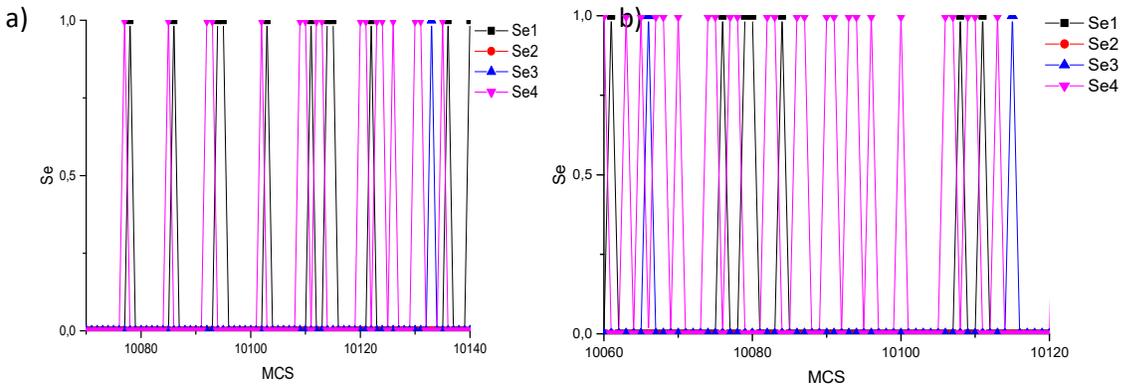


Figure 18: a) The ratio of the occurrence of the scenarios Se1, Se2, Se3 and Se4 versus the injection rate α the extraction rate $\beta=0.7$, b) The cross correlation between the ratio of occurrence of the scenarios Se1 and Se4 versus the injection rate α of the extraction rate $\beta=0.7$



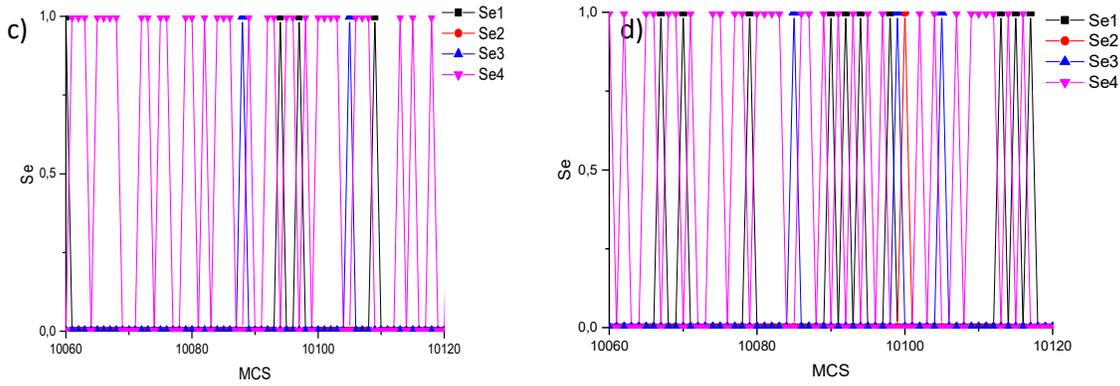


Figure 19: The occurrence in time of the different scenarios for $\beta=0.7$ and $Pe=0.1$ and a) $\alpha=0.2$ b) $\alpha=0.5$ c) $\alpha=0.7$ d) $\alpha=0.95$: this occurrence is quantified by 0 if the scenario does not occur and 1 if it does

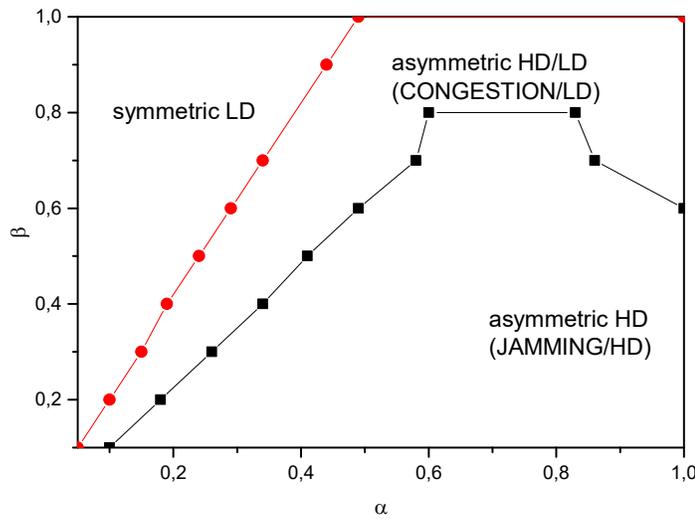


Figure 20: Phase Diagram Of The Global System (1st Road/2nd Road)

