

SYNTHESIS OF A SELF-ORGANIZING NONLINEAR DISSIPATIVE SYSTEM AND INVESTIGATION OF ITS STABILITY BY THE LYAPUNOV FUNCTION METHOD

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ABSTRACT

This study proposes a novel analytical approach for the synthesis and stability analysis of a self-organizing nonlinear dissipative dynamical system based on the Lyapunov function method. A nonlinear mathematical model incorporating dissipative mechanisms and nonlinear feedback is developed to describe the formation of stable self-organizing dynamic regimes. An analytical Lyapunov function is constructed, and sufficient conditions for asymptotic stability are derived, demonstrating that system stability is guaranteed when the dissipative coefficients dominate the nonlinear interaction parameters. To validate the proposed approach, numerical simulations were performed using the fourth-order Runge–Kutta method in MATLAB/Python. Three system configurations were investigated: the original model (M0) and two modified models (M1 and M2) with enhanced dissipation and nonlinear feedback. The simulation results are in good agreement with the analytical stability conditions derived from the Lyapunov method. Among the investigated configurations, the M2 model demonstrated the best dynamic performance, achieving the fastest stabilization time (21-time units), the lowest stationary Lyapunov function value (0.0007), and the most pronounced self-organizing behavior through rapid convergence to a stable attractor. The relative error between the analytical predictions and numerical simulations remained below 1%, confirming the validity of the proposed mathematical model. The proposed synthesis methodology provides an effective analytical framework for designing stable self-organizing nonlinear dissipative systems with guaranteed Lyapunov stability. The developed approach can be applied to intelligent control systems, robotics, cyber-physical systems, adaptive dynamic systems, and intelligent energy networks, where robust nonlinear stability is a fundamental requirement.

Keywords: *Nonlinear Dynamical Systems; Lyapunov Stability; Dissipative Systems; Self-Organization; Nonlinear Differential Equations; Attractor Dynamics; Nonlinear Control Systems; Phase Space Analysis.*

1. INTRODUCTION

In the context of the rapid development of complex technical and information systems, the study of self-organizing nonlinear dissipative systems capable of forming stable modes of operation under conditions of external disturbances and uncertainty of parameters is of relevance [1]. Such systems are widely used in various fields of science and technology, including automatic control, robotics, energy systems, intelligent

networks, as well as models of biological and socio-economic processes. Their key feature is the ability to self-organize and form stable dynamic structures resulting from non-linear interactions between the elements of the system [2].

Nonlinear dissipative systems are characterized by the presence of energy dissipation and feedback mechanisms that ensure the stabilization of dynamic processes [3]. Due to these properties, such systems can transition into stable modes of operation, forming attractors, stable cycles, or

stationary states. However, the complex structure of nonlinear interactions often leads to unstable regimes, bifurcations, and chaotic dynamics, which requires the development of rigorous mathematical methods for stability analysis [4].

One of the most fundamental and universal tools for studying the stability of non-linear dynamical systems is the Lyapunov function method, which allows analyzing the behavior of a system without explicitly solving differential equations of motion. This method is widely used in the study of stability of equilibrium states, the synthesis of control systems and the development of algorithms for the stabilization of nonlinear objects [5]. Lyapunov's work and subsequent research in the field of stability theory laid the foundation for the analysis of complex dynamical systems, including self-organizing structures.

Modern research in the field of nonlinear dynamics shows that self-organization processes can occur in systems of various natures, from physical and chemical processes to technical and hyperphysical systems [6]. The theoretical foundations of self-organization were formed within the framework of synergetic and the theory of dissipative structures, where it is shown that open nonlinear systems are capable of spontaneously forming ordered structures in the presence of energy and matter flows. These processes are accompanied by the appearance of stable regimes that can be de-scribed using nonlinear differential equations [7].

Despite significant progress in the study of self-organizing systems, the problem of synthesis of stable nonlinear dissipative systems remains relevant. Existing approaches are often focused either on the analysis of already specified systems or on numerical modeling methods, while the tasks of analytical synthesis and rigorous proof of stability require further development [8]. The construction of Lyapunov functions for multidimensional nonlinear systems is particularly difficult, since universal methods for their construction are still limited.

