

PERFORMANCE ANALYSIS OF INFINITE-FAILURE NHPP SOFTWARE RELIABILITY MODELS USING POWER-TYPE LIFETIME DISTRIBUTIONS

SEUNG KYU PARK

Professor, Department of Electronic Engineering, Namseoul University, 91 Daehak-ro, Seonghwan-eup, 31020, Seobuk-gu, Cheonan-si, Chungnam, Korea
E-mail: skpark@nsu.ac.kr

ABSTRACT

This study analyzed and evaluated the performance of an infinite-failure NHPP reliability model by applying Power-type lifetime distributions (Weibull, Extended-Weibull, Rayleigh, Weibull-Extension) that can reflect the characteristics of failure rates changing in the form of powers of time and are suitable for various reliability change patterns. For this study, software failure times were utilized, and parameter calculation of the presented model was solved using the MLE procedure. By utilizing various performance analysis methods (selection of the best-fitting model using MSE and R^2 , estimation power for the true value using $m(t)$, analysis of instantaneous failure intensity using $\lambda(t)$, and evaluation of future reliability using $\hat{R}(\tau)$), it was confirmed that the Weibull-Extension model was the most efficient among the competing models. Therefore, this study newly elucidated the reliability performance of Power-type distribution models, which had been lacking in previous research. Furthermore, it is expected that if software developers efficiently utilize these research results, they can use them as basic testing data necessary for failure rate analysis during the early stages of development. These results could serve as useful information for meaningful design guidelines and testing.

Keywords: *Extended-Weibull, Infinite-Failure NHPP, Power-type, Rayleigh, Weibull, Weibull-Extension.*

1. INTRODUCTION

As digital innovation based on Artificial Intelligence (AI) rapidly spreads across industries, the importance of intelligent software as a core infrastructure and strategic technology in various application fields is becoming increasingly prominent. In particular, in AI-converged software systems where high reliability is required, even a simple defect in the software can lead to the collapse of the entire system and massive economic losses. Based on this, technologies for quantitatively evaluating and analyzing software reliability have recently emerged as an essential research topic. Probabilistic models incorporating the Non-Homogeneous Poisson Process (NHPP) have been primarily used as a representative method for evaluating software reliability. NHPP models can effectively reflect failure rates that change over time and are suitable for mathematically modeling failure patterns observed in actual software operating environments. Existing NHPP reliability research has mainly been conducted on Exponential-type and Log-type distribution models, but these models have limitations in that they can be applied restrictively to specific failure patterns [1]. In contrast, NHPP models based on Power-type distributions have the advantage of being able to explain a wider variety of

reliability failure patterns because they can reflect the characteristic that the failure rate changes in the form of a power of time. In this regard, Yang [2] applied a finite-failure NHPP model to an Exponential-type distribution and assessed the efficiency of the software reliability model in various ways. Furthermore, Yang and Kim [3] analyzed and evaluated the efficiency of Log-based distributions using an infinite-failure NHPP reliability model with a new method. Therefore, Bae [4] newly assessed the efficiency of the NHPP-based reliability model by extending the existing prior research to the Log-type distribution. On the other hand, Xiao and Dohi [5] newly evaluated the efficiency of the Weibull-type distribution through goodness-of-fit testing and predictive power analysis using the NHPP-based software reliability model. Accordingly, Yang and Park [6] presented research results that newly examined the performance of the model by extending the infinite-failure NHPP model to the Weibull-Extension and Flexible Weibull-Extension distributions. Similarly, Kim [7] analyzed reliability attributes by applying an infinite-failure NHPP model with a Power-law structure to a statistical process control method in an environment where software defects are continuously detected.

Therefore, this study aims to extend existing research to a Power-type life distribution capable of explaining various reliability failure patterns, apply it to an infinite-failure NHPP software reliability model, and then analyze the efficiency of the presented model. According to the findings of this study, we also intend to present the model with the best performance.

2. RELATED RESEARCH

2.1.1 NHPP Model

The NHPP model is a widely used stochastic framework for modeling time-dependent failure behaviors in software reliability engineering. By utilizing the mean value function ($m(t)$), the NHPP model quantitatively describes the cumulative failure process and varying failure-occurrence patterns over time. Based on observed failure data, the model enables the prediction of future software failures and reliability trends. Depending on the formulation of $m(t)$, NHPP-based models can be classified into several categories, such as Exponential-type, Log-type, and Power-type distribution models. Moreover, NHPP models are particularly effective in capturing the dynamic and non-stationary failure properties of practical software systems, making them highly applicable to reliability assessment, failure prediction, and software quality evaluation.

For an NHPP model, $N(t)$ represents the accumulated count of failures up to time t , and $m(t)$ denotes the average number of failures that may occur up to that point. As a measure of reliability growth, $m(t)$ characterizes the expected failure behavior of the software and plays an essential role in forecasting future failures. That is, the resulting NHPP-based model is expressed in Equation (1).

$$P\{N(t) = n\} = \frac{[m(t)]^n \cdot e^{-m(t)}}{n!} \quad (1)$$

Note that $n = 0, 1, 2, \dots, \infty$.

