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PERFORMANCE EVALUATION OF WEIBULL-TYPE LIFETIME DISTRIBUTIONS BASED ON INFINITE FAILURE NHPP SOFTWARE RELIABILITY MODEL

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ABSTRACT

This study focuses on evaluating the performance of NHPP-based software reliability models under infinite failure conditions by applying Weibull-type lifetime distributions, which are suitable for modeling time-dependent failure rates. Software failure time data were used for the analysis, and model parameters were estimated using the MLE approach. To quantitatively assess model performance, several evaluation criteria were employed: model efficiency measured by MSE and R^2, predictive accuracy assessed using the mean value function, failure occurrence rate using the intensity function, and system reliability based on the reliability function. The results demonstrate that the proposed Weibull-Extension model exhibits the highest performance among the models considered. These findings suggest that the Weibull-Extension model offers a more effective alternative for software reliability analysis and can be applied even in complex development environments. Therefore, this study contributes to the field by newly identifying the reliability performance of Weibull-type models, which have been underexplored in previous research, and is expected to support developers in early-stage failure rate estimation during the software development lifecycle.
Keywords: *Extended-Weibull, Infinite-Failure NHPP, Musa-Okumoto, Weibull, Weibull-Extension, Weibull-Type.*

1. INTRODUCTION

In the era of the Fourth Industrial Revolution, driven by Artificial Intelligence (AI), advanced software technologies are required to autonomously perform real-time decision-making and control tasks. Consequently, AI-based software must exhibit high levels of accuracy, predictability, and operational stability. Particularly, since AI systems are capable of learning and evolving over time, initial defects in software can escalate into more critical failures as the system matures. Due to these characteristics, the development of defect-free, highly reliable software has emerged as a key issue in the AI-driven digital convergence landscape of the Fourth Industrial Revolution. To address this challenge, software developers are increasingly conducting reliability analyses from the early stages of development to validate whether the system meets user requirements prior to deployment. In order to perform such analyses effectively, it is essential not only to conduct formal reliability testing but also to predict and eliminate possible defect patterns in advance. As a result, extensive research has been conducted on predicting software failures using failure time data collected during system operation. A variety of software reliability models have been developed to advance this research area [1]. In this regard, Kim and Shin [2] evaluated the efficiency of software reliability models employing flexible extended Weibull distributions through the Non-Homogeneous Poisson Process (NHPP) modeling. Furthermore, Xiao and Dohi [3] proposed Weibull-type distributions and assessed their effectiveness in NHPP-based reliability modeling. Subsequently, Kim [4] examined the reliability characteristics of NHPP software reliability models employing Makeham and extended Weibull distributions. Building on this, Yang [5] newly evaluated the reliability performance of finite-failure NHPP software reliability models based on the Weibull lifetime distribution. Park [6] also investigated the cost-efficiency of a software development model employing the Weibull distribution. Furthermore, Yang and Park [7] applied finite failure NHPP reliability models utilizing the Weibull-Extension

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distribution, which is categorized as a Weibull-type distribution, and proposed a novel interpretation of the models' performance properties.

Therefore, this study aims to expand the applicability of Weibull-type lifetime distributions in various forms and apply them to infinite failure NHPP software reliability models. Furthermore, an optimization technique is proposed to evaluate the reliability performance, and the most effective model among those proposed is identified.

2. RELATED RESEARCH

2.1.1 NHPP Model

In NHPP-based systems, when the failure time intervals are non-uniform, the number of failures up to time t, N(t), follows a Poisson distribution and reflects the non-homogeneous characteristics of the failure occurrence process. Accordingly, NHPP models are well suited to represent the dynamic behavior of software failure processes and are considered highly effective for software reliability modeling.

In particular, N(t) denotes the total accumulated failures by time t, and m(t) represents the expected average failure count by that time. Accordingly, m(t) serves as a key predictive metric for estimating the likelihood of future failures over time.