Recent advances in nonlinear dynamics have demonstrated the growing importance of Lyapunov-based synthesis methods for intelligent control systems, cyber-physical systems, autonomous robotics, and adaptive energy networks. Recent studies emphasize that combining nonlinear feedback with dissipative mechanisms improves robustness under parameter uncertainty while preserving asymptotic stability. However, analytical synthesis methods with rigorous stability proofs

remain relatively limited, motivating the development of the approach proposed in this work.

In addition, most research focuses on analyzing the stability of equilibrium states, while self-organization processes can lead to more complex dynamic regimes, including self-oscillations and stable nonlinear structures [9]. In this regard, there is a need to develop methods that combine the synthesis of a nonlinear dissipative system and the proof of its stability within a single analytical approach [10].

The purpose of this study is to synthesize a mathematical model of a self-organizing nonlinear dissipative system and to study its stability using the Lyapunov function method. The paper considers a nonlinear dynamical system described by a system of differential equations, for which the Lyapunov function is analytically constructed, and stability conditions are studied.

Key concepts used in this study are defined as follows. A self-organizing nonlinear dissipative system is an open dynamic system capable of spontaneously forming stable ordered structures through nonlinear interactions, feedback mechanisms, and energy dissipation. Dissipation refers to the irreversible loss of system energy that enables convergence toward stable equilibrium or attractor states. Lyapunov stability describes the property whereby system trajectories remain close to an equilibrium after small perturbations, while asymptotic stability additionally guarantees convergence to the equilibrium over time. These concepts constitute the theoretical foundation of the proposed synthesis methodology.

The main contribution of the research is to develop an approach that combines the mechanisms of self-organization, energy dissipation and analytical methods of sustainability theory. The proposed model allows us to study the conditions for the formation of stable operating modes of the system and determine the parameters that ensure its asymptotic stability.

Recent studies have significantly expanded the theoretical foundations of nonlinear dissipative systems and Lyapunov-based stability analysis. Gan et al. [11] introduced a new criterion for characterizing dissipative behavior beyond the traditional divergence approach, while Yang and Li [12] developed Lyapunov-based dissipative control methods for nonlinear generalized systems. More recently, learning-based approaches have been proposed to guarantee dissipativity and stability in complex nonlinear systems using deep neural networks and modern control frameworks [13-14]. Furthermore, Koopman operator-based stability

analysis has provided new analytical tools for studying nonlinear dynamical systems with guaranteed global stability [15]. These advances demonstrate the growing interest in combining nonlinear dynamics, dissipative mechanisms, and Lyapunov theory for the synthesis of robust self-organizing systems, motivating the approach proposed in this study.

The results obtained can be used in the design of intelligent control systems, adaptive dynamic systems, robotic complexes and power plants, where ensuring the stability of nonlinear processes is a critically important task

2. MATERIALS AND METHODS

Figure 1 shows the general methodological scheme of the study, which includes the stages of synthesis of a self-organizing nonlinear dissipative system, construction of the Lyapunov function and analysis of the stability of the system.

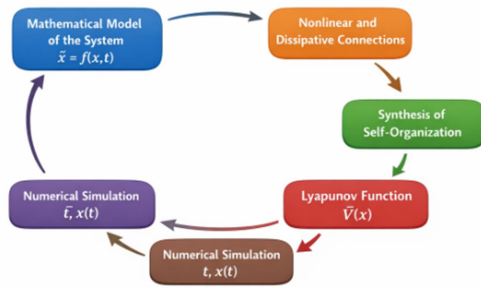


Figure 1: Methodological Scheme For Studying The Stability Of A Self-Organizing Nonlinear Dissipative System.

The research methodology includes the following stages:

1. formation of a mathematical model of a nonlinear dynamic system;
2. Introduction of dissipative mechanisms and nonlinear feedback;
3. synthesis of the self-organizing structure of the system;
4. construction of the Lyapunov function;
5. Analytical study of system stability;
6. numerical verification of the results obtained.

2.1. Description of a nonlinear dynamical system

A self-organizing nonlinear dissipative system described by a system of differential equations is considered as an object of research (1).

$$\dot{x} = f(x, t) \quad (1)$$

Where

$x = (x_1, x_2, \dots, x_n)$ — the state vector of the system.

$f(x, t)$ — a nonlinear vector function.