That is, $m(t)$ is an average function representing the expected cumulative failure count at time t . Thus, differentiating $m(t)$ with respect to time t yields $\lambda(t)$, which serves as a key indicator of instantaneous failure intensity in reliability performance analysis. Accordingly, the relationship between the two functions can be represented by Equations (2) and (3).

$$m(t) = \int_0^t \lambda(s) ds \quad (2)$$

$$\frac{dm(t)}{dt} = \lambda(t) \quad (3)$$

2.1.2 Infinite-Failure NHPP Reliability Model

The infinite-failure NHPP reliability model is a stochastic software reliability framework designed to describe failure processes in which failures continuously occur over time. Since $m(t)$ diverges as time progresses, the model is particularly appropriate for analyzing dynamic software systems operating in practical environments characterized by continuous fault generation and evolving system conditions. In particular, infinite-failure NHPP models are commonly formulated on the basis of Weibull-based power-type structures, enabling effective representation of varying failure-occurrence patterns, including increasing, decreasing, and fluctuating trends. Due to these properties, the models have been extensively applied to reliability assessment and fault prediction for long-running AI-driven systems and other highly complex software systems.

Therefore, this study aims to analyze infinite-failure NHPP models based on Power-type lifetime distributions by considering the practical failure situations in which additional defects may occur even during the failure repair process. Consequently, Equation (4) presents the mean value function ($m(t)$), which serves as a key indicator of reliability performance in the infinite-failure NHPP models, and $F(t)$ refers to the cumulative distribution function [8].

$$m(t) = -\ln(1 - F(t)) \quad (4)$$

In the same way, $\lambda(t)$ can be obtained by differentiating $m(t)$ as shown in Equation (5), and $f(t)$ represents the probability density function.

$$\lambda(t) = \frac{dm(t)}{dt} = \frac{f(t)}{(1 - F(t))} \quad (5)$$

In reliability analysis, $m(t)$ serves as an important indicator for predicting system reliability. In contrast, $\lambda(t)$ characterizes the instantaneous rate of failure occurrence at a specific time point. Based on these two reliability-related functions, the likelihood function associated with the NHPP-based model is given by Equation (6).

$$L_{NHPP}(\theta | \underline{x}) = \left(\prod_{i=1}^n \lambda(x_i) \right) \exp[-m(x_n)] \quad (6)$$

2.2 Infinite Failure NHPP: Weibull (Crow-AM SAA) Model

The Weibull (Crow-AMSAA) NHPP model is an infinite-failure-based power-law model in which the cumulative count of failures, $m(t)$, follows a power-law pattern over time rather than a linear growth structure. Therefore, the model serves as the fundamental model of the Power-type NHPP family, where failure occurrences increase in the form of t^b . Moreover, because the failure occurrence rate can be represented through a single shape parameter within the power-law structure, this model has been extensively employed in reliability analysis and failure prediction. Owing to its wide applicability to software reliability analysis, the model is highly appropriate for examining time-dependent failure patterns in complex software systems.

Accordingly, the reliability performance-related functions are expressed in Equations (7) and (8) [9].

$$m(t) = \int_0^t abt^{b-1} dt = at^b \quad (7)$$

$$\lambda(t) = abt^{b-1} \quad (8)$$

In this model, $a > 0$ denotes the scale parameter, whereas $b > 0$ corresponds to the shape parameter.

Subsequently, substituting the above equation into Equation (6), simplifying by applying a logarithm to both sides, the log-likelihood function is solved by Equation (9).

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln a + n \ln b + \sum_{i=1}^n (b-1) \ln x_i - a_n^b \quad (9)$$

Consequently, the maximum likelihood estimates ($\hat{a}_{MLE}$, $\hat{b}_{MLE}$) for the Weibull distribution model are obtained from Equation (9) through the MLE procedure. The corresponding nonlinear estimation equations are then solved numerically by employing the bisection algorithm. Subsequently, the final parameter estimation formulas are expressed in Equations (10) and (11).

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial a} = \frac{n}{\hat{a}} - x_n^b = 0 \quad (10)$$

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = -\hat{a} x_n^b \ln x_n + \frac{n}{b}$$

$$+ \sum_{i=1}^n \ln x_i = 0 \quad (11)$$

2.3 Infinite Failure NHPP: Extended-Weibull Model

The Extended-Weibull NHPP model is an infinite-failure-based model designed to enhance the flexibility of the conventional Weibull power structure while preserving its original power-law characteristics. In particular, this model retains the fundamental t^b -based structure of the Weibull NHPP model and extends it by introducing additional parameters capable of representing a wider variety of failure occurrence patterns. As such, this model is especially appropriate for systems influenced by multiple concurrent failure mechanisms and for environments exhibiting complex failure-rate behaviors. For this reason, it is extensively employed in the reliability field, capable of expressing various failure patterns.

Therefore, the reliability performance-related functions are expressed in Equations (12) and (13) [10].

$$m(t) = \int_0^t \lambda(\omega) d\omega = \frac{a}{b} t^b \quad (12)$$

$$\lambda(t) = at^{b-1} \quad (13)$$

In this model, $a > 0$ denotes the scale parameter, whereas $b > 0$ corresponds to the shape parameter.

Thereafter, substituting the above equation into Equation (6), simplifying by applying a logarithm to both sides, the log-likelihood function is given by Equation (14).

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln a + (b-1) \sum_{i=1}^n x_i - \frac{a}{b} x_n^b \quad (14)$$

Consequently, the maximum likelihood estimators ($\hat{a}_{MLE}$ and $\hat{b}_{MLE}$) may be determined using a numerical procedure such as the bisection method from the following equations. Therefore, the final parameter estimation equations are given in Equations (15) and (16).

