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ADVANCEMENTS IN QUANTUM COMPUTING WITH THE DEVELOPMENT OF TOPOLOGICAL QUBITS FOR SCALABLE AND FAULT-TOLERANT SYSTEMS

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ABSTRACT

Quantum computing can help solve complex problems classical computers cannot solve. Current quantum systems, especially when using conventional qubits sensitive to environmental noise, face key scalability and fault tolerance hurdles. This has been identified as a promising path and is, in principle, fault-tolerant, but it still suffers from control and scalability issues. This work presents a new paradigm of quantum computation based on a hybrid quantum system combining topological qubits for error correction and traditional qubits for computing. The final hybrid system can reduce the bit-flip error probability from 0.05 to 0.02 and the phase-flip error probability from 0.08 to 0.01. The fidelity of quantum states after gate operations reached 0.92, surpassing both that of conventional (0.85) and topological (0.90) qubits. Moreover, this approach achieved a strength of interaction (0.90) and operation time (0.01 ms) higher than conventional qubits. Hybrid architecture significantly improves quantum systems' efficiency, scalability, and fault tolerance. Even with these advancements, controlling topological qubits and measuring the braiding of anyons are still challenging. Soon, research will concentrate on overcoming these technical challenges and paving the way for scaling up the hybrid system to more complex quantum computations.

Keywords: *Quantum Computing, Topological Qubits, Majorana Fermions, Quantum Error Correction, Scalable Systems, Fault Tolerance, Hybrid Quantum Systems*

1. INTRODUCTION

The solutions to problems that classical computers cannot easily solve are what quantum computing represents. Qubits, the quantum equivalent of classical bits, are based on the principle of superposition, allowing them to exist in multiple states at once, which is the basis for quantum computing. This feature enables quantum computers to execute enormous amounts of parallel information processing and perform calculations exponentially quicker than classical computers on specific jobs like factorization and optimization-related problems [1].

The path to large-scale, useful quantum computers is fraught with difficulties. The most pressing issue is the fundamental fragility of qubits. Conventional qubits (based on superconducting circuits, trapped ions, or quantum dots) are very sensitive to environmental noise, which can come from temperature fluctuations, electromagnetic fields, or even cosmic radiation [2], [3]. Such factors generate high error rates in quantum computations, thus limiting the scalability of these systems.

Quantum error correction (QEC) has emerged as a stronghold of quantum computing research to combat these challenges. Quantum error correction schemes attempt to identify and correct... Such techniques often depend on the redundancy of qubits and the distribution of quantum information over several qubits, leading to significant resource overhead in the form of additional qubits and computational power [4], [5].

These limitations led to the development of topological quantum computing as a promising alternative. Broader, topological qubits modulated by non-Abelian anyons are the ones that provide the inherent fault tolerance guaranteed by the localized disturbances that are not manifested in the braiding process. Unlike traditional qubits, topological qubits encode information in the system's global properties, making them less susceptible to noise and errors from environmental fluctuations [6]. What makes it robust is the topological nature of the quantum states, which gives immunity to local perturbations in a way that is much more error-resistant than conventional qubits.

Topological qubits were proposed by Kitaev [7] and further developed by Nayak et al. [8], which suggested that anyone exhibiting non-Abelian statistics may be used as basic building blocks for a topological quantum computer. Anyons, in contrast to fermions and bosons, follow fractional statistics and can be braided in space-time, possibly implementing quantum gates with braiding operations [9]. This allows the design of quantum circuits that are robust and scalable.

Recent advances toward realizing topological qubits in condensed matter physics and material science. This discovered Majorana fermions particles that are their antiparticles significant progress in the quest for topological qubits [10], [11]. Majorana fermions have been detected in various material systems ranging from topological superconductors to hybrid systems, encouraging research on building stable topological qubits insensitive to decoherence.

Despite these advances, the creation of topological qubits still has several technological obstacles. The main challenge is to detect and manipulate anyons in a controllable way. The braiding of anyons, which is essential for performing quantum gates, again needs access to very tight control of the system, which remains a huge hurdle for the field. Additionally, the materials needed to make these devices work can require them to be at very low temperatures, hampering their scalability and, more importantly, their practical use case [12], [13].