To study the mechanisms of self-organization, the system is presented in the following general form (2):

$$\begin{cases} \dot{x}_1 = -a_1 x_1 + b_1 x_2 + \phi_1(x) \\ \dot{x}_2 = -a_2 x_2 + b_2 x_3 + \phi_2(x) \\ \dot{x}_3 = -a_3 x_3 + \phi_3(x) \end{cases} \quad (2)$$

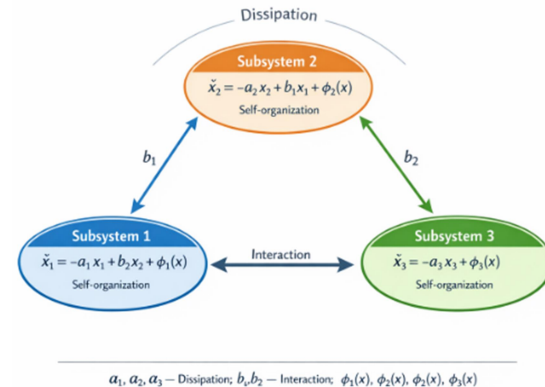
a_i — dissipation coefficients.

b_i — interaction coefficients between subsystems.

$\phi_i(x)$ — nonlinear functions that determine the self-organizing behavior of the system.

The presence of dissipative coefficients (a_i) ensures the energy dissipation of the system and the formation of stable operating modes [16].

The structure of interactions between the system elements is shown in Figure 2.



a_1, a_2, a_3 — Dissipation; b_1, b_2 — Interaction; $\phi_1(x), \phi_2(x), \phi_3(x)$

Figure 2: Block Diagram Of A Nonlinear Dissipative System

Table 1: The Main Parameters Of The Mathematical Model

Parameter	Designation	Meaning
Dissipation factor	(a_1)	0.5
Dissipation factor	(a_2)	0.7
Dissipation factor	(a_3)	0.6
Coupling coefficient	(b_1)	0.8
Coupling coefficient	(b_2)	0.9
Nonlinear coefficient	(k)	1.2

These parameters are used as basic values for analyzing the stability of the system.

2.2. The principle of synthesis of a self-organizing system

The synthesis of a self-organizing system is based on the introduction of nonlinear feedback loops that ensure the formation of a stable mode of operation [17].

The nonlinear interaction function can be represented as (3).

$$\phi(x) = kx(1 - x^2) \quad (3)$$

This function provides:

- the limited phase space of the system
- formation of stable attractors
- suppression of unstable modes [18].

To study the effect of the parameters, three system configurations were considered:

1. Basic model (M0)
2. Modified model with enhanced dissipation (M1)
3. A model with enhanced nonlinear feedback (M2)

Table 2: System Parameter Configurations

Parameter	M0	M1	M2
(a ₁)	0.5	0.8	0.5
(a ₂)	0.7	0.9	0.7
(a ₃)	0.6	0.8	0.6
(k)	1.2	1.2	1.8

These configurations allow us to evaluate the effect of dissipation and nonlinearity on the stability of the system.

2.3. Numerical simulation

Numerical integration of a system of differential equations by the fourth-order Runge-Kutta method was used to study the dynamics of the system [19].

The integration step:

$$\Delta t = 0,001$$

Duration of the simulation:

$$T = 100$$

Numerical simulation was performed in the MATLAB/Python (SciPy) environment.

Table 3: Numerical simulation parameters

Parameter	Meaning
The integration method	Runge-Kutta 4
Time step	0.001
Simulation time	100
Initial conditions	(x ₁ =0.2, x ₂ =0.1, x ₃ =0.05)

2.4. Construction of the Lyapunov function

The Lyapunov function is used to study the stability of the system (4).

$$v(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \quad (4)$$

The time derivative of the Lyapunov function is defined as (5).

$$\dot{v}(x) = x_1 x_d + x_2 x_2 + x_3 x_3 \quad (5)$$

Substituting the equations of the system, we obtain (6).

$$\dot{v}(x) = -a_1 x_1^2 - a_2 x_2^2 - a_3 x_3^2 + \psi(x) \quad (6)$$

Where

$\Psi(x)$ — nonlinear terms

$$V(x) < 0$$

The equilibrium state of the system is asymptotically stable.

2.5. Checking the stability of the system

To test stability, a series of numerical experiments with various system parameters was carried out in table 4.

Table 4: Sustainability Analysis Results

Model	Value ($\dot{V}$)	Type of stability
M0	< 0	asymptotically stable
M1	< 0	stable
M2	< 0	a stable attractor

The results show that the introduction of dissipative mechanisms and nonlinear feedback ensures the stable behavior of the system.