Thus, the NHPP-based reliability model is defined as shown in Equation (1).

$$P\{N(t) = n\} = \frac{[m(t)]^n \cdot e^{-m(t)}}{n!} \quad (1)$$

Note that $n = 0,1,2,\cdots,\infty$.

Therefore, Equation (2) defines the mean value function m(t), which predicts the expected number of failures.

$$m(t) = \int_0^t \lambda(s)ds \quad (2)$$

Accordingly, the instantaneous failure rate λ(t) is obtained by differentiating Equation (2) with respect to time, as shown in Equation (3).

$$\frac{dm(t)}{dt} = \lambda(t) \quad (3)$$

2.1.2 NHPP Software Reliability Model

The NHPP-based software reliability model is a probabilistic approach that utilizes the function m(t) to predict future software failures by using the cumulative number of observed failures. This study aims to analyze the reliability characteristics of Weibull-type distribution models applied to software failure time data using an NHPP framework.

The analysis is based on the infinite failure condition, in which failures are assumed to occur continuously, even during the repair process. The infinite failure model assumes that software failures can continue to occur throughout the operational life of the system. Even after a failure is repaired, new failures may arise, making this modeling approach particularly useful for long-running or large-scale complex systems. Therefore, in terms of reliability performance, m(t) serves as an attribute function representing the expected number of failures, while λ(t) represents the instantaneous failure rate as an attribute function. In addition, F(t) denotes the cumulative distribution function, and f(t) denotes the probability density function.

Thus, the function m(t) that determines the reliability performance of the infinite failure NHPP model can be derived as shown in Equation (4) [8].

$$m(t) = -\ln(1 - F(t)) \quad (4)$$

Furthermore, the intensity function λ(t) is derived from m(t) as shown in Equation (5).

$$\lambda(t) = m(t)' = \frac{f(t)}{(1 - F(t))} \quad (5)$$

Accordingly, m(t) denotes the expected number of cumulative failures over time, serving as a predictive metric of system reliability, while λ(t) represents the failure intensity function, indicating the rate at which failures are expected to occur at any given time.

Based on m(t) and λ(t), which characterize the system's reliability properties, the likelihood function of the infinite failure NHPP model is formulated as shown in Equation (6).

$$L_{NHPP}(\Theta|\underline{x}) = \left(\prod_{i=1}^{n} \lambda(x_i)\right) exp[-m(x_n)] \quad (6)$$

Note that $\underline{x} = (x_1, x_2, x_3 \cdots x_n)$

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2.2 NHPP Musa-Okumoto Model

The Musa-Okumoto model is a fundamental infinite failure NHPP model used to analyze software failure occurrences. This model assumes an infinite number of inherent faults in the software, with the failure intensity decreasing over time. It is characterized by increasingly longer intervals between failures as time progresses and exhibits a logarithmic decay in failure intensity. Thus, this model is considered suitable for large-scale, complex systems where software failures become less frequent over time.

Accordingly, the attribute functions governing the reliability performance are defined in Equations (7) and (8) [9].

$$m(t) = a \ln(1 + bt) \tag{7}$$

$$\lambda(t) = \frac{ab}{(1 + bt)} = h(t) \tag{8}$$

Note that both a>0 and b>0 denote the scale and shape parameters, respectively. Additionally, h(t) denotes the hazard function that characterizes the system's failure behavior.

Thus, the log-likelihood function constructed using m(t) and λ(t), which characterize the system's reliability performance, is presented in Equation (9).

$$lnL_{NHPP}(\Theta|\underline{x}) = nlna + nlnb - \sum_{i=1}^{n} \ln(1 + bx_i) - a\, ln(1 + b\, x_n) \tag{9}$$

Specifically, the parameter estimators ($\hat{a}_{MLE}$, $\hat{b}_{MLE}$) of the Musa-Okumoto basic model considered in this study are obtained by applying the Maximum Likelihood Estimation (MLE) approach to Equation (9), followed by a numerical solution using the bisection method. Accordingly, the final expressions for estimating the model parameters are provided in Equations (10) and (11).