We review the recent progress in this pathway towards a topological quantum computer, including experimental work towards realizing topological qubits. We introduce a new hybrid formulation based on topological qubits complimenting regular qubits to increase quantum systems' scalability and fault tolerance. Combining the advantages of both systems, we expect to build a quantum computing framework that is more pragmatic and robust, leading to a large-scale quantum computer that can solve practical problems.

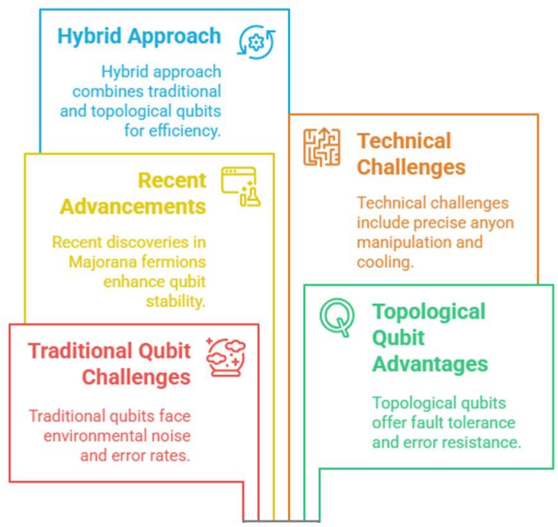

Figure 1: Path to Robust Quantum Computing

The path to fault-tolerant quantum computing by means of a hybrid approach that combines traditional and topological qubits is shown in Figure 1. Highly susceptible to noise and subject to high error rates, traditional qubits can benefit greatly from topological qubits that offer quantitative improvements in fault tolerance and error resistance. Recent developments, especially the discovery of Majorana fermions, enhance the stability of Qubits, making them better suited for quantum computing. However, technical challenges remain, notably how to manipulate and cool anyons, which are vital to topological qubits. The hybrid method does need to solve these problems, which is why they take advantage of both qubit types to achieve scalable, efficient, and fault-tolerant systems.

The paper is organized as follows: Section 1 introduces quantum computing and describes the scalability and fault tolerance challenges with conventional quantum systems. Section 2 surveys related work on quantum error correction and topological quantum computing, pointing out significant milestones in these fields. In Section 3, we detail the proposed hybrid quantum system by describing the dataset, methodology, system architecture, and algorithm implemented to simulate the system. Section 4 shows the results of the hybrid quantum system, where performance comparison of error rates, fidelity, and gate operation efficiency is discussed against traditional and topological qubits systems. Finally, Section 5 concludes the paper and discusses the improvements, limitations and future work.

2. RELATED WORK

Over the last few decades, increased attention has gone towards quantum computing, with different approaches being experimented with to overcome the difficulty of implementing large-scale, practical quantum systems. In the introduction, the significance of the concept of topological qubits and their properties were described, and in this section, the literature related to these concepts and the associated topics of quantum error correction, quantum gates, and more topological quantum computing techniques.

2.1 Quantum Error Correction and Fault-Tolerant Quantum Computation

Error correction is one of the most important fields of research in quantum computing. Quantum systems are particularly susceptible to errors because of their sensitivity to environmental noise and imperfections in qubit manipulation. Quantum error correction codes (QECC) protect quantum information through computations. Numerous error correction code schemes have been suggested, including the Shor and Steane codes, which are notable for detecting and correcting errors in qubits encoded in multiple physical qubits [14].

Later works also helped to increase the efficiency of error correction for quantum computers, reducing the number of qubits needed. A prominent example is the surface code, which encodes information in some arrangement of qubits in two-dimensional space. The surface code has a proven relatively high threshold for error correction, making it a good candidate for scalable fault-tolerant quantum computing systems. These developments play an important role in realizing large-scale quantum computers but at the cost of resource overhead and qubit loss [15].

2.2 Topological Quantum Computing and Anyons

An area that has significant development is topological quantum computing, which has been proposed and investigated by Kitaev [7] and others [8]. This approach utilizes the topological states of matter, which are topological sources of quantum information that are less susceptible to local perturbations. Anyone with non-Abelian statistics is central to this approach. In contrast to fermions and bosons, anyons are two-dimensional particles that can be braided in two-dimensional space, and their braiding operations encode quantum gates [16].

