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PERFORMANCE ANALYSIS OF INFINITE FAILURE NHPP SOFTWARE RELIABILITY MODEL USING LOG-BASED LIFETIME DISTRIBUTIONS

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ABSTRACT

This study analyzes the performance of an infinite-failure NHPP software reliability model using log-based lifetime distributions (Gompertz, Log-Logistic, and Pareto), in which the failure rate varies nonlinearly over time. Software failure-time data are used for model construction, and the parameters are estimated using maximum likelihood estimation (MLE). Through various evaluation methods, including model efficiency assessed using MSE and R^2, predictive accuracy evaluated by the m(t) function, failure occurrence intensity examined using the λ(t) function, and model reliability analyzed through the $\hat{R}(\tau)$ function, the Log-Logistic model was confirmed to be the most effective among the proposed models. These findings indicate that the proposed approach can be effectively applied to reliability assessment across diverse software environments. Consequently, this study clarifies the previously underexplored reliability characteristics of log-based lifetime distributions and is expected to provide developers with a fundamental analytical technique for assessing software failure rates during the early stages of development.

Keywords: *Gompertz, Infinite Failure, Log-based, Log-Logistic, NHPP, Pareto.*

1. INTRODUCTION

The rapid progress of artificial intelligence (AI) technologies has led to a sharp rise in the demand for intelligent software systems with high reliability. In particular, such AI systems require not only functional accuracy but also high reliability that ensures failure-free operation over long-term deployment. Consequently, evaluating the reliability of AI-based software systems calls for new analytical approaches that incorporate failure occurrence patterns, which play a critical role in determining the reliability performance of the models. Accordingly, software developers have increasingly focused on Non-Homogeneous Poisson Process (NHPP) models, which are well suited for analyzing time-varying failure rates within various reliability modeling frameworks. In particular, the infinite-failure NHPP model, which does not impose a limit on the number of potential defects, is highly appropriate for capturing the characteristics of large-scale intelligent software systems considered in this study. Consequently, extensive research has been conducted using infinite-failure NHPP-based software reliability models to predict future failures by applying observed failure-time data collected during system operation [1]. Building on this line of research, Kim [2] utilized an infinite-failure NHPP reliability framework to investigate and compare the modeling properties of the Lomax and Gompertz distributions, providing an evaluation of their efficiency. Yang and Kim [3] expanded this perspective by applying an infinite-failure NHPP model to a software lifetime problem and offering a novel analysis of the model's cost characteristics derived from the intensity function. Accordingly, Yoo [4] examined the factors influencing the performance of infinite-failure NHPP software reliability models by analyzing properties associated with Weibull-type distributions. With respect to related infinite-failure models, Yang and Park [5] conducted a new reliability analysis of software by employing an extended Weibull distribution. Subsequently, Kim and Kim [6] evaluated the efficiency by comparing the extended Weibull distribution with the Makeham lifetime distribution. Based on these studies, Yang [7] applied Weibull-family lifetime distributions and presented new analytical results regarding the performance characteristics of NHPP reliability models.

Therefore, this study applies log-based property distributions to an infinite-failure NHPP model to analyze nonlinear failure patterns that cannot be adequately explained by conventional Weibull-based models. Additionally, based on this modeling framework, we propose an optimization approach for assessing reliability performance and identify the best-performing model among the alternatives.

2. RELATED RESEARCH

2.1.1 Infinite Failure NHPP Software Reliability Model

NHPP software reliability analysis predicts future failures based on the m(t) function, which is calculated from the observed failure history. In this analytical framework, N(t) denotes the total number of failures that have occurred by time t. Accordingly, N(t) is assumed to follow a Poisson probability distribution parameterized by m(t), as indicated in Equation (1). Thus,

$$P\{N(t)=n\}=\frac{[m(t)]^n\cdot e^{-m(t)}}{n!} \quad (1)$$

Note that $n = 0,1,2,\cdots,\infty$.