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial a} = \frac{n}{\hat{a}} - \frac{1}{b} x_n^b = 0 \quad (15)$$

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = \sum_{i=1}^n x_i - \frac{a}{b} x_n^b \ln x_n$$

$$+ \frac{a}{b^2} x_n^b = 0 \quad (16)$$

2.4 Infinite Failure NHPP: Rayleigh Model

The Rayleigh NHPP model is a special case of the Weibull model and is classified as a Power-type NHPP model in which the cumulative count of failures, $m(t)$, follows a t^2 -based structure. As an infinite-failure-based NHPP model, it is capable of describing systems whose failure occurrences increase over time. Furthermore, because the failure intensity continuously rises as operating time progresses, this model is particularly suitable for systems exhibiting increasingly frequent failures. Although the Rayleigh model was originally utilized for functional modeling of electromagnetic wave phenomena, it has recently been widely adopted in reliability analysis as a well-known special case of the Power-type NHPP family.

As explained above, the Rayleigh distribution is obtained as a special case of the Weibull distribution by setting its shape parameter (α) to 2. Accordingly, by substituting $b = \frac{1}{2\beta^2}$ and simplifying the resulting expression, $F(t)$ of the Rayleigh distribution is expressed as Equation (17).

$$F(t) = 1 - \exp(-bt^\alpha) = 1 - \exp(-bt^2) \quad (17)$$

Therefore, the reliability performance-related functions are expressed in Equations (18) and (19) [11].

$$m(t) = -\ln[1 - F(t)] = bt^2 \quad (18)$$

$$\lambda(t) = m'(t) = \frac{f(t)}{[1 - F(t)]} = 2bt \quad (19)$$

Subsequently, substituting the above equation into Equation (6), simplifying by applying a logarithm to both sides, the log-likelihood function is given by Equation (20).

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln 2 + n \ln b + \sum_{i=1}^n \ln x_i - bx_n^2 = 0 \quad (20)$$

Consequently, the maximum likelihood estimators ($\hat{b}_{MLE}$) may be computed using a numerical procedure such as the bisection method from the following equations.

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - x_n^2 = 0 \quad (21)$$

2.5 Infinite Failure NHPP: Weibull-Extension Model

The Weibull-Extension NHPP model is an infinite-failure-based model designed to model complex failure patterns by extending the power structure of the basic Weibull distribution. In other words, it is a power-type NHPP model capable of representing diverse failure patterns through the incorporation of additional parameters or functions into the basic Weibull NHPP model while preserving its power structure (t^b). Therefore, by reflecting actual conditions where multiple causes of failure act simultaneously, it is useful for analyzing systems where the rate of failure occurrence is not constant over time. Furthermore, by making the shape of the failure rate function more flexible, it is possible to efficiently represent non-monotonic failure rate curves and various lifetime characteristics that are difficult to express with the existing Weibull model. For these reasons, the Weibull-Extension NHPP model has been widely applied to predicting complex failure behaviors in integrated software and hardware reliability systems.

Based on the Weibull-Extension model, the probability distribution functions are expressed in Equations (22) and (23), respectively.

$$F(t) = 1 - \exp \left[-\frac{\lambda}{a} \left(1 - e^{(at)^b} \right) - 1 \right] \quad (22)$$

$$f(x) = \lambda b (at)^{b-1} \exp \left[(at)^b + \frac{\lambda}{a} \left(1 - e^{(at)^b} \right) \right] \quad (23)$$

In this model, $a > 0$ denotes the scale parameter, whereas $b > 0$ corresponds to the shape parameter.

Accordingly, the reliability performance-related functions are expressed in Equations (24) and (25) [12].

$$m(t) = -\ln(1 - F(t)) = \frac{\lambda}{a} (e^{(at)^b} - 1) \quad (24)$$

$$\lambda(t) = \frac{f(t)}{(1 - F(t))} = \lambda b (at)^{b-1} \exp[(at)^b] \quad (25)$$

Following this, substituting the above equation into Equation (6), simplifying by applying a logarithm to both sides, the log-likelihood function is given by Equation (26).

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln \lambda + n \ln b + \sum_{i=1}^n (ax_i)^b$$

$$+(b-1) \sum_{i=1}^n \ln(ax_i) - \frac{\lambda}{a} (e^{(ax_n)^b} - 1) \quad (26)$$

In Equation (26), the shape parameter (b) was set to 1 to facilitate numerical convergence. Thus, the parameter estimators ($\hat{\lambda}_{MLE}$, $\hat{a}_{MLE}$) of this model are obtained by applying the MLE method to Equation (26), and the resulting nonlinear equations are numerically solved using the bisection method. Accordingly, the final parameter estimation equations are given in Equations (27) and (28).

$$\frac{\partial \ln L_{NHPP}(\theta|x)}{\partial \lambda} = \frac{n}{\hat{\lambda}} - \frac{1}{a} [e^{(ax_n)^b} - 1] = 0 \quad (27)$$

$$\frac{\partial \ln L_{NHPP}(\theta|x)}{\partial a} = (b-1) \frac{n}{\hat{a}} + b a^b \sum_{i=1}^n x_i^b + \frac{\lambda}{a^2} [e^{ax} - 1 - a^b b x_n^b e^{(ax_n)^b}] = 0 \quad (28)$$

3. RELIABILITY PERFORMANCE ANALYSIS

In this work, reliability properties of the models were examined based on failure-time data accumulated during software testing. Thus, to quantitatively analyze the reliability performance of the presented models, this work develops and implements an optimization-based reliability evaluation algorithm. The proposed framework consists of the following five systematic procedures.