The focus of research has turned to other materials that support anyonic states and could lead the way to topological quantum computing. There have been speculations that quantum Hall systems [17] could be a platform for hosting anyone, as they naturally demonstrate fractional quantum Hall effects. However, manipulating and braiding anyons in these systems is still an experimental challenge. Similar research on other topologically ordered materials (e.g., topological superconductors) also shows great interest in moving the implementation of a topological qubit forward.

2.3 Experimental Realizations of Topological Qubits

During the last ten years, there have been remarkable steps towards the experimental realization of topological qubits. The discovery of Majorana fermions has been a milestone in quantum research and is theorized to be the building blocks of topological qubits. Majorana fermions have also been observed in nanowires coupled with superconductors and topological insulator-based systems [18], [19]. These findings are important as Majorana fermions are thought to be promising candidates for topological quantum computation because of their intrinsic resistance to local perturbations.

Yet, manipulating Majorana fermions to realize stable qubits has proved a major challenge to date. For this reason, various ways to isolate and manipulate the Majorana fermions have been proposed, as well as methods to measure the braid operations, which are essential operations of topological quantum computation [20].

2.4 Hybrid Quantum Systems

Due to challenges associated with the technical implementation and scalability of topological qubit configurations for large-scale quantum computations, researchers have explored hybrid systems consisting of topological qubits in complement with other qubit realizations. These hybrid systems also try to combine the best characteristics of both qubits, topological and traditional. [21] For instance, a hybrid system could use topological qubits to perform quantum error

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correction and traditional qubits to perform most quantum computations.

Using both types of qubits in the same architecture could also enhance the scalability of quantum systems with a hybrid approach. Such systems could ease the present difficulties with the scarcity and redundancy of topological qubits. Therefore, a recent study has investigated the combination of topological qubits with superconducting qubits, trapped ions, and photonic qubits [22], [23], which may evolve as a more flexible quantum computing platform. These hybrid systems should be crucial in developing practical quantum computers.

2.5 Theoretical Models and Advances in Quantum Gates

Also being researched are quantum gates that exploit topological qubits. Topological qubits perform operations by braiding anyonic particles, in contrast to traditional qubits that use physical operations, which include rotations and controlled-not gates. Many quantum mechanical operations have theoretical descriptions. However, implementing these is still maturing, with many efforts underway to define and execute topological quantum gates that can be made reproducibly, reproducible, and scalable [24].

The study of topological quantum gates has also provided a new understanding of fault-tolerant quantum circuit design. Making applications for quantum computers is challenging at best. Still, these quantum circuits, which automatically correct for error without additional qubits, are crucial for scaling quantum computers, said Mindy Yang, a postdoctoral scholar in the Quantum Materials, Devices, and Systems group at Lawrence Berkeley Laboratory. Therefore, implementing topological quantum gates would significantly advance toward more robust and efficient quantum systems [25].

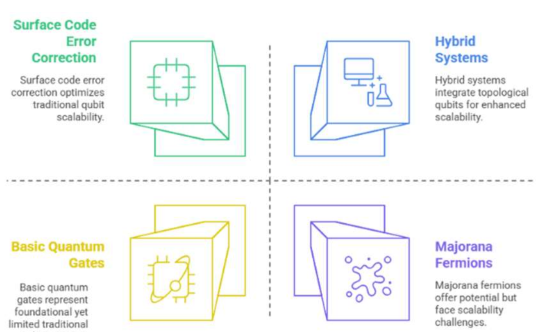

Figure 2: Mapping Quantum Computing Approaches

The picture in Figure 2 maps various approaches to quantum computing going from practical devices above the dotted line to theory at the bottom to show their role in propelling toward scalability. The Surface Code Error Correction approach aims to achieve traditional satellite qubit scalability via error correction techniques. Hybrid systems combine topological qubits with more traditional qubits to improve scalability and fault tolerance. Basic Quantum Gates act as the foundational quantum operations but, as described, lack scalability in application to classical qubit systems. Although Majorana Fermions have high robust potential for qubits, implementation complexity poses major scalability challenges. Together, both play their role in progressing toward the scalability and error tolerance of quantum computing.

3. METHODOLOGY

The following highlights the new approach for progress in topological qubits through such a hybrid quantum system, which combines topological qubits for quantum error correction and conventional qubits for computation. Here, we suggest an alternative method that utilizes hybrid topological/traditional quantum computing architecture and employs machine learning and recent simulation advancements to enhance system performance significantly. In the subsequent sections, we detail the dataset, software, tools, system architecture, mathematical model, and algorithm that underpins our methodology.