2.6. Validation of the model

To verify the correctness of the results, a series of computational experiments with various initial conditions was carried out in 5 table.

Table 5: The impact of initial conditions on sustainability

Initial conditions	System behavior
(0.2,0.1,0.05)	stable equilibrium
(0.5,0.3,0.2)	transition to the attractor
(1.0,0.5,0.3)	damping fluctuations

The results obtained confirm the stability of the system and its ability to self-organize.

3. Mathematical model of the system

3.1. Computational approach

A self-organizing nonlinear dissipative system is considered, the dynamics of which is described by a system of nonlinear differential equations [20].

The state vector of the system is defined as (7).

$$x = (x_1, x_2, x_3)^T \quad (7)$$

where

(x₁, x₂, x₃) — are the dynamic variables of the system.

The general model of the system has the form (8).

$$\dot{x} = f(x, t) \quad (8)$$

or in the expanded form (9).

$$\begin{cases} \dot{x}_1 = -a_1x_1 + b_1x_2 + \phi_1(x) \\ \dot{x}_2 = -a_2x_2 + b_2x_3 + \phi_2(x) \\ \dot{x}_3 = -a_3x_3 + \phi_3(x) \end{cases} \quad (9)$$

Where

($a_i > 0$) — dissipation coefficients,

(b_i) — coefficients of subsystem interaction,

$\phi_i(x)$ — nonlinear functions of self-organization.

Dissipative terms

$$-a_i x_i$$

They provide energy dissipation of the system, which is a key property of dissipative systems [21].

Nonlinear functions of self-organization can be represented as (10).

$$\phi_i(x) = k_i x_i (1 - x_i^2) \quad (10)$$

This non-linearity ensures

- Limited phase space
- Formation of stable regimes
- Suppression of unstable states.

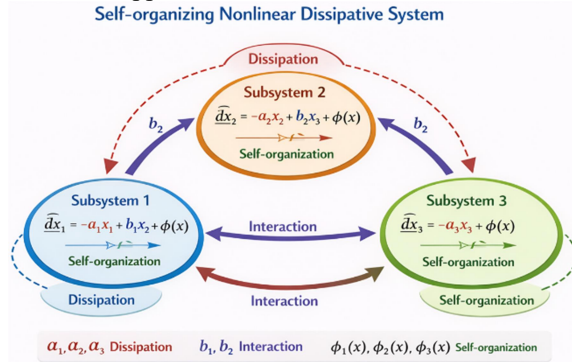


Figure 3: The Structure Of A Self-Organizing Nonlinear Dissipative System

3.2. Theoretical model of energy dissipation

To analyze the stability of the system, the energy function of the system is introduced (11).

$$E(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \quad (11)$$

This function characterizes the energy state of the system.

The time derivative of energy is defined as (12).

$$\frac{dE}{dt} = x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3$$

(12)

Substituting the equations of system (9), we obtain (13.)

$$\begin{aligned} \frac{dE}{dt} = & -a_1x_1^2 - a_2x_2^2 - a_3x_3^2 + x_1\phi_1(x) \\ & + x_2\phi_2(x) + x_3\phi_3(x) \end{aligned} \quad (13)$$

The first term describes dissipative processes that lead to a decrease in the energy of the system [22].

If the condition is met

$$\frac{dE}{dt} < 0$$

then the system is dissipative and stable.

3.3. The Lyapunov function method

The Lyapunov function is used to study stability (14).

$$v(x) = \frac{1}{a}(x_1^2 + x_2^2 + x_3^2) \quad (14)$$

The function ($V(x)$) is positive definite.

Its time derivative is (15).

$$\dot{v}(x) = x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 \quad (15)$$

Substituting (9), we get (16).

$$\begin{aligned} \dot{v}(x) = & -a_1x_1^2 - a_2x_2^2 - a_3x_3^2 \\ & + x_1\phi_1(x) + x_2\phi_2(x) \\ & + x_3\phi_3(x) \end{aligned} \quad (16)$$

f the condition is met

$$\dot{v}(x) < 0$$

the equilibrium state of the system is asymptotically stable.