Step 1: Verification of data usability using observed software failure times.

Step 2: Calculation of model parameters ($\hat{a}$, $\hat{b}$) through the MLE procedure.

Step 3: Assessment of model efficiency using model selection criteria (MSE and R²).

Step 4: Analysis of reliability performance-related functions including $m(t)$, $\lambda(t)$, and $\hat{R}(\tau)$.

Step 5: Overall reliability performance assessment of the considered models.

This study utilizes a dataset containing 30 software failure occurrences accumulated during 187.35 hours of operation. The corresponding failure-time records are summarized in Table 1 [13].

The cited dataset comprises 30 software failure events observed over a cumulative execution time of 187.35 hours, where the failures were observed at irregular intervals under desktop computer system operating conditions.

3.1. Step 1: Verification of Data Usability Using Observed Software Failure Times

Prior to model development, a Laplace trend analysis was conducted to verify the adequacy of the

Table 1: Software Failure Time.

Failure count	Failure time (hours)	Failure time (hours) × 10 ⁻²
1	4.79	0.0479
2	7.45	0.0745
3	10.22	0.1022
4	15.76	0.1576
5	26.10	0.261
6	35.59	0.3559
7	42.52	0.4252
8	48.49	0.4849
9	49.66	0.4966
10	51.36	0.5136
11	52.53	0.5253
12	65.27	0.6527
13	69.96	0.6996
14	81.70	0.817
15	88.63	0.8863
16	107.71	1.0771
17	109.06	1.0906
18	111.83	1.1183
19	117.79	1.1779
20	125.36	1.2536
21	129.73	1.2973
22	152.03	1.5203
23	156.40	1.564
24	159.80	1.598
25	163.85	1.6385
26	169.60	1.696
27	172.37	1.7237
28	176.00	1.76
29	181.22	1.8122
30	187.35	1.8735

failure-time dataset presented in Table 1 for software reliability assessment. According to the Laplace criterion, values falling between -2 and 2 imply that the dataset does not exhibit statistically significant trends or abnormal observations, thereby supporting its reliability and consistency for subsequent analysis. As illustrated in Figure 1, all trend-test

values are fall within the acceptance region of -2 to 2. This finding indicates the absence of extreme data points and confirms that the failure-time dataset in Table 1 satisfies the assumptions required for the reliability modeling performed in this study.

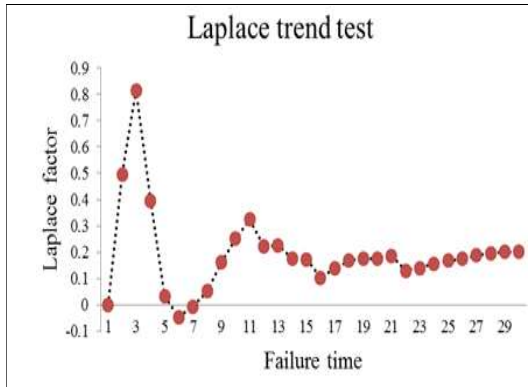


Figure 1: Laplace Trend Analysis.

3.2. Step 2: Calculation of Model Parameters ($\hat{a}$, $\hat{b}$) Through the MLE Procedure

The parameter estimates $\hat{a}_{MLE}$ and $\hat{b}_{MLE}$ of the presented model were obtained through the MLE procedure, and the corresponding estimates are presented in Table 2 [14].

3.3. Step 3: Assessment of Model Efficiency Using Model Selection Criteria (MSE and R^2)

3.3.1. Mean Square Error (MSE)

The MSE is a widely adopted statistical criterion for assessing the predictive capability of reliability models. Smaller MSE values reflect reduced discrepancies between observed and estimated results, thereby indicating enhanced model accuracy and goodness-of-fit. Thus, the MSE values summarized in Table 3 were utilized as a

Table 2: MLE-based parameter estimates.

Type	NHPP Model	MLE
Basic	Weibull	$\hat{a}_{MLE} = 22.6889$ $\hat{b}_{MLE} = 0.44490$
Power-type Lifetime Distributions	Extended-Weibull	$\hat{a}_{MLE} = 16.4988$ $\hat{b}_{MLE} = 1.09080$
	Rayleigh	$\hat{b}_{MLE} = 8.5470$
	Weibull-Extension	$\hat{\lambda}_{MLE} = 0.06830$ $\hat{a}_{MLE} = 15.0109$

quantitative reference measure for selecting the most appropriate model. Consequently, a model with a lower MSE is judged to exhibit superior goodness-of-fit. The MSE formula used for model evaluation is expressed in Equation (29) [15].

$$MSE = \frac{\sum_{i=1}^n (m(x_i) - \hat{m}(x_i))^2}{n - k} \quad (29)$$

Note that $\hat{m}(x_i)$ refers to the cumulative failure count through time x_i , as obtained from $m(t)$.