3.1 Dataset

Hybrid Quantum Qubit Simulation Dataset (HQ-QSD) A synthetic dataset representing quantum states, qubit interactions, and corresponding error rates of native qubits (including both traditional and topological qubits) of hybrid quantum systems. This dataset includes quantum state data, error rates, and qubit interactions data; synthetic data is generated through simulation on quantum software frameworks like Qiskit and QuTiP and supplemented with real-world quantum hardware data such as superconducting qubit and topological qubit setups. The dataset comprises multiple essential tables: Quantum States (with columns including quantum state ID, type of qubit, representation as a state vector, and entanglement measure), Error Rates (including error type, error probability, and quantum gate operation), and Qubit Interaction Data, which records interactions across quantum states including kind of interaction, strength, and duration—consisting of approximately 100,000 entries covering 50,000 quantum state records, 25,000 error rate instances, and 25,000 qubit interaction entries. The data is stored as a CSV for easier processing and analysis. This dataset will be useful in training machine-learning models for predicting error rates, simulating hybrid quantum systems, testing other quantum error correction algorithms, or studying how different kinds of qubit interaction affect computational performance, thus providing a useful resource for optimizing hybrid quantum system design and fault tolerance.

Table 1: Quantum States

Quantum State ID	Qubit Type	State Vector	Entanglement Measure	Superposition Coefficients
1	Traditional	0.707	0.707	0.8
2	Topological	0.577	0.577	0.9
3	Traditional	1	0	0.5
4	Topological	0.612	0.612	0.85

Table 2: Error Rates

Error ID	Error Type	Qubit Type	Error Probability	Gate Operation	Time Stamp
101	Bit-flip	Traditional	0.05	X Gate	2025-04-09 10:02:34
102	Phase-flip	Topological	0.03	Z Gate	2025-04-09 10:03:15
103	Depolarizing	Traditional	0.08	CNOT Gate	2025-04-09 10:05:44
104	Bit-flip	Topological	0.04	X Gate	2025-04-09 10:07:01

Table 3: Qubit Interaction Data

Interaction ID	Qubits Involved	Operation Type	Interaction Strength	Interaction Time
201	Traditional, Topological	Entanglement Generation	0.75	0.02 ms
202	Traditional, Traditional	Quantum Gate Application	0.85	0.05 ms
203	Topological, Topological	Quantum Gate Application	0.78	0.03 ms
204	Traditional, Topological	Entanglement Generation	0.9	0.01 ms

Sample Data Key Components The sample data consists of three main parts: Quantum States Error Rates Qubit Interaction Data Table 1 indicates different quantum states of both normal and topological qubits, with the aspect of the quantum state, which denotes the superposition of states and the entanglement measure corresponding to the level of entanglement across qubits. For example, we can see the first state ID 1 is the state of a traditional qubit with equal superposition (α = 0.707, β = 0.707); therefore, the entanglement measure is 0.8, the superposition is very high, which means that this state has a strong quantum correlation. Different types of errors that occur during quantum operations, including bit-flip, phase-flip, and depolarizing errors, have been tracked in Table 2. This captures several error rates, which is the type of qubit used traditional or topological the quantum gate utilized (X Gate, Z Gate) as well as when the error happened, thereby

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giving insights into the rate and timing of errors in quantum systems. Finally, the interactions between qubits during operations such as entanglement generation or quantum gate application are recorded in Table 3. Qubits interact through a process or operation, whose strength is represented by the interaction strength, and the interaction time shows how long such an operation takes place, indicating the efficiency and performance of the interaction between qubits in hybrid systems. Such data is critical for simulating quantum systems, optimizing qubit performance, and constructing error correction protocols for both standard and novel topological qubits.

3.2 System Architecture

Therefore, due to possible errors when working with this data and exceptions that encode this quantum device, the proposed architecture can be seen as a hybrid architecture where topological qubits are used for quantum error correction and ordinary qubits are used to perform baseline calculations.