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial a} = \frac{n}{\hat{a}} - ln(1 + b\, x_n) = 0 \tag{10}$$

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} \frac{x_i}{(1 + bx_i)} - \frac{ab}{(1 + b\, x_n)} = 0 \tag{11}$$

2.3 NHPP Weibull (Crow-AMSAA) Distribution Model

The Weibull distribution model is a non-homogeneous failure rate model in which the failure rate follows a Weibull distribution. It is widely used to model systems where the failure rate varies over time. In this model, the failure intensity is estimated based on the parameters of the Weibull distribution, making it highly effective for predicting software failure occurrences. The Crow-AMSAA model, an extension of the Weibull distribution model, is specifically designed for software reliability analysis and is commonly applied to characterize failure patterns in software systems. Therefore, both the Weibull and Crow-AMSAA models share a fundamentally similar structure and can be classified as infinite failure NHPP models.

Accordingly, the attribute functions governing the reliability performance are defined in Equations (12) and (13) [10].

$$m(t) = \int_0^t abt^{b-1}\, dt = at^b \tag{12}$$

$$\lambda(t) = abt^{b-1} \tag{13}$$

Thus, substituting into Equation (6), followed by a logarithmic transformation, yields the log-likelihood function presented in Equation (14).

$$lnL_{NHPP}(\Theta|\underline{x}) = nlna + nlnb + \sum_{i=1}^{n} (b - 1) \ln x_i - a_n^b \tag{14}$$

That is, the parameter estimators ($\hat{a}_{MLE}$, $\hat{b}_{MLE}$) for the Weibull distribution model considered in this study are obtained by applying the MLE approach to Equation (14), followed by a numerical solution using the bisection method. Therefore, the final equations for computing the parameters are provided in Equations (15) and (16).

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial a} = \frac{n}{\hat{a}} - x_n^b = 0 \tag{15}$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = -\hat{a}x_n^b \ln x_n + \frac{n}{\hat{b}} + \sum_{i=1}^{n} \ln x_i = 0 \tag{16}$$

2.4 NHPP Extended-Weibull Distribution Model

The Extended-Weibull distribution model is an infinite-failure NHPP model that extends the conventional Weibull distribution to flexibly capture the time-dependent behavior of failure rates. As such, the Extended-Weibull distribution is derived from the standard Weibull distribution. This model is well-suited for systems where the failure rate follows a complex temporal pattern, rather than exhibiting a simple monotonic increase or decrease.

Owing to its flexibility, the model has found extensive application in reliability analyses of both hardware and software systems.

Accordingly, the attribute functions governing the reliability performance are defined as shown in Equations (17) and (18) [11].

$$m(t) = \int_0^t \lambda(\omega)\, d\omega = \frac{a}{b} t^b \quad (17)$$

$$\lambda(t) = at^{b-1} \quad (18)$$

Note that both a>0 and b>0 denote the scale and shape parameters, respectively.

Thus, substituting into Equation (6), followed by a logarithmic transformation, yields the log-likelihood function presented in Equation (19).

$$\ln L_{NHPP}(\Theta|\underline{x}) = nlna + (b-1)\sum_{i=1}^{n} x_i - \frac{a}{b} x_n^b \quad (19)$$

That is, the Extended-Weibull model parameters ($\hat{a}_{MLE}$, $\hat{b}_{MLE}$) are estimated by applying the MLE approach to Equation (19), followed by solving the resulting equations via the bisection method. Accordingly, Equations (20) and (21) present the final formulations for parameter estimation.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial a} = \frac{n}{\hat{a}} - \frac{1}{b} x_n^b = 0 \quad (20)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \sum_{i=1}^{n} x_i - \frac{a}{\hat{b}} x_n^b \ln x_n + \frac{a}{b^2} x_n^b = 0 \quad (21)$$

2.5 NHPP Weibull-Extension Distribution Model

The Weibull-Extension distribution model is an infinite-failure NHPP model developed by extending the conventional Weibull distribution into a more advanced form to accurately model systems with time-varying failure behavior. This distribution enhances modeling flexibility by introducing an additional parameter into the standard Weibull formulation, thereby enabling more refined representation of systems with time-dependent failure rates.