In terms of model reliability performance, m(t) serves as the mean value function describing the expected number of failures, whereas λ(t) characterizes the instantaneous failure rate. Accordingly, their relationship is expressed in Equations (2) and (3).

$$m(t)=\int_0^t \lambda(s)ds \quad (2)$$

$$\frac{dm(t)}{d(t)}=\lambda(t) \quad (3)$$

In this work, the analysis is conducted under an infinite-failure NHPP model, which assumes that software faults may continue to occur even during the debugging process which imposes no upper bound on the number of latent defects. Also, if θ represents the residual faults detectable up to testing time t, and F(t) stands for the cumulative distribution function, f(t) indicates the associated probability density function, then the m(t) governing the model reliability is given as shown in Equation (4) [8].

$$\mathrm{m(t)}=-\ln[1-F(t)] \quad (4)$$

Therefore, the intensity function can be derived from the mean value function as expressed in Equation (5).

$$\lambda(\mathrm{t})=\frac{dm(t)}{dt}=\frac{f(t)}{[1-F(t)]} \quad (5)$$

Using the relationships established in Equations (4) and (5), the likelihood function of the infinite-failure NHPP model is given by Equation (6).

$$L_{NHPP}(\Theta|\underline{x})=\left(\prod_{i=1}^{n}\lambda(x_i)\right)exp[-m(x_n)] \quad (6)$$

Note that $\underline{x}=(x_1,x_2,x_3,\cdots,x_n)$

2.2 Infinite Failure NHPP: Gompertz Model

The Gompertz distribution has been extensively applied in fields such as reliability engineering due to its capability to represent systems exhibiting an exponentially increasing failure rate over time. Owing to this characteristic, the Gompertz distribution serves as a suitable baseline distribution for NHPP models whose failure intensity grows exponentially with time. Accordingly, the Gompertz distribution can be employed as a lifetime model, and its distribution function with shape parameter α is given by the following expression [9].

$$\mathrm{F(t)}=1-exp\left[-\alpha\left(e^{\beta t}-1\right)\right] \quad (7)$$

$$\mathrm{f(t)}=\alpha\beta e^{\beta t}e^{\alpha}exp\left[-\alpha\left(e^{\beta t}\right)\right] \quad (8)$$

Note that $\alpha>0,\ \beta>0$.

Thus, the corresponding intensity function can be derived as follows.

$$\lambda(\mathrm{t})=\frac{f(t)}{[1-F(t)]}=\alpha\beta e^{\beta t} \quad (9)$$

Upon converting both sides of Equation (9) into logarithmic form, the Gompertz model yields a log-based distribution (ln λ(t)) for the failure rate. Such a log-based lifetime distribution is capable of capturing nonlinear variations in the failure rate, making it suitable for representing both the rapid decrease in failures during early testing and the stabilization phase during long-term operation. Consequently, the Gompertz NHPP model is categorized as an infinite-failure model with log-based distribution properties. Based on Equation (7), the corresponding m(t) is expressed as follows.

$$\mathrm{m(t)}=-\ln[1-F(t)]=\alpha\left(e^{\beta t}-1\right) \quad (10)$$

Consequently, by substituting Equation (10) into Equation (6), taking the logarithm of both sides, and rearranging, the log-likelihood function as presented in Equation (11) is obtained.

$$\ln L_{NHPP}(\Theta|\underline{x})= nln\alpha+nln\beta-\beta\sum_{i=1}^{n}lnx_i$$

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$$-\alpha\left(e^{\beta x_n}-1\right) \quad (11)$$

Accordingly, by differentiating the log-likelihood function with respect to the parameters α and β, the estimators $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$, which satisfy Equations (12) and (13), can be computed using the bisection method.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \alpha}=\frac{n}{\alpha}-\left(e^{\beta x_n}-1\right)=0 \quad (12)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \beta}=\frac{n}{\beta}+\sum_{i=1}^{n} x_i-\alpha x_n e^{\beta x_n}=0 \quad (13)$$

2.3 Infinite Failure NHPP: Log-Logistic Model

The Log-Logistic distribution has been extensively applied in reliability and failure-time studies because it effectively models systems where the failure rate rises during the early phase and declines thereafter. Given these properties, it is well suited as a baseline lifetime distribution for NHPP models characterized by log-based patterns. Thus, the Log-Logistic distribution may be adopted as a lifetime model, and the corresponding distribution function with shape parameter (k) is given by the following expression [10].

$$F(t)=\frac{(\tau t)^k}{[1+(\tau t)^k]} \quad (14)$$

$$f(t)=\frac{\tau k(\tau t)^{k-1}}{[1+(\tau t)^k]^2} \quad (15)$$

Note that $\tau>0,\ k>0$.