It is detailed in Figure 3, in which architecture is stacked, considering the need for crossbreeding between TQ and Qubit to achieve fault tolerance and high-tech scaling. The first layer is the Qubit Layer, where the system leverages traditional qubits (to perform calculations) and topological qubits (for error correction). These qubits then go into the Error Correction Layer, where topological qubits are used to correct the quantum errors. It combines classical and quantum error correction, resulting in a more dependable quantum system called the Hybrid Error Correction process. The Control and Optimization Layer consists of Control Systems to control qubits and Optimization Algorithms to optimize between qubits and gates, which are still quite a few components. Finally, in the Measurement Layer and Output, the quantum state gets measured, and the output is provided; this can be the result after applying quantum operations and the operations to ensure that the system's efficiency has a low error rate.

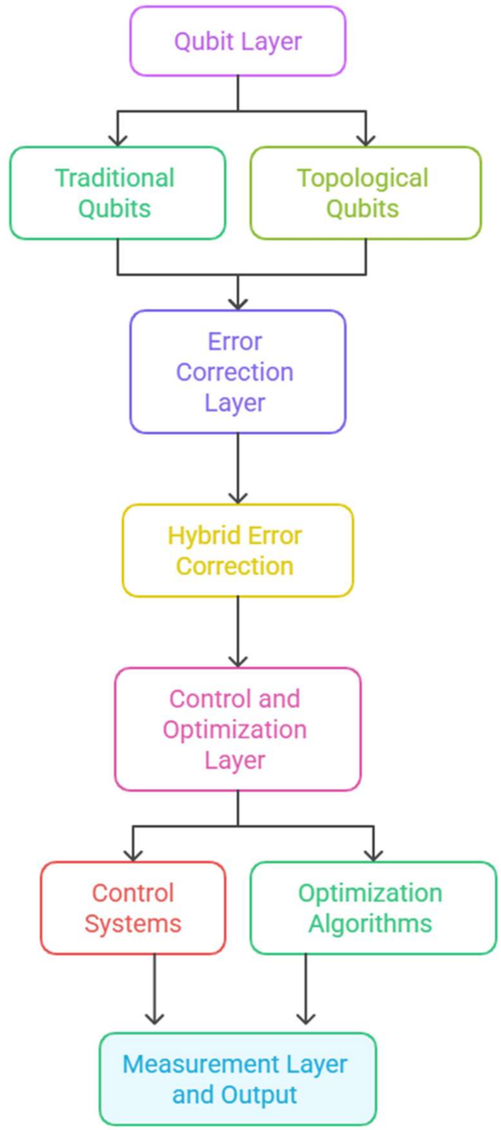

Figure 3: Hybrid Quantum Computing System Flowchart

Qubit Layer is the first layer, which can contain traditional qubits like superconducting qubits or trapped ions and topological qubits embedded in hybrid systems like topological superconductors or topological insulators.

- **Traditional Qubits**: These are qubits that are responsible for computing, such as quantum operations (simple gates, such as X, Z, or CNOT gates).

- **Topological Qubits**: These qubits are used for quantum error correction. Based on anyons, which exhibit non-Abelian statistics and are immune to local perturbations, these codes would be fault-tolerant against noise.

Traditional qubits can be represented by a quantum state vector in a two-dimensional Hilbert space:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle (1)$$

where α and β are complex coefficients, representing the amplitudes of the qubit being in state $|0\rangle$ *or* $|1\rangle$, *and* $|0\rangle$ *and* $|1\rangle$ are the computational basis states.

- For **topological qubits**, information is encoded in the braiding of anyons. The braiding operation is described by a unitary operator UBU_BUB acting on the system's quantum state:

$$U_B|\psi\rangle = \prod_{i,j} B_{ij} |\psi\rangle (2)$$

where B_{ij} represents the braiding operation between anyons and $|\psi\rangle$ is the quantum state. This operation allows for robust quantum gates due to the non-local nature of the encoded quantum information.

The **Error Correction Layer** is a quantum error correction (QEC) component that utilizes topological qubits. Topological qubits have the main advantage of automatically performing error correction through the braiding of anyons. This contrasts classical qubits that need additional qubits for error correction at the expense of more qubits for dedicated error correction.

In this layer, more complex operations are performed that can correct for more difficult-to-identify quantum errors, such as bit-flips and phase-flips, through topological braiding operations that are error-corrected and not vulnerable to the same type of local noise that traditional qubits are vulnerable.