As such, the Weibull-Extension model is particularly effective for analyzing systems in which the failure rate changes in a complex manner over time. Owing to these advantages, it has been widely applied in the reliability analysis of hybrid systems consisting of both hardware and software components, especially for predicting intricate failure patterns.

Accordingly, the probability distribution function of the Weibull-Extension model is formulated in Equations (22) and (23).

$$F(\mathrm{t}) = 1 - \exp\left[-\frac{\lambda}{a}\left(1 - e^{(at)^b}\right) - 1\right] \quad (22)$$

$$f(x) = \lambda \mathrm{b}(at)^{b-1} \exp\left[(at)^b + \frac{\lambda}{a}\left(1 - e^{(at)^b}\right)\right] \quad (23)$$

Note that both a>0 and b>0 denote the scale and shape parameters, respectively.

Therefore, the attribute functions governing the reliability performance are defined as shown in Equations (24) and (25) [12].

$$m(\mathrm{t}) = -\ln(1 - \mathrm{F(t)}) = \frac{\lambda}{a}\left(e^{(at)^b} - 1\right) \quad (24)$$

$$\lambda(\mathrm{t}) = \frac{f(t)}{(1 - F(t))} = \lambda \mathrm{b}(at)^{b-1} \exp\left[(at)^b\right] \quad (25)$$

Accordingly, substituting into Equation (6), followed by a logarithmic transformation, yields the log-likelihood function presented in Equation (26).

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\lambda + nlnb + \sum_{i=1}^{n}(ax_i)^b + (b-1)\sum_{i=1}^{n} ln(ax_i) - \frac{\lambda}{a}\left(e^{(ax_n)^b} - 1\right) \quad (26)$$

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In Equation (26), to facilitate numerical convergence, the scale parameter b is fixed at 1. Therefore, by differentiating the model with respect to the parameters λ and a, the maximum likelihood estimators ($\hat{\lambda}_{MLE}$, $\hat{a}_{MLE}$) satisfying Equations (27) and (28) can be obtained using numerical methods.

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial\lambda} = \frac{n}{\hat{\lambda}} - \frac{1}{a}\left[e^{(ax_n)^b} - 1\right] = 0 \qquad (27)$$

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial a} = (b-1)\frac{n}{\hat{a}} + b\,a^b\sum_{i=1}^{n} x_i^b + \frac{\lambda}{a^2}\left[e_n^{ax} - 1 - a^b\,b\,x_n^b\,e^{(ax_n)^b}\right] = 0 \qquad (28)$$

3. RELIABILITY PERFORMANCE ANALYSIS

This study proposes an optimization-based algorithm for quantitatively assessing the reliability performance of the applied model.

Accordingly, this algorithm was systematically developed and implemented through the following five-step procedure.

Step 1: Assessment of data reliability utilizing software failure time data.

Step 2: Estimation of model parameters $(\hat{\theta}, \hat{b})$ using the MLE approach.

Step 3: Selection of an effective model based on criteria such as MSE and R^2.

Step 4: Analysis of performance attribute functions (m(t), λ(t), and $\hat{R}(\tau)$).

Step 5: Reliability performance evaluation of the proposed models.

To assess reliability performance, software failure time data collected during the development process were utilized, as shown in Table 1 [13].

This dataset consists of 30 software failures that occurred at irregular intervals over a total operational time of 187.35 hours under normal system operation.

Table 1: Software Failure Time.