Accordingly, the corresponding intensity function is given as follows.

$$\lambda(t)=\frac{\tau k(\tau t)^{k-1}}{[1+(\tau t)^k]}=\frac{\tau^k k(t)^{k-1}}{[1+(\tau t)^k]} \quad (16)$$

Accordingly, the Log-Logistic NHPP model can be classified as an infinite-failure model capable of representing the logarithmic increase and subsequent decrease in the failure rate over time. Furthermore, by applying the distribution function in Equation (14), the corresponding mean value function can be expressed as follows.

$$m(t)=-\ln(1-F(t)=\ln[(1+(\tau t)^k] \quad (17)$$

In the present work, the shape parameter that defines the form of the failure-time distribution is fixed at 2. Accordingly, the corresponding log-likelihood function is calculated as follows.

$$\ln L_{NHPP}(\Theta|\underline{x})=kn\ln\tau+n\ln k+(k-1)\sum_{i=1}^{n}\ln x_i$$
$$-\sum_{i=1}^{n}\ln[1+(\tau x_i)^k]-\ln[1+(\tau x_n)^k] \quad (18)$$

By taking partial derivatives with respect to the parameters τ and k yields the estimating Equations (19) and (20), and the corresponding estimators $\hat{\tau}_{MLE}$ and $\hat{k}_{MLE}$ can be numerically solved through the bisection method.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \tau}=\frac{kn}{\hat{\tau}}-\sum_{i=1}^{n}\frac{{x_i}^k\,k\,\tau^{k-1}}{[1+(\hat{\tau}x_i)^k]}$$
$$-\frac{k\,\tau^{k-1}{x_n}^k}{[1+(\hat{\tau}x_n)^k]}=0 \quad (19)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial k}=n\ln\tau+\frac{n}{k}-\sum_{i=1}^{n}\frac{(\tau x_i)^k(\ln\tau x_i)}{[1+(\hat{\tau}x_i)^k]}$$
$$+\sum_{i=1}^{n}\ln x_i-\frac{(\tau x_n)^k(\ln\tau x_n)}{[1+(\hat{\tau}x_n)^k]}=0 \quad (20)$$

2.4 Infinite Failure NHPP: Pareto Model

The Pareto distribution is widely used in reliability engineering and lifetime analysis, particularly for systems in which failures occur frequently at early stages but gradually stabilize over time. Given this characteristic, the Pareto distribution is well suited as a baseline distribution for NHPP models that exhibit high initial failure rates followed by rapid stabilization. Accordingly, when used as a lifetime distribution with shape parameter α, its distribution function is defined as follows [11].

$$F(t)=1-\left[\frac{\beta^\alpha}{(\beta+t)^\alpha}\right] \quad (21)$$

$$f(t)=\left[\frac{\alpha\beta^\alpha}{(\beta+t)^{(1+\alpha)}}\right] \quad (22)$$

Accordingly, the resulting intensity function can be derived as presented below.

$$\lambda(t)=\frac{\alpha}{(\beta+t)} \quad (23)$$

Thus, the Pareto NHPP model is well suited for characterizing systems in which a small number of factors generate a disproportionately large number of failures, and it is therefore classified as an infinite-failure model exhibiting log-based lifetime distribution. Moreover, using Equation (23), the corresponding m(t) is obtained as follows.

$$\mathrm{m(t)} = \alpha \ln\left(1 + \frac{t}{\beta}\right) \quad (24)$$

Accordingly, taking the logarithm of both sides and simplifying the resulting expression yields the log-likelihood function, as presented in Equation (25).

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\alpha - \sum_{i=1}^{n} ln(\beta + x_i) - \alpha \ln\left(1 + \frac{x_n}{\beta}\right) \quad (25)$$

Therefore, taking partial derivatives with respect to the parameters α and β yields the estimating Equations (26) and (27), and the corresponding estimators $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$ can be calculated through the bisection method.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \alpha} = \frac{n}{\hat{\alpha}} - \ln\left(1 + \frac{x_n}{\beta}\right) = 0 \quad (26)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \beta} = -\sum_{i=1}^{n} \frac{1}{(\beta + x_i)} - \alpha\left(\frac{1}{\beta + x_n} - \frac{1}{\beta}\right) \quad (27)$$

3. PERFORMANCE ANALYSIS OF THE MODELS

To demonstrate the analysis, the proposed models are evaluated using the failure-time data summarized in Table 1. This dataset consists of 41 failures observed over a total operating time of 1197.945 time units [12].

The analysis framework for the proposed software reliability models consists of the following steps.

Table 1: Software Failure Time.