Figure 2 illustrates the variation in MSE over the 30 observed failures throughout the testing period. As shown in Figure 2, the Weibull-Extension model maintains the smallest MSE values across all 30 failure occurrences. This finding confirms its superior predictive effectiveness compared with the alternative models.

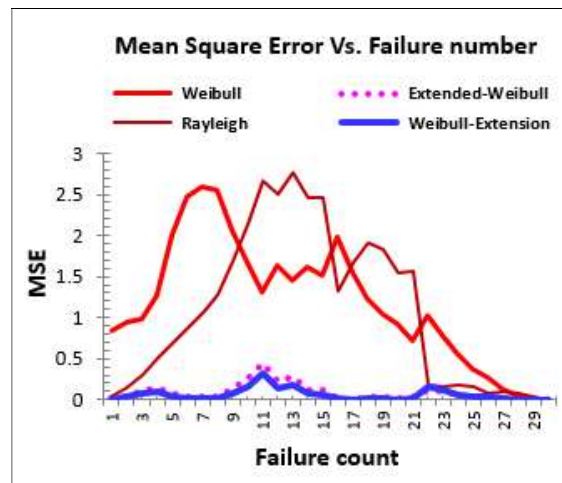


Figure 2: Comparative Analysis Using MSE.

3.3.2. Coefficient of Determination (R^2)

R^2 is a statistical measure that represents the explanatory power of a model by indicating how well the observed data are explained by the model. A larger R^2 value demonstrates a better fit of the model to the observed data, indicating superior predictive capability. Therefore, the R^2 measure is mathematically expressed in Equation (30) [16].

$$R^2 = 1 - \frac{\sum_{i=1}^n (m(x_i) - \hat{m}(x_i))^2}{\sum_{i=1}^n (m(x_i) - \sum_{j=1}^n m(x_j)/n)^2} \quad (30)$$

In this work, R^2 was employed as an evaluation criterion to analyze the effectiveness and goodness-

of-fit of the presented models. Accordingly, Table 3 summarizes the MSE and R^2 measures employed to evaluate model performance and identify the most suitable model.

Table 3: Model Selection Criteria.

Type	NHPP Model	MSE	R^2
Basic	Weibull	35.5399	0.8873
Power-type Lifetime Distributions	Extended-Weibull	2.5268	0.9919
	Rayleigh	32.3356	0.8975
	Weibull-Extension	1.7466	0.9944

3.4. Step 4: Analysis of Reliability Performance-Related Functions Including $m(t)$, $\lambda(t)$, and $\hat{R}(\tau)$

3.4.1. Mean Value Function ($m(t)$)

$m(t)$ denotes the expected cumulative count of failures occurring by time t and plays a fundamental role in assessing reliability behavior and failure-growth characteristics. As a key reliability metric, $m(t)$ is frequently employed to evaluate the performance of software reliability models [17].

Figure 3 illustrates the variation in $m(t)$ over time, reflecting the model's capability to accurately estimate the actual failure process. As illustrated in Figure 3, the Weibull-Extension and Extended-Weibull models show predictions that most closely match the observed values. These findings indicate that both models achieve higher predictive accuracy and superior performance compared with the other competing models.

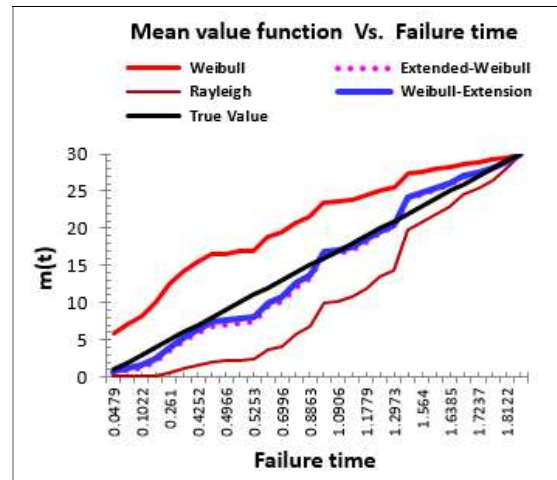


Figure 3: Performance Comparison Using $m(t)$.

3.4.2. Intensity Function ($\lambda(t)$)

$\lambda(t)$ describes the instantaneous rate of failure occurrence and plays a central role in evaluating the behavior of NHPP models. By utilizing the $\lambda(t)$ function, it is possible to analyze the reliability status of the system through the analysis of failure patterns and to predict the intensity of future failures. Therefore, $\lambda(t)$ serves as a reliability measure that can analyze the reliability performance.

Figure 4 shows how $\lambda(t)$ changes over time. The Weibull model shows a high failure rate initially, but as failures are repaired over time, it eventually demonstrates a trend of gradually decreasing failure rates. Consequently, these results demonstrate the effectiveness of this model.

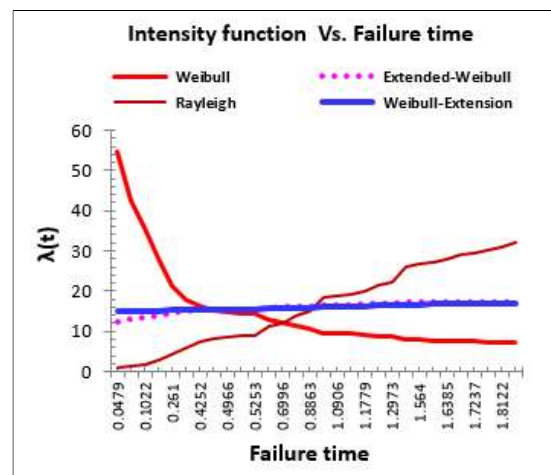


Figure 4: Performance Comparison Using $\lambda(t)$.