Failure number	Failure time (hours)	Failure time (hours)$\times 10^{-2}$
1	4.79	0.0479
2	7.45	0.0745
3	10.22	0.1022
4	15.76	0.1576
5	26.10	0.261
6	35.59	0.3559
7	42.52	0.4252
8	48.49	0.4849
9	49.66	0.4966
10	51.36	0.5136
11	52.53	0.5253
12	65.27	0.6527
13	69.96	0.6996
14	81.70	0.817
15	88.63	0.8863
16	107.71	1.0771
17	109.06	1.0906
18	111.83	1.1183
19	117.79	1.1779
20	125.36	1.2536
21	129.73	1.2973
22	152.03	1.5203
23	156.40	1.564
24	159.80	1.598
25	163.85	1.6385
26	169.60	1.696
27	172.37	1.7237
28	176.00	1.76
29	181.22	1.8122
30	187.35	1.8735

3.1. Step 1: Assessment of Data Reliability Utilizing Software Failure Time Data.

To assess the suitability of the failure time data employed in the reliability analysis, the Laplace Trend Test was performed using the dataset shown in Table 1.

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The results of this test are illustrated in Figure 1. Typically, when the result of the trend test falls between −2 and 2, it indicates the absence of significant outliers, suggesting that the data are stable and thus suitable for reliability analysis.

As shown in Figure 1, the result of the trend test falls within the range of −2 to 2, indicating that no extreme values are present. Accordingly, the failure time data presented in Table 1 can be regarded as appropriate for this study.

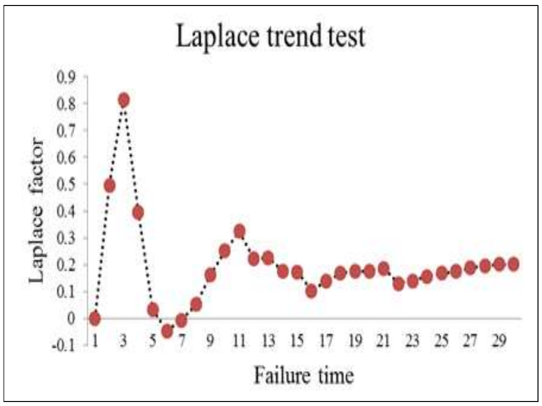

Figure 1: Results of Laplace Trend Test.

3.2. Step 2: Estimation of Model Parameters $(\hat{\theta}, \hat{b})$ Using the MLE Approach.

The parameter estimates ($\hat{a}_{MLE}$, $\hat{b}_{MLE}$) of the proposed model were obtained using the MLE approach, and the estimated results are summarized in Table 2 [14].

Table 2: Parameter estimates using MLE.

Type	NHPP Model	MLE	
		$\hat{a}$	$\hat{b}(\hat{\lambda})$
Basic	Musa-Okumot	21.3144	1.6471
Weibull-type Lifetime Distribution	Weibull	22.6889	0.4449
	Extended-Weibull	16.4988	1.0908
	Weibull-Extension	0.0683	15.0109

3.3. Step 3: Selection of an Effective Model Based on Criteria Such as MSE and R².

3.3.1. Mean Square Error (MSE)

The MSE is one of the key metrics used to quantitatively evaluate the predictive performance of a model. A smaller MSE value indicates lower prediction error, which in turn implies a more effective model in terms of goodness-of-fit. Accordingly, the MSE values provided in Table 2 were employed as reference data to identify the efficient model. That is, a model with a smaller MSE is considered to have a better fit, and the MSE can be derived as shown in Equation (29) [15].

$$MSE = \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{n-k} \tag{29}$$

Note that $\hat{m}(x_i)$ denotes the total accumulated failures.

Figure 2 illustrates the variation in MSE corresponding to the total of 30 failures observed throughout the entire failure time period. It can be observed that the Weibull-Extension model exhibits the lowest MSE values across the full failure range, indicating superior predictive performance. Also, the Extended-Weibull model maintains low MSE values, indicating strong goodness-of-fit.