Failure number	Failure time (hours)	Failure time (hours)$\times 10^{-1}$
1	5.649	0.5649
2	8.92	0.892
3	20.29	2.029
4	29.955	2.9955
5	34.715	3.4715
6	75.95	7.595
7	78.171	7.8171
8	78.625	7.8625
9	83.022	8.3022
10	89.114	8.9114
11	89.804	8.9804
12	92.96	9.296
13	93.66	9.366
14	110.655	11.0655
15	111.988	11.1988
16	122.545	12.2545
17	127.045	12.7045
18	128.712	12.8712
19	128.99	12.899
20	131.768	13.1768
21	131.829	13.1829
22	141.712	14.1712
23	164.212	16.4212
24	342.85	34.285
25	356.144	35.6144
26	399.144	39.9144
27	446.494	44.6494
28	476.644	47.6644
29	497.144	49.7144
30	497.661	49.7661
31	591.161	59.1161
32	665.644	66.5644
33	686.444	68.6444
34	765.944	76.5944
35	772.977	77.2977
36	774.944	77.4944
37	791.561	79.1561
38	815.978	81.5978
39	837.145	83.7145
40	861.945	86.1945
41	1197.945	119.7945

Step 1: Software failure data are first verified through the Laplace trend test.

Step 2: Model parameters are then obtained via maximum likelihood estimation (MLE).

Step 3: Model selection is performed by calculating R^2 and MSE.

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Step 4: Performance metrics such as m(t), λ(t), and reliability $\hat{R}(\tau)$ are evaluated.
Step 5: The optimal model is identified based on comparative performance results.

To validate the reliability of the software failure-time data, the Laplace trend test was performed, as shown in Figure 1.

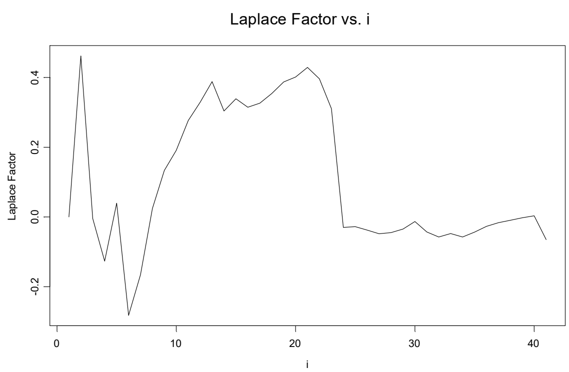

Figure 1: Result of Laplace Trend Test.

It is generally accepted that when the trend test results of the cited data remain within the bounds of −2 and 2, the data exhibit no extreme values and are considered stable, making them appropriate for reliability assessment [13]. As shown in Figure 1, the results lie between −2 and 2, confirming that no extreme values are present. That is, to explain it again, the failure-time data referenced in Table 1 are applicable to this study.

Additionally, using the MLE method, the parameters (α, β) of the proposed model were derived, and the estimated values are presented in Table 2 [14].

3.1. Selection of an Efficient Model

In NHPP-based software reliability models, the effectiveness of a model is often measured by the mean squared error (MSE), which serves as a fundamental performance indicator. A smaller MSE indicates a smaller deviation from the true values, thereby implying a more effective model in terms of goodness-of-fit.

Thus, this study uses the MSE results summarized in Table 2 as the evaluation criterion for effective model selection. Specifically, models with smaller MSE values are regarded as more appropriate, as expressed in Equation (28) [15].

$$MSE = \frac{\sum_{i=1}^{n} \left(m(x_i) - \hat{m}(x_i)\right)^2}{n-k} \tag{28}$$

Note that $\hat{m}(x_i)$ is the estimated count of accumulated failures by time x_i, n is the number of failures included in the referenced data, and k denotes the number of model parameters used.

Table 2: Parameter Estimates using MLE.

Type	NHPP model	MLE	
		$\hat{\alpha}(\hat{k})$	$\hat{\beta}(\hat{\tau})$
Log-based Lifetime Distributions	Gompertz	3.7235	0.0005
	Log-Logistic	2.0(*Fixed*)	0.0690
	Pareto	6.3458	1.8757

Figure 2 presents the trend of MSE across all 41 observed failure times, showing its variation across failure counts.

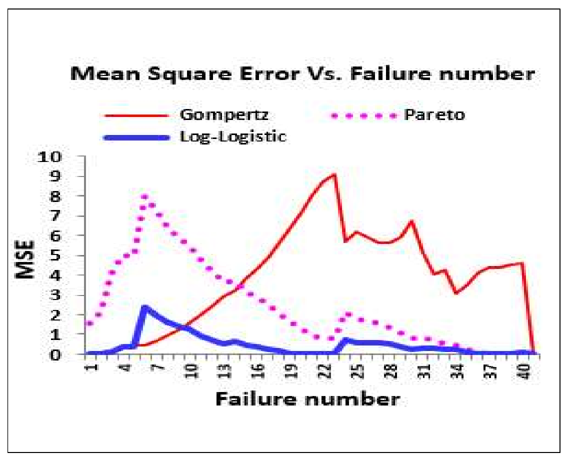

Figure 2: Efficiency Analysis using MSE.