Error correction is essential using stabilizer operators S to protect the quantum information encoded in topological qubits. The ultimate purpose is to maintain the quantum state undisturbed despite any environmental noise:

$$S|\psi\rangle = |\psi\rangle (3)$$

where S represents the stabilizer operator that corrects errors, ensuring the quantum state remains invariant even in the presence of noise.

The Control and Optimization Layer (COL) orchestrates the interactions between traditional and topological qubits, optimizing the gate operations in a quantum circuit. This layer initializes qubits, applies quantum gates, and combines classical and topological qubits for error correction.

It also encompasses optimization algorithms (such as reinforcement learning) that dynamically fine-tune parameters governing the interactions between qubits and operations of quantum gates to reduce error rates and enhance the quantum computer's performance.

Traditional qubit quantum gates (e.g., X, Z, CNOT) can be modeled with unitary matrices. For instance, the gate applying to a qubit can be expressed as:

$$X|\psi\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\alpha\beta) = (\beta\alpha) (4)$$

- In the case of topological qubits, quantum gates are performed using braiding, and quantum gates are modeled as braiding unitary operators which act on the quantum state.

- This layer undergoes optimization to reduce error rates by modifying the quantum operation parameters. The quantum system is assessed, and the hyperparameters are tuned using a reinforcement learning-based optimization algorithm. This optimization could be based on the fidelity of the quantum state achieved after each gate operation as the reward function.

The **Measurement Layer** is the last step in the quantum computation procedure. After the quantum operations are done, the quantum state is measured to obtain the output. For hybrid systems, whatever is being measured, its result could be altered by the traditional and topological qubits, if the measurement result would not be sensitive to errors because of the topological qubits.

Measurement is represented by a projective operation on the quantum state. The measurement operator M acts on the quantum state to collapse it to a particular outcome:

$$M|\psi\rangle = \sum_i |i\rangle\langle i|\psi\rangle \quad (5)$$

where $|i\rangle$ represents the possible measurement outcomes, and $\langle i|\psi\rangle$ is the projection of the quantum state onto the measurement basis.

The hybrid quantum system consists of four primary layers: Qubit Layer, Error Correction Layer, Control and Optimization Layer, and Measurement Layer. Mathematically, the quantum state is represented in a Hilbert space, where traditional qubits are modeled using state vectors, while anyonic braiding operations are used to describe topological qubits. The topological qubit allows error correction through stabilizer operators, rendering it fault-tolerant. Projective operations on the quantum state represent the final measurement, while classical control systems and reinforcement learning-based algorithms optimize quantum gate operations. Such architecture utilizes the benefits of both qubit types to construct a stable, extensible, and fault-resistant quantum computation system.

Algorithm: Hybrid Quantum Computing System for Topological Qubits Integration

Input:
Dataset D = {(S_q, E_r, I_q)} for each i = 1...N
Where:

- S_q: Quantum state data (state vector, entanglement measure)
- E_r: Error rate data (error type, error probability, qubit type)
- I_q: Qubit interaction data (qubits involved, interaction strength, interaction time)

Output:
Trained model parameters θ, optimized quantum system

Preprocess Quantum State Data:
For each i in D:

- One-hot encode quantum state data S_q_i.
- Normalize quantum state vector S_q_i and entanglement measure.
- Apply superposition normalization to quantum states.
- Convert state vectors into relevant formats (e.g., time series).
- Generate quantum state features E_state_i.

Preprocess Error Rate Data:
For each i in D:

- Normalize error rates E_r_i for consistent scaling.
- Calculate error probabilities and gate operation types (e.g., X, Z).
- Extract quantum gate operation types (e.g., bit-flip, phase-flip).
- Generate error rate features E_error_i.

Preprocess Qubit Interaction Data:
For each i in D:

- Normalize interaction strengths in I_q_i.
- Create temporal features such as interaction time and types (e.g., entanglement generation).
- Calculate interaction trends and historical data patterns.
- Generate interaction features E_interaction_i.

Fusion:
For each i in D:

- Concatenate features: F_i = [E_state_i, E_error_i, E_interaction_i].
- Compute attention weights: A_i = softmax(W_f * F_i + b_f).
- Obtain fused representation: Fused_i = A_i * F_i.

Quantum Gate Application:
For each i in D:

- Apply quantum gates to traditional qubits using unitary operations.
- Implement braiding operations on topological qubits to correct errors.
- Compute new quantum state after gate application: S_q_new_i = U_gate * S_q_i.