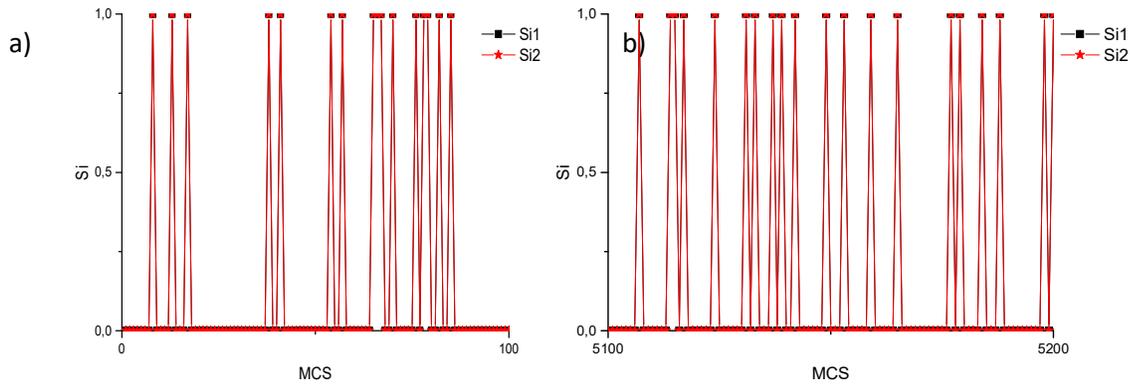


Figure 21: The Occurrence In Time Of The Different Injection Scenarios For $B=0.7$ And $Pe=0.1$ And $A=0.2$ A) At The Beginning Of The Simulation B) Throughout The Simulation: This Occurrence Is Quantified By 0 If The Scenario Does Not Occur And 1 If It Does

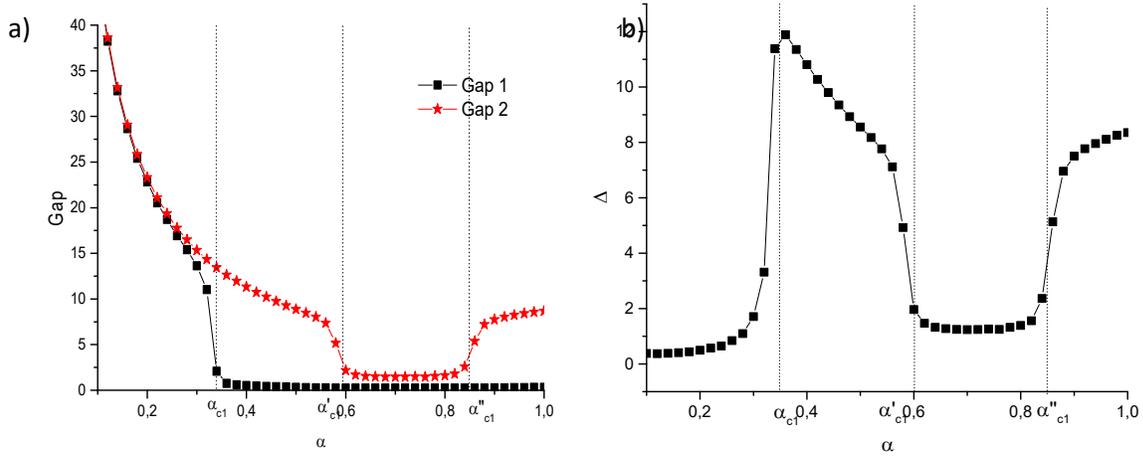


Figure 22: a) The average gap of the first road Gap1 and of the second road Gap2 versus the injection rate α for the extraction rate $\beta=0.7$ b) the difference between the average gap of the first road Gap1 and of the second road Gap2: $\Delta(GAP) = GAP2 - GAP1$

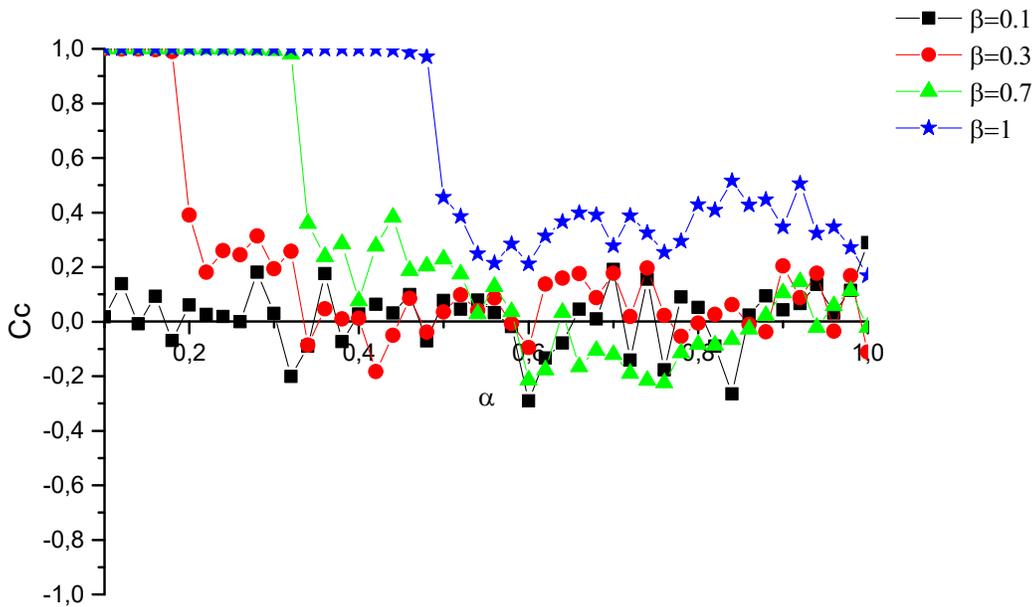


Figure 23: The Cross Correlation Between The Flow Of The First Road And The Second Road Versus The Injection Rate α For Different Values Of The Extraction Rate β