Overall, the results indicate that the Log-Logistic model consistently exhibits the smallest MSE across the entire failure range, thereby demonstrating superior predictive performance.

Table 3 shows detailed data analyzing the trend of changes in MSE values according to the total number of failures (41 times). This data is used as reference data for selecting an efficient model.

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Table 3: Trend Analysis of MSE.

Failure number	MSE		
	Gompertz	Log-Logistic	Pareto
1	0.0191	0.0030	1.5662
2	0.0816	0.0066	2.1262
3	0.1612	0.1074	4.1170
4	0.2743	0.3568	5.0025
5	0.4423	0.3842	4.9200
6	0.4367	2.3489	7.9798
7	0.6593	1.9992	7.2540
8	0.9414	1.5930	6.4462
9	1.2379	1.3936	5.9188
10	1.5576	1.2610	5.4903
11	1.9750	0.9487	4.7991
12	2.4120	0.7439	4.2596
13	2.9239	0.5110	3.6569
14	3.2419	0.6369	3.6808
15	3.8227	0.4296	3.1342
16	4.2921	0.3937	2.8912
17	4.8984	0.2707	2.4848
18	5.6000	0.1439	2.0427
19	6.3777	0.0495	1.6162
20	7.1502	0.0088	1.2825
21	8.0308	0.0043	0.9464
22	8.7189	0.0143	0.7835
23	9.1097	0.0039	0.7621
24	5.6806	0.7561	2.1166
25	6.1563	0.5867	1.7774
26	5.9364	0.6011	1.6597
27	5.5948	0.6114	1.5415
28	5.6408	0.5170	1.3170
29	5.9119	0.3863	1.0611
30	6.7026	0.2143	0.7587
31	5.1106	0.3125	0.7836
32	4.0533	0.3313	0.7146
33	4.2295	0.2127	0.5132
34	3.0892	0.2155	0.4453
35	3.5200	0.1009	0.2667
36	4.0991	0.0260	0.1288
37	4.3503	0.0011	0.0485
38	4.4045	0.0067	0.0083
39	4.5354	0.0417	0.0019
40	4.5645	0.1029	0.0302
41	0.0000	0.0001	0.0000

The coefficient of determination (R^2) serves as an indicator of how well the differences between actual and predicted values are explained by the model. When evaluating model efficiency, a larger R^2 value suggests improved performance, indicating better explanatory power [16]. It is defined as

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n} \left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \quad (29)$$

Note that $\hat{m}(x_i)$ denotes the estimated cumulative failures obtained from m(t) evaluated up to time x_i.

Table 4 shows that the Log-Logistic model demonstrates superior efficiency compared with the other models.

Table 4: Criteria for Efficient Model Selection.

Type	NHPP Model	Model Comparison	
		MSE	R^2
Log-based Lifetime Distributions	Gompertz	157.944	0.7283
	Log-Logistic	18.6370	0.9679
	Pareto	96.3340	0.8343

3.2. Analysis of the Mean Value Function (m(t))

Table 5 provides a comparative summary of the equations used to compute the mean value functions m(t) for the log-based characteristic distributions.

Table 5: Mean Value Function ($m(t)$).

Type	NHPP model	m(t)
Log-based Lifetime Distributions	Gompertz	$\alpha\left(e^{\beta t} - 1\right)$
	Log-Logistic	$ln[1 + (\tau t)^k]$
	Pareto	$\alpha \ln\left(1 + \frac{t}{\beta}\right)$

Figure 3 illustrates the trend of m(t), representing the model's performance in predicting the true values. m(t) is an essential metric that reflects the ability of the model to predict the true values over time, providing critical information for assessing model characteristics and conducting reliability analysis [17]. Consequently, m(t), reflecting the model's reliability characteristics, becomes an essential tool for testing and assessing the model's reliability performance.