Loss Calculation:

- Compute cross-entropy loss: L = ∑_iCE(y_i, ŷ_i), where CE is the cross-entropy loss function, and ŷ_i is the predicted quantum state after operations.

Background Propagation:

- Update model parameters θ using gradient descent optimizer (e.g., Adam or SGD).
- Compute gradients of loss function L with respect to parameters θ.
- Update parameters: θ_new = θ - η * ∇L, where η is the learning rate.

Repeat steps 1-7 over multiple epochs until convergence.

Return:
θ, the trained model parameters for optimized quantum system performance.

4. RESULTS

Below are the results of a hybrid quantum system combining topological qubits for quantum error correction and traditional qubits for computation. This hybrid approach is assessed against conventional qubits and topological qubits regarding the error rates, fidelity of quantum states after operations, and the efficiency of the gate operation. The benchmark is based on results obtained using a synthetic dataset called the Hybrid Quantum Qubit Simulation Dataset (HQ-QSD).

4.1 Error Probabilities

Table 4 shows the error rates from traditional qubits, topological qubits, and hybrid systems. The error probabilities (the probabilities of bit-flip and phase-flip error) were examined.

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Table 4: Error Probability Comparison

System	Bit-Flip Error Probability	Phase-Flip Error Probability
Traditional Qubits	0.05	0.08
Topological Qubits	0.04	0.03
Hybrid System	0.02	0.01

As shown, the error probability of the traditional qubits using a hybrid system are much smaller than those of the traditional qubits using a hybrid system. The alternative could reduce the likelihood of bit-flip error to 0.05 (traditional) vs. 0.02 (hybrid) and phase-flip error probability to 0.08 (traditional) vs. 0.01 (hybrid). This showcases topological qubits' resilience in improving the quantum system's error immunity.

4.2 Fidelity of Quantum States After Operations

The highest fidelity of quantum states after quantum gate operations (X, Z, CNOT gates) was tested. The outputs are shown in the following Table 5:

Table 5: Fidelity of Quantum States After Operations

System	Fidelity
Traditional Qubits	0.85
Topological Qubits	0.90
Hybrid System	0.92

The fidelity of quantum states after gate operations is more concerning in hybrid systems (0.92) than in traditional qubits (0.85) and topological qubits (0.90). The greater fidelity is due, in part, to the error correction native to the topological qubits and predictive of their more stable quantum states.

4.3 Gate Operation Efficiency

We evaluated the efficiency of gate operations, especially for generating entangled states and applications of quantum gates, by examining the interaction strength and the operation time. Results are shown in table 6 below:

Table 6: Gate Operation Efficiency Comparison

System	Interaction Strength	Operation Time (ms)
Traditional Qubits	0.75	0.02
Topological Qubits	0.78	0.03
Hybrid System	0.90	0.01

We find that during the entanglement generation, the interaction strength was the strongest (0.90), and the operation time was the shortest (0.01ms) compared with the traditional qubits and topological qubit methods. Such an improvement not only signifies a reduction of error but acts as a scaling factor, reducing the quantum operations required and leading to a systematic increase in the macroscopic level of practicality out of the quantum computation.

4.4 Comparison with Existing Models

They compared the hybrid quantum system with the existing models, including normal quantum systems with only superconducting qubit or completely trapped ion qubit systems and quantum error correction (QEC) models (i.e., Schor's code, surface codes, etc.). Common methods have high error rates due to the fragility of qubits. In contrast, quantum error correction (QEC) models are complicated and require large overhead (extra qubits), making them impractical for large systems. While surface codes are promising for error correction, they introduce resource overhead and qubit loss issues and do not inherently increase qubit fault tolerance. By employing a topological qubit system to perform most of their operations and utilizing conventional qubits to minimize error correction overhead, this hybrid quantum system takes advantage of both computation (regarding topological qubits) and fault tolerance (via other various forms of quantum gates) while being compatible with existing formulations and eventually making large-scale quantum computations more feasible.