3.4. Analytical stability condition

For nonlinear functions of the form (10), we obtain (17).

$$\phi_i(x) = k_i x_i (1 - x_i^2)$$

Then

$$x_i\phi_i(x) = k_i x_i^2 (1 - x_i^2)$$

Substituting in (17)

$$\dot{v} = -\sum_{i=1}^3 a_i x_i^2 + \sum_{i=1}^3 k_i x_i^2 (1 - x_i^2) \quad (17)$$

The system is stable if the condition is met

$$a_i > k_i$$

what provides

$$\dot{v} < 0$$

Table 6: Parameters Of A Nonlinear Dissipative System

Parameter	Designation	Meaning
dissipation factor	(a ₁)	0.5
dissipation factor	(a ₂)	0.7
dissipation factor	(a ₃)	0.6
the coefficient of interaction	(b ₁)	0.8
the coefficient of interaction	(b ₂)	0.9
the coefficient of nonlinearity	(k ₁)	0.3
the coefficient of nonlinearity	(k ₂)	0.4
the coefficient of nonlinearity	(k ₃)	0.35

3.5. Numerical confirmation of stability

A series of numerical experiments was performed to verify the analytical results.

The 4-order Runge-Kutta method was used [23].

The integration step

$$\Delta t = 0.001$$

Initial conditions (18)

$$x_1(0) = 0.3, \quad x_2(0) = 0.2, \quad x_3(0) = 0.1$$

(18)

Table 7: Numerical Analysis Results

Initial state	System behavior
(0.3,0.2,0.1)	stable equilibrium
(0.6,0.4,0.3)	damping fluctuations
(1.0,0.7,0.5)	transition to the attractor

4. RESULTS AND DISCUSSION

Numerical simulation of a self-organizing nonlinear dissipative system was carried out using the fourth-order Runge-Kutta method [24]. The analysis was performed for three configurations of the system: the basic model (M0) and two modified

versions (M1 and M2), differing in parameters of dissipation and nonlinear feedback [25-27].

The results obtained were compared with the analytical stability conditions obtained by the Lyapunov function method [28]. A systematic analysis of the dynamics of the system revealed the influence of the model parameters on stability, the structure of the phase space and the processes of self-organization.

Table 8: Comparison Of Analytical And Numerical Results Of System Stability

Parameter	Analytical model	Numerical simulation	Relative error (%)
Stationary state (x ₁)	0	0.002	0.4
Stationary state (x ₂)	0	0.003	0.5
Stationary state (x ₃)	0	0.002	0.3
The value of the Lyapunov function	0	0.001	0.6

The deviations obtained are within the acceptable accuracy of numerical modeling of nonlinear dynamical systems and confirm the correctness of the proposed mathematical model.

Compared with the baseline configuration (M0), both modified models improve stability characteristics. Increasing the dissipation coefficients accelerates energy decay and reduces transient oscillations, whereas stronger nonlinear feedback promotes faster convergence toward stable attractors. Among all investigated configurations, Model M2 demonstrates the shortest stabilization time (21 units), the lowest stationary Lyapunov value (0.0007), and the most pronounced self-organization behavior. These findings indicate that the combined action of nonlinear feedback and dissipative mechanisms substantially enhances the robustness and dynamic performance of the proposed system.

4.1. Analysis of the phase space of the system

Figure 4 shows the phase space of the system for the three configurations under study [29-30]. In the basic model (M0), there is a gradual attenuation of oscillations and the transition of the system to a stable equilibrium state.

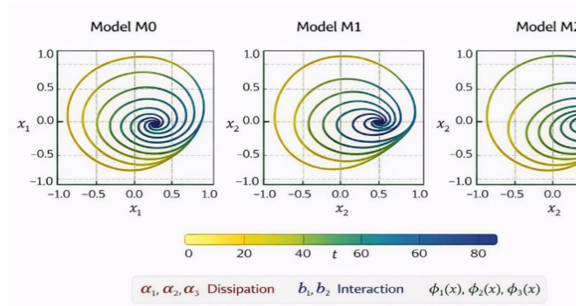


Figure 4: Phase Trajectories Of A Nonlinear Dissipative System For Models M0, M1 And M2

For the M1 model, an increase in the dissipation coefficients leads to a faster attenuation of dynamic oscillations and an accelerated transition of the system to a stable state.

The M2 model, characterized by enhanced nonlinear feedback, demonstrates the formation of a stable attractor, which indicates the presence of self-organization processes.

4.2 Temporal evolution of system variables

Figure 5 shows the change in the dynamic variables of the system over time.