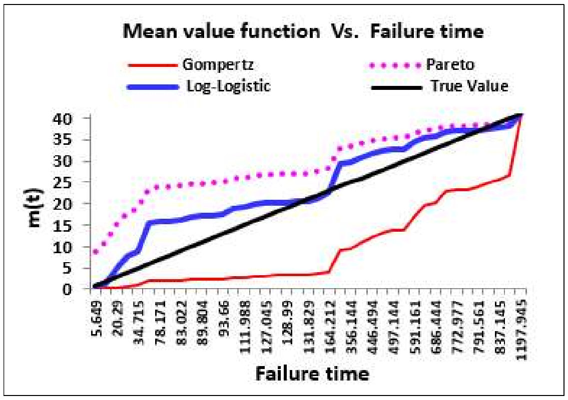

Figure 3: Performance Analysis using m(t).

As shown in Figure 3, the results indicate that the Log-Logistic model exhibits the smallest error relative to the true values. Therefore, it can be concluded that this model demonstrates the best performance among the proposed models.

3.3. Analysis of the Intensity Function (λ(t))

In the NHPP model, λ(t) is a function representing the instantaneous failure rate over time and serves as a key indicator that quantitatively expresses the process of change in software reliability. Consequently, λ(t) directly reflects the reliability growth of the software and, together with m(t), is used as a key performance indicator to evaluate the reliability performance of the model.

Table 6 presents a summary of the results obtained by comparing the λ(t) calculation formulas of each proposed model.

Table 6: Intensity Function (λ(t)).

Type	NHPP model	λ(t)
Log-based Lifetime Distributions	Gompertz	$\alpha\beta e^{\beta t}$
	Log-Logistic	$\frac{\tau^k k(t)^{k-1}}{[1+(\tau t)^k]}$
	Pareto	$\frac{\alpha}{(\beta+t)}$

The intensity function λ(t) represents the rate at which failures occur at time t, and therefore serves as a key indicator of the dynamic characteristics of a model's failure pattern. Accordingly, λ(t), together with m(t), provides essential information for evaluating the reliability performance [18].

Figure 4 presents the trend of the instantaneous failure intensity over the entire operational period.

As shown in the figure, the Log-Logistic model exhibits a relatively low initial failure rate, which rises sharply as time progresses. However, as failures are repaired, the rate gradually decreases, reflecting a pattern consistent with typical software failure behavior.

This result confirms that the Log-Logistic model demonstrates the most effective performance among the evaluated models.

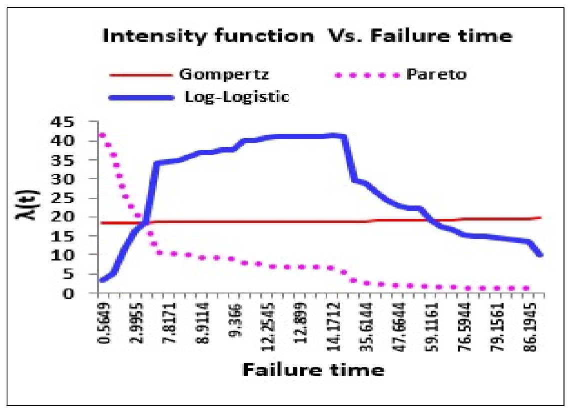

Figure 4: Performance Analysis using λ(t).

Conversely, the Pareto model displays a high initial failure intensity followed by a monotonic decline, indicating its poorer goodness-of-fit.

Table 7 shows the detailed results of the analysis based on the m(t) and λ(t) functions to estimate the expected number of failures and failure rate across all 41 failure points.

Here, each observed failure count is considered the reference (true) value for model evaluation.

Table 7: Trend Analysis of Reliability Performance.