Table 4 illustrates the reliability performance trends analyzed by $m(t)$ and $\lambda(t)$ using the failure count recorded during 187.35 hours of testing.

Table 4: Reliability Performance Trend Analysis.

Failure count	Reliability Performance Attributes							
	m(t)				$\lambda(t)$			
	Weibull	Extended-Weibull	Rayleigh	Weibull-Extension	Weibull	Extended-Weibull	Rayleigh	Weibull-Extension
1	5.8708	0.5498	0.0196	0.7202	54.5283	12.5206	0.8188	15.0601
2	7.1456	0.8901	0.0474	1.1212	42.6719	13.0330	1.2735	15.0875
3	8.2247	1.2567	0.0893	1.5395	35.8038	13.4125	1.7470	15.1160
4	9.9725	2.0156	0.2123	2.3785	28.1522	13.9505	2.6940	15.1734
5	12.4818	3.4944	0.5822	3.9530	21.2764	14.6044	4.4615	15.2809
6	14.3285	4.9011	1.0826	5.4078	17.9116	15.0215	6.0838	15.3803
7	15.5087	5.9508	1.5453	6.4762	16.2272	15.2661	7.2684	15.4532
8	16.4422	6.8678	2.0096	7.4007	15.0859	15.4493	8.2889	15.5164
9	16.6175	7.0487	2.1078	7.5823	14.8875	15.4828	8.4889	15.5288
10	16.8683	7.3124	2.2546	7.8464	14.6119	15.5302	8.7795	15.5468
11	17.0382	7.4942	2.3585	8.0284	14.4304	15.5620	8.9795	15.5592
12	18.7663	9.4972	3.6412	10.0193	12.7917	15.8719	11.1573	15.6952
13	19.3547	10.2440	4.1832	10.7566	12.3083	15.9722	11.9590	15.7456
14	20.7377	12.1327	5.7050	12.6125	11.2928	16.1988	13.9658	15.8723
15	21.5027	13.2595	6.7139	13.7151	10.7938	16.3190	15.1504	15.9476
16	23.4512	16.4018	9.9158	16.7778	9.6866	16.6104	18.4119	16.1568
17	23.5815	16.6262	10.1659	16.9960	9.6198	16.6292	18.6427	16.1717
18	23.8461	17.0873	10.6888	17.4444	9.4868	16.6672	19.1162	16.2024
19	24.4034	18.0831	11.8585	18.4120	9.2173	16.7459	20.1350	16.2684
20	25.0891	19.3544	13.4317	19.6467	8.9041	16.8409	21.4290	16.3528
21	25.4745	20.0915	14.3845	20.3624	8.7363	16.8934	22.1760	16.4017
22	27.3372	23.8867	19.7548	24.0480	7.9999	17.1384	25.9880	16.6534
23	27.6840	24.6366	20.9068	24.7768	7.8751	17.1826	26.7350	16.7032
24	27.9502	25.2214	21.8257	25.3454	7.7816	17.2162	27.3162	16.7420
25	28.2632	25.9194	22.9460	26.0244	7.6743	17.2554	28.0085	16.7884
26	28.7002	26.9132	24.5847	26.9916	7.5287	17.3095	28.9914	16.8544
27	28.9078	27.3930	25.3943	27.4589	7.4613	17.3350	29.4649	16.8863
28	29.1771	28.0229	26.4752	28.0727	7.3755	17.3678	30.0854	16.9283
29	29.5590	28.9307	28.0689	28.9579	7.2568	17.4140	30.9777	16.9887
30	29.9997	29.9998	30.0000	30.0015	7.1240	17.4666	32.0256	17.0600

3.4.3. Reliability Function ($\hat{R}(\tau)$)

The reliability function ($\hat{R}(\tau)$) is a key indicator for evaluating the predictive reliability performance of NHPP models, representing the

probability that no failure will occur during mission time τ following the current point in time. By utilizing the $\hat{R}(\tau)$ function, future reliability can be predicted to assess the likelihood of future failures and evaluate the stability of the system. Thus, $\hat{R}(\tau)$

can serve as a performance indicator for evaluating model reliability in conjunction with the $\lambda(t)$ and $m(t)$ functions.

In this work, the proposed models were evaluated in terms of reliability performance by employing the $\hat{R}(\tau)$ function with a mission time setting of 116 h. By definition, reliability refers to the probability that a failure occurs in the testing point ($x_n = 187.35 \times 10^{-2}$ hours) and that no failure occurs in the confidence interval ($x_n, x_n + \tau$).

Based on the above explanation, $\hat{R}(\tau)$ is obtained in the form of Equation (31) [18].

$$\hat{R}(\tau|x_n) = \exp[-\{m(x_n + \tau) - m(x_n)\}] \quad (31)$$

Figure 5 illustrates the variation in reliability over the specified mission time.