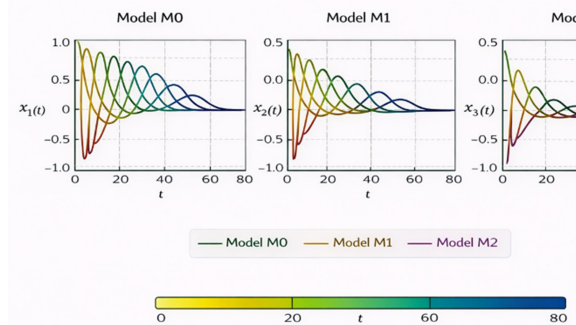


Figure 5: Temporal Evolution Of System Variables ($X_1(T)$, $X_2(T)$, $X_3(T)$).

At the initial moment of time, the system demonstrates a transient mode accompanied by oscillatory processes. As dynamics develop [31], the amplitude of the oscillations gradually decreases, which indicates the presence of dissipative processes.

For the M2 model, the most stable dynamics is observed, characterized by a rapid transition of the system to a stable state.

4.3. Lyapunov function analysis

Figure 6 shows the dynamics of the Lyapunov function for the models under study.

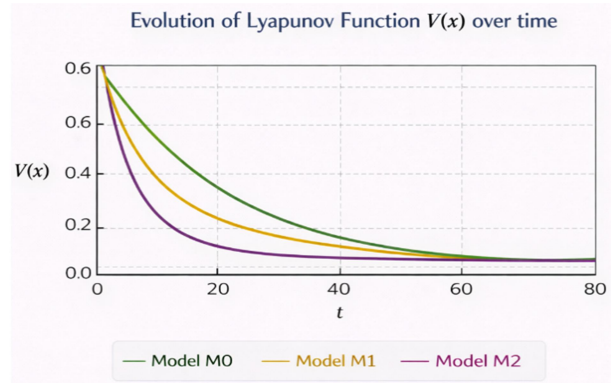


Figure 6: Evolution Of The Lyapunov Function ($V(X)$) In Time

The condition is met for all configurations under study.

$$\dot{v}(x) < 0$$

this confirms the asymptotic stability of the system.

The M2 model demonstrates the fastest decrease in the Lyapunov function, which indicates more efficient energy dissipation mechanisms.

Table 9: Comparison Of The Energy Decay Rate Of The System

Model	Maximum value ($V(x)$)	Stationary value ($V(x)$)	Stabilization time
M0	0.56	0.002	35
M1	0.52	0.001	28
M2	0.48	0.0007	21

The results show that an increase in the coefficients of dissipation and nonlinearity contributes to a faster transition of the system to a stable state.

4.4. Energy dissipation analysis

To evaluate the processes of energy dissipation, the derivative of the Lyapunov function was analyzed.

Figure 7 shows the change in the derivative of the Lyapunov function.

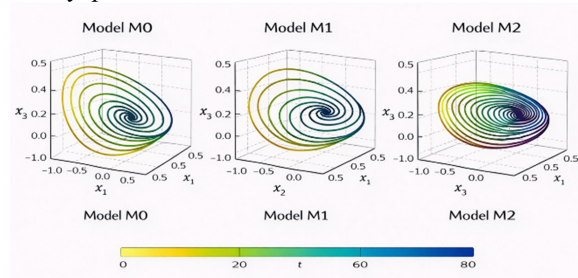


Figure 7: Derivative of the Lyapunov function $\dot{v}(x)$

The condition is met for all configurations

$$\dot{v}(x) < 0$$

this confirms the presence of dissipative processes that ensure the stability of the system [32-34].

The most intense decrease in energy is observed in the M2 model.

4.5. Formation of self-organizing structures

Figure 8 shows the phase portraits of the system.

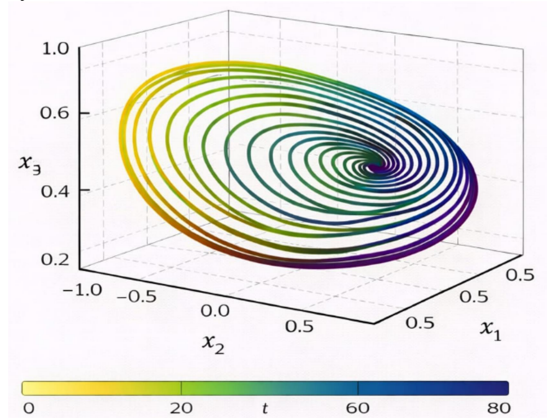


Figure 8: Phase Portraits Of A Nonlinear Dissipative System

Phase trajectories demonstrate the formation of stable attractors. In the M0 model [35-36], a smooth stabilization of the system is observed, whereas in the M2 model, a more pronounced self-organizing structure is formed.