Failure Time (hours)	Reliability Performance					
	m(t)			λ(t)		
	Gompertz	Log-Logistic	Pareto	Gompertz	Log-Logistic	Pareto
5.649	0.13664	0.65769	8.81556	18.50451	3.22249	41.60086
8.92	0.21596	1.49370	11.10612	18.50786	5.07694	36.68439
20.29	0.49278	5.04617	15.67130	18.51948	11.36924	26.00252
29.955	0.72946	7.73025	17.96776	18.52936	16.41273	20.84340
34.715	0.84649	8.87084	18.85214	18.53423	18.75716	18.98797
75.95	1.87327	15.57107	23.64121	18.57646	34.04253	10.72075
78.171	1.92924	15.82997	23.81977	18.57874	34.59570	10.47509
78.625	1.94069	15.88205	23.85566	18.57921	34.70551	10.42626
83.022	2.05173	16.37233	24.19314	18.58372	35.71268	9.97583
89.114	2.20602	17.01268	24.63290	18.58997	36.94445	9.41245
89.804	2.22353	17.08258	24.68084	18.59067	37.07239	9.35263
92.86	2.30115	17.38614	24.88892	18.59381	37.61168	9.09656
93.66	2.32149	17.46404	24.94228	18.59463	37.74560	9.03183
110.655	2.75575	18.98384	25.98127	18.61208	39.93742	7.84573
111.988	2.78998	19.09334	26.05599	18.61345	40.06093	7.76574
122.545	3.06200	19.91850	26.61866	18.62430	40.82422	7.18555
127.045	3.17843	20.24942	26.84412	18.62893	41.04358	6.96377
128.712	3.22163	20.36912	26.92565	18.63065	41.11026	6.88506
128.99	3.22884	20.38893	26.93914	18.63093	41.12064	6.87210
131.768	3.30094	20.58468	27.07244	18.63379	41.21314	6.74527
131.829	3.30253	20.58893	27.07534	18.63385	41.21494	6.74254
141.712	3.55993	21.25376	27.52788	18.64402	41.38958	6.32728
164.212	4.15121	22.61192	28.45164	18.66719	41.07904	5.54921
342.85	9.11561	29.43019	33.08559	18.85217	29.69467	2.80784
356.144	9.50499	29.78338	33.32571	18.86601	28.90730	2.70827
399.144	10.78422	30.84189	34.04547	18.91085	26.56239	2.42961
446.494	12.22844	31.88322	34.75372	18.96034	24.31433	2.18234
476.644	13.16791	32.49032	35.16670	18.99192	23.04544	2.04952
497.144	13.81567	32.88160	35.43290	19.01342	22.24722	1.96808
497.661	13.83210	32.89126	35.43947	19.01397	22.22772	1.96611
591.161	16.88217	34.49128	36.52826	19.11236	19.14819	1.66471
665.644	19.42708	35.59429	37.27906	19.19110	17.21174	1.48354
686.444	20.15666	35.88032	37.47378	19.21314	16.73542	1.43978
765.944	23.02368	36.89905	38.16739	19.29764	15.12542	1.29391
772.977	23.28342	36.98402	38.22525	19.30514	14.99719	1.28242
774.944	23.35625	37.00765	38.24134	19.30723	14.96170	1.27924
791.561	23.97463	37.20489	38.37565	19.32495	14.66821	1.25301
815.978	24.89365	37.48734	38.56799	19.35102	14.25654	1.21636
837.145	25.70042	37.72543	38.73014	19.37364	13.91732	1.18628
861.945	26.65773	37.99686	38.91499	19.40018	13.53923	1.15287
1197.945	41.00630	41.05752	41.00000	19.76336	9.87266	0.83450

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3.4. Analysis of the Reliability Function $(\widehat{R}(\tau))$

In the NHPP model, $\hat{R}(\tau)$ is a reliability measure that represents the probability that no failure occurs during a future mission time, based on the estimated parameters. Specifically, it denotes the probability that no failure occurs during the mission time τ following the final observed failure time x_n. Therefore, $\hat{R}(\tau)$ serves as an important indicator for predicting future reliability. Accordingly, this study aims to evaluate future reliability using $\hat{R}(\tau)$.

Figure 5 shows the reliability performance evaluated for specific future mission times. In this work, reliability is defined as the probability that no failure occurs within the interval $[x_n,\ x_n+\tau]$, where $x_n = 1197.945$ and τ denotes the future mission period.

Accordingly, the reliability associated with the future mission time is given as follows [19].

$$\hat{R}(\tau|x_n) = \exp[-\{m(x_n+\tau) - m(x_n)\}] \qquad (30)$$

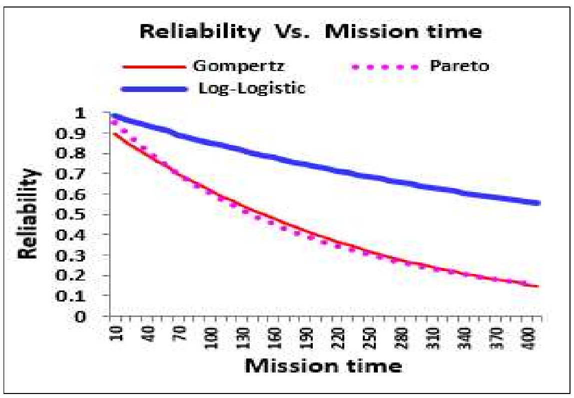

Figure 5: Performance Analysis using $\hat{R}(\tau)$.