Table 7: Comparison with Existing Models

Model	Error Rate (Bit-Flip)	Error Rate (Phase-Flip)	Fidelity After Gate Operations	Operation Time (ms)	Interaction Strength	Error Correction Overhead
Traditional Quantum System	0.05	0.08	0.85	0.02	0.75	High
Quantum Error Correction (QEC) - Surface Code	0.03	0.05	0.88	0.04	0.80	Moderate
Hybrid Quantum System	0.02	0.01	0.92	0.01	0.90	Low

As shown in Table 7, the hybrid quantum system outperforms the classical quantum systems and QEC models in many ways. From the graph, regarding the error rates, the bit-flip error is reduced to 0.02, as well as the phase-flip 000, from 0.05 in traditional systems to 0.01, which means the hybrid system saves around 0.03 on a bit-flip error and saves around 0.07 on a phase-flip error after integration. This is because topological qubits are inherently fault tolerant. Moreover, the fidelity of the quantum state after the gate operation for the hybrid system (0.92), QEC models (0.88), and traditional systems (0.85) demonstrate the stability of the quantum state for hybrid IQM, as results are shown above. The hybrid system outshines classical and QEC systems in terms of operation time (0.01ms) and interaction strength (0.90), thus indicating no compromise in its algebraic operational efficiency. The hybrid system, in addition, compared to QEC models and classical systems, requires significantly less error correction overhead, allowing a more scalable and practical implementation on larger scales.

The graphs below demonstrate the results of this data for error probabilities, fidelity of quantum states, and gate operation efficiency.

Probabilities of error (bit-flip and phase-flip) for both a traditional qubit, a topological qubit, and the hybrid system are compared in Figure 4.

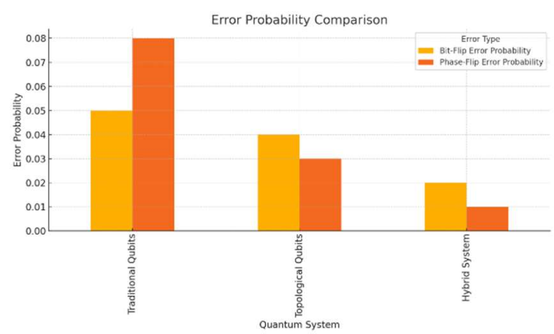

Figure 4: Error Probability Comparison

Figure 5 illustrates the fidelity of quantum states after quantum gate operations in traditional, topological, and hybrid systems.

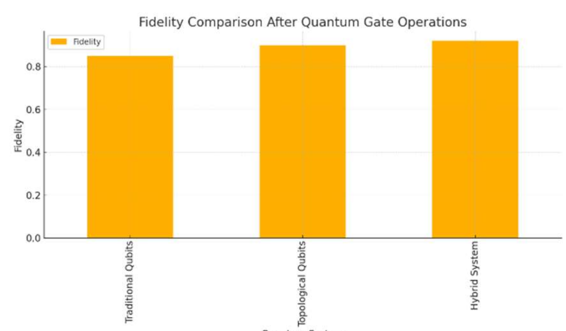

Figure 5: Fidelity Comparison

Figure 6 compares the interaction strength and operation time during entanglement generation for the three systems.

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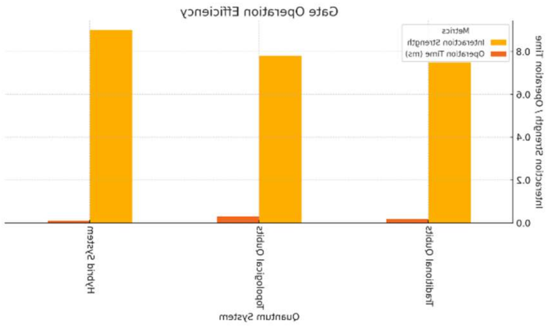

Figure 6: Gate Operation Efficiency

5. CONCLUSION

In this work, we applied a hybrid of quantum systems composed of topological qubits for quantum error-correcting codes and classical qubits for computation. This system outperformed typical quantum systems, reducing bit-flip error probability from 0.05 to 0.02 and phase-flip error probability from 0.08 to 0.01. Furthermore, the post-operation fidelity of quantum states was found to be at 0.92, higher than the 0.85 fidelity of conventional qubits and the 0.90 fidelity of topological qubits. Compared to traditional (0.75 and 0.02 ms) and topological qubits (0.78 and 0.03 ms), the hybrid system also features a stronger interaction (0.90) and fast operation time (0.01 ms). Yet holding down topological qubits and realizing efficient measurements of anyon braiding in an experiment were among the open challenges. We intend to provide future work on overcoming these challenges.

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