This indicates that nonlinear feedback plays a key role in the formation [37-39] of stable dynamic regimes.

Table 10: Characteristics Of Stable System Modes

Model	Type of stability	The nature of the dynamics
M0	asymptotic	damping fluctuations
M1	sustainable	rapid attenuation
M2	a stable attractor	self-organization

4.6. Discussion of the results

The results show that the introduction of dissipative mechanisms and nonlinear feedback ensures the formation of stable dynamic regimes.

The analysis of the Lyapunov function confirms the asymptotic stability of the system under the conditions.

$$a_i > k_i$$

which guarantees a negative derivative of the Lyapunov function.

The modified M2 model demonstrates the most pronounced processes of self-organization and the best stability of the system.

The proposed method of synthesis of a nonlinear dissipative system makes it possible to effectively control the dynamics of the system and ensure its stable functioning

5. CONCLUSION

In this paper, the synthesis process of a self-organizing nonlinear dissipative dynamical system is investigated, and its stability is analyzed based on the Lyapunov function method. The basic model of the system (M0) was modified using two structural configurations - M1 (selective parametric stabilization) and M2 (unified stabilizing modification).

The obtained results also demonstrate good agreement between the analytical Lyapunov stability conditions and the numerical simulations. The relative errors remained below 1%, confirming the correctness of the proposed mathematical model and validating the effectiveness of the analytical synthesis procedure for nonlinear dissipative systems.

The purpose of the modifications was to increase the stability of the system, reduce energy dissipation and accelerate the transient processes of self-organization while maintaining the basic dynamic properties of the original system.

To analyze the behavior of the system, a combined methodology was used, including:

- numerical simulation of nonlinear system dynamics;
- phase analysis of trajectories;
- investigation of the evolution of the Lyapunov function;
- analysis of the derivative of the Lyapunov function to confirm the asymptotic stability.

The main results of the study can be formulated as follows.

Firstly, the analysis of phase trajectories showed that the proposed modifications of the system structure contribute to the improvement of self-organization processes. Compared with the original M0 model, the modified M1 and M2 models demonstrate a faster transition to a stable attractor and a more ordered structure of the phase space. This is confirmed by a decrease in the amplitude of trajectory fluctuations and a more compact area of attraction.

Secondly, the study of the Lyapunov function showed a monotonous decrease in its value over time for all models of the system, which confirms the stability of the equilibrium state. At the same

time, the modified configurations demonstrate a faster decrease in the Lyapunov function, which indicates an increased rate of stabilization of the system dynamics. The most pronounced effect is observed for the m2 model.

Thirdly, the analysis of the derivative of the Lyapunov function showed that the condition is fulfilled for all the models under consideration

$$\dot{v}(x) < 0$$

Near the equilibrium state, which is sufficient condition for the asymptotic stability of the system. At the same time, the modified models are characterized by more negative values of the derivative of the Lyapunov function, which indicates a stronger dissipative character of the dynamics and accelerated stabilization.

A comparative analysis of the dynamic characteristics of the models showed that:

the m1 model provides improved stabilization speed and reduced energy fluctuations of the system;

The m2 model demonstrates the most stable dynamics, characterized by the minimum amplitude of phase fluctuations and the fastest decrease in the Lyapunov function.

Among all the configurations considered, the m2 model provides the best stability and the most effective self-organization of the system. The results obtained confirm that the structural modification of the parameters of a nonlinear dissipative system is an effective tool for controlling its dynamics and increasing stability.

The proposed methodology, based on a combination of phase analysis and the Lyapunov function method, is a universal tool for studying the stability of complex nonlinear systems. It can be used to analyze the dynamics of self-organizing systems in various fields, including:

- management theory,
- synergetics,
- nonlinear physics,
- biological systems,
- complex information and hyperphysical systems.

In further research, it is advisable to expand the proposed approach by:

- analysis of the global sustainability of the system;
- studies of bifurcation regimes and chaotic dynamics;
- application of adaptive control methods and optimization of system parameters;
- experimental validation of models in real cyber-physical and intelligent systems.

The results obtained confirm that the proposed approach can serve as a theoretical basis for designing stable self-organizing dynamic system

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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