As shown in Figure 5, the reliability of the Log-Logistic model remains relatively high throughout the mission time, in contrast to the other models, which exhibit a decreasing trend over time. Accordingly, the Log-Logistic model is identified as the best-performing model due to its superior reliability.

Table 8 shows in detail the results for reliability estimates calculated by inputting mission time into the proposed model, categorized by mission time. In the reliability analysis, $\hat{R}(\tau)$ has a value of "1 or less." A higher value indicates higher reliability of the model and is ultimately considered an efficient model.

Table 8: Trend Analysis of $\hat{R}(\tau)$.

Mission Time (hours)	$\hat{R}(\tau)$		
	Gompertz	Log-Logistic	Pareto
10	0.89407	0.98351	0.94869
20	0.85889	0.96743	0.90041
30	0.82490	0.95174	0.85494
40	0.79209	0.93643	0.81212
50	0.76040	0.92148	0.77175
60	0.72982	0.90689	0.73370
70	0.70032	0.89265	0.69779
80	0.67185	0.87873	0.66391
90	0.64439	0.86514	0.63191
100	0.61791	0.85186	0.60169
110	0.59238	0.83889	0.57313
120	0.56777	0.82621	0.54612
130	0.54405	0.81381	0.52058
140	0.52121	0.80170	0.49641
150	0.49920	0.78985	0.47353
160	0.47801	0.77826	0.45186
170	0.45761	0.76692	0.43133
180	0.43798	0.75583	0.41188
190	0.41908	0.74498	0.39343
200	0.40090	0.73436	0.37593
210	0.38342	0.72397	0.35933
220	0.36661	0.71379	0.34357
230	0.35045	0.70383	0.32860
240	0.33492	0.69408	0.31438
250	0.31999	0.68453	0.30087
260	0.30566	0.67517	0.28803
270	0.29189	0.66600	0.27582
280	0.27867	0.65702	0.26420
290	0.26598	0.64822	0.25315
300	0.25380	0.63959	0.24263
310	0.24212	0.63114	0.23261
320	0.23092	0.62285	0.22307
330	0.22017	0.61473	0.21397
340	0.20987	0.60676	0.20531
350	0.20000	0.59895	0.19704
360	0.19055	0.59128	0.18916
370	0.18149	0.58377	0.18165
380	0.17281	0.57639	0.17447
390	0.16451	0.56915	0.16762
400	0.15656	0.56205	0.16108
410	0.14896	0.55509	0.15484

3.5. Assessment of Reliability Performance

In this study, both the goodness-of-fit evaluated using reference indicators (MSE, R^2) and the reliability performance assessed using reliability functions (m(t), λ(t), $\hat{R}(\tau)$) were comprehensively analyzed. Accordingly, the optimal model was identified based on these evaluation results [20].

Table 9 shows the relative comparison results of the proposed models by evaluation item based on the research findings. The results confirm that the Log-Logistic model has the best performance.

Table 9: Assessment of Reliability Performance.

NHPP model	Efficiency		Performance Attributes		
	MSE	R^2	m(t)	λ(t)	$\hat{R}(\tau)$
Gompertz	Worst	Worst	Bad	Worst	Good
Log-Logistic	Best	Best	Best	Best	Best
Pareto	Bad	Good	Good	Good	Good

4. CONCLUSION

If software developers can develop a predictive model during the early testing phase based on collected failure-time data, future failure occurrences can be predicted, thereby enhancing product reliability. Accordingly, this study applies log-based lifetime distributions, suitable for reliability analysis, to an infinite-failure NHPP model and evaluates the performance of the proposed models using software failure time data**,** identifying their reliability characteristics.

The results of this study can be summarized as follows.
First, analysis of the reference metrics for model selection (MSE, R^2) indicated that the Log-Logistic model exhibited the best goodness of fit.

Second, the evaluation results for the performance functions m(t) and λ(t) revealed that the Log-Logistic model outperformed all other models by achieving the highest true value predictive accuracy and the lowest error rate.

Third, reliability analyses demonstrated that although all models showed declining reliability over time, the Log-Logistic model consistently retained the highest reliability, indicating its superior performance**.**

In conclusion, the Log-Logistic model was identified as the optimal model among the proposed models. Thus, this work provides basic data that can be helpful to software developers in the early stages. Moreover, the findings also imply that future research should focus on selecting appropriate distribution models for the software sector, while conducting a thorough assessment of reliability-related performance metrics.

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