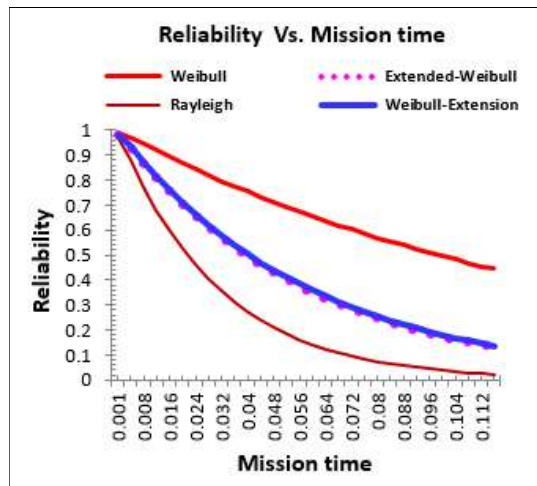


Figure 5: Performance Comparison Using $\hat{R}(\tau)$.

As a result of the analysis, a downward trend in reliability was observed for all models throughout the testing period, but the Weibull model exhibited the highest reliability among the competing models, thereby demonstrating the best performance.

The resulting reliability values for various mission times are summarized in Table 5, enabling a detailed examination of the reliability characteristics of the presented models. To improve readability and facilitate comparison, the mission times were scaled down by a factor of 10^{-3} .

3.5. Step 5: Overall Reliability Performance Assessment of the Considered Models

The model performance was assessed based on the analysis results that included both evaluation metrics (MSE, R^2) for assessing model efficiency and attribute functions related to reliability performance ($m(t)$, $\lambda(t)$, $\hat{R}(\tau)$). The comparative results for all examined models are presented in Table 6. According to research results, the Weibull-

Table 5: Analysis of $\hat{R}(\tau)$ Trends.

Mission Time (hours)	$\hat{R}(\tau)$			
	Weibull	Extended-Weibull	Rayleigh	Weibull-Extension
0.001	0.9929	0.9827	0.9685	0.9831
0.004	0.9719	0.9325	0.8796	0.9340
0.008	0.9447	0.8696	0.7736	0.8724
0.012	0.9182	0.8109	0.6801	0.8148
0.016	0.8925	0.7561	0.5977	0.7610
0.02	0.8676	0.7050	0.5252	0.7108
0.024	0.8434	0.6574	0.4614	0.6638
0.028	0.8198	0.6130	0.4052	0.6199
0.032	0.7970	0.5716	0.3557	0.5790
0.036	0.7748	0.5329	0.3122	0.5407
0.04	0.7533	0.4969	0.2740	0.5049
0.044	0.7324	0.4633	0.2403	0.4715
0.048	0.7121	0.4320	0.2108	0.4403
0.052	0.6924	0.4028	0.1848	0.4112
0.056	0.6732	0.3755	0.1620	0.3840
0.06	0.6546	0.3501	0.1419	0.3585
0.064	0.6366	0.3264	0.1244	0.3348
0.068	0.6190	0.3043	0.1089	0.3126
0.072	0.6020	0.2837	0.0954	0.2919
0.076	0.5854	0.2645	0.0835	0.2726
0.08	0.5693	0.2466	0.0730	0.2545
0.084	0.5537	0.2299	0.0639	0.2376
0.088	0.5385	0.2143	0.0559	0.2218
0.092	0.5238	0.1998	0.0489	0.2071
0.096	0.5094	0.1863	0.0427	0.1934
0.1	0.4955	0.1736	0.0373	0.1805
0.104	0.4820	0.1619	0.0326	0.1685
0.108	0.4689	0.1509	0.0285	0.1573
0.112	0.4561	0.1407	0.0249	0.1469
0.116	0.4437	0.1311	0.0217	0.1371

Extension model was found to yield more favorable overall results than the other models.

Table 6: Evaluation of Reliability Performance.

NHPP Model	Model Criteria		Performance Property		
	MSE	R^2	$m(t)$	$\lambda(t)$	$\hat{R}(\tau)$
Weibull	Worst	Good	Worst	Good	Best
Extended-Weibull	Good	Best	Good	Worst	Good
Rayleigh	Worst	Good	Worst	Worst	Good
Weibull-Extension	Best	Best	Best	Worst	Good

Consequently, by integrating these performance evaluation results at the outset of the development process, software developers can obtain meaningful design guidelines and leverage testing-related information to improve development strategies, planning efficiency, and overall software quality assurance.

4. CONCLUSION

By exploiting failure-time information during the early stages of software development, developers can construct reliability models in advance and thereby enhance the dependability of the developing software system. This proactive approach is expected to improve overall software quality and reduce reliability-related faults. Accordingly, this study applied Power-type lifetime distributions, which are capable of describing diverse reliability growth patterns and are well suited for reliability analysis, to infinite-failure NHPP models. Based on the observed failure-time data, the considered models were analyzed in terms of their predictive capability and reliability attributes, followed by a comprehensive assessment of their overall effectiveness.

The main conclusions of this study are as follows:

First, based on the comparative assessment of performance metrics, including MSE and R^2 , the Weibull-Extension model was found to outperform the competing models in terms of goodness-of-fit and predictive capability.

Second, an analysis of the reliability-related attribute functions, including $m(t)$ and $\lambda(t)$, revealed that the Weibull model exhibited appropriate failure-intensity behavior. However, the Weibull-Extension

model demonstrated the highest accuracy in predicting the observed failure data and was therefore identified as the most effective model.

Third, the performance analysis based on the reliability function ($\hat{R}(\tau)$), which represents the probability of successful system operation, showed that the reliability of all models decreased over time. Despite the declining reliability trend, the Weibull model consistently yielded the highest reliability estimates over the entire observation period, demonstrating its superior performance.

In conclusion, the Weibull-Extension model was confirmed as the most effective model among the candidate infinite-failure NHPP models considered in this study. The proposed approach can be extended to various systems characterized by time-dependent failure-rate behavior. However, because failure-generation mechanisms may vary according to the physical and environmental conditions of different industrial applications, further research is required to explore optimized models tailored to specific domains and to develop corresponding reliability evaluation frameworks.

Acknowledgements

Funding for this paper was provided by Namseoul University.

REFERENCES:

- [1] T. J. Yang, "Performance Analysis on the Reliability Attributes of NHPP Software Reliability Model Applying Exponential and Inverse-Exponential Lifetime Distribution", *Journal of Theoretical and Applied Information Technology*, Vol. 100, No. 22, 2022, pp. 6645-6656.
- [2] T. J. Yang, "Comparative Analysis on the Reliability Performance of NHPP Software Reliability Model Applying Exponential-type Lifetime Distribution", *International Journal of Performance Engineering*, Vol. 18, No. 10, 2022, pp. 679-689.
- [3] T. J. Yang and H. C. Kim, "Comparative Software Infinite NHPP Reliability Growth Model Based on the Intensity Function of Log-Linear and Log-Power Forms", *Smart Computing Review*, Vol. 4, No. 6, December 2014, pp. 451-458.
- [4] H. J. Bae, "Performance Analysis of Log-type Lifetime Distribution Based on Infinite Failure NHPP Software Reliability Model", *Journal of*

- Theoretical and Applied Information Technology*, Vol. 104, No. 8, 2026, pp. 186-196.
- [5] X. Xiao and T. Dohi, "On the Role of Weibull-type Distributions in NHPP-based Software Reliability Modeling", *International Journal of Performability Engineering*, Vol. 9, No. 2, 2013, pp. 123-132.
- [6] T. J. Yang and J. G. Park, "A Comparative Study of the Software NHPP Based on Weibull Extension Distribution and Flexible Weibull Extension Distribution", *International Journal of Soft Computing*, Vol. 11, No. 4, 2016, pp. 276-281.
- [7] H. C. Kim, "The Assessing Comparative Study for Statistical Process Control of Software Reliability Model Based on Musa-Okumoto and Power-law Type", *Journal of Korea Institute of Information, Electronics, and Communication Technology*, Vol. 8, No. 6, 2015, pp. 483-490.
- [8] T. J. Yang, "Comparative Study on the Performance Evaluation of Infinite Failure NHPP Software Reliability Model with Log-Type Distribution Property", *ARP Journal of Engineering and Applied Sciences*, Vol. 17, No. 11, 2022, pp. 1209-1218.
- [9] T. H. Yoo, "A Performance Valuation for NHPP Software Reliability Model Depend on Weibull-type Distribution", *International Journal of Pure and Applied Mathematics*, Vol. 118, No. 19, 2018, pp. 1021-1033.
- [10] K. G. Manton, E. Stallard and J. W. Vaupel, "Alternative Models for the Heterogeneity of Mortality Risks Among the Aged", *Journal of the American Statistical Association*, Vol. 81, No. 395, 1986, pp. 635-644.
- [11] T. J. Yang, "A Study on the Reliability Performance Analysis of Finite Failure NHPP Software Reliability Model Based on Weibull Life Distribution", *International Journal of Engineering Research and Technology*, Vol. 12, No. 11, 2019, pp. 1890-1896.
- [12] M. Xie, Y. Tang, T. N. Goh, "A Modified Weibull Extension with Bathtub-Shaped Failure Rate Function", *Reliability Engineering and System Safety*, Vol. 76, 2002, pp. 279-285.
- [13] Y. Hayakawa, G. Telfar, "Mixed Poisson-type Processes with Application in Software Reliability", *Mathematical and Computer Modelling*, Vol. 31, 2000, pp. 151-156.
- [14] H. C. Kim and J.B. Kim, "A Performance Comparative Evaluation for NHPP Software Reliability Model Considering Makeham and Extended Weibull Distribution", *Journal of Engineering and Applied Sciences*, Vol. 12, Special Issue 10, 2017, pp. 8909-8912.
- [15] Y. C. Ra, "A Comparative Analysis on the Performance of Finite Failure NHPP Software Reliability Model Based on Rayleigh-type Lifetime Distribution", *Journal of Theoretical and Applied Information Technology*, Vol. 99, No. 24, 2021, pp. 6162-6172.
- [16] H. J. Bae, "Performance Assessment of NHPP Software Reliability Models Applying PARETO-type Lifetime Distribution Properties", *Journal of Theoretical and Applied Information Technology*, Vol. 103, No. 7, 2025, pp. 2857-2867.
- [17] S. K. Park, "Comparative Analysis on the Reliability Attributes of Finite Failure NHPP Software Reliability Model with Exponential Distribution Characteristics", *Journal of Theoretical and Applied Information Technology*, Vol. 100, No. 18, 2022, pp. 5220-5231.
- [18] T. H. Park, "A Comparative Analysis on the Performance Attributes of a Software Development Cost Model Based on Weibull Lifetime Distribution", *Journal of Theoretical and Applied Information Technology*, Vol. 101, No. 13, 2023, pp. 5229-5239.