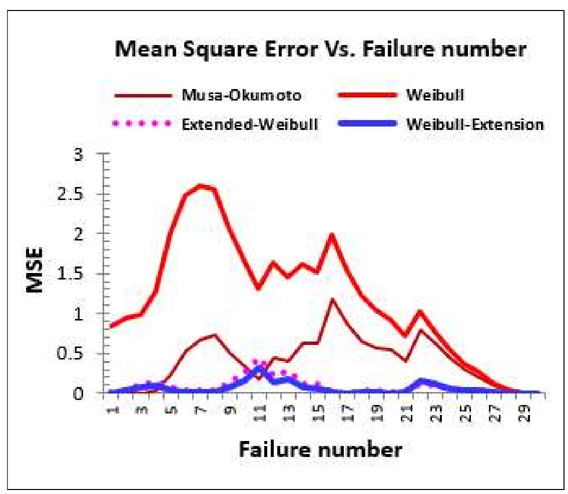

Figure 2: Efficiency Analysis of MSE.

3.3.2. Coefficient of Determination (R^2)

R^2 is an indicator of a model's explanatory power, showing how well the model accounts for the observed data. Thus, a higher R^2 value indicates a greater agreement between the predicted and actual values, suggesting better model efficiency. Accordingly, it is defined in Equation (30) [16].

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n}\left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \quad (30)$$

To evaluate both the efficiency and the suitability of the model, the present study employed this value as an evaluation criterion. Consequently, Table 3 presents the calculated MSE and R² values, which serve as reference indicators for selecting an efficient model.

Table 3: Model Efficiency Analysis.

Type	NHPP Model	MSE	R^2
Basic	Musa-Okumoto	12.0372	0.9618
Weibull-type Lifetime Distribution	Weibull	35.5399	0.8873
	Extended-Weibull	2.5268	0.9919
	Weibull-Extension	1.7466	0.9944

3.4. Step 4: Analysis of Performance Attribute Functions (m(t), λ(t), and $\hat{R}(\tau)$).

3.4.1 Mean Value Function ($\mathbf{m(t)}$)

m(t) represents the expected cumulative number of failures up to time *t*, serving as essential data for analyzing the failure pattern and reliability of the model. Accordingly, m(t) functions as a critical metric for evaluating the reliability performance of the model [17].

Figure 3 illustrates the trend of m(t), highlighting its ability to predict the true values accurately. Therefore, the simulation results reveal that the Weibull-Extension and Extended-Weibull models yield the lowest prediction errors relative to the true values. This finding suggests that these models possess superior estimation accuracy and demonstrate more efficient performance compared to the other proposed models.

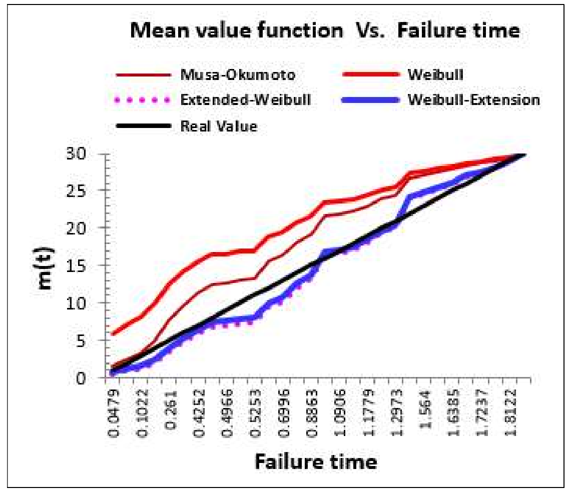

Figure 3: Performance Trend of m(t).

3.4.2 Intensity Function ($\boldsymbol{\lambda(t)}$)

λ(t) is a function that indicates how repeatedly a failure occurs at time t, and is important data that represents the dynamic characteristics of the failure pattern of the model [18].
Figure 4 presents the trend of the intensity function, showing the variation in instantaneous failure occurrence over the failure time span. Accordingly, the simulation results indicate that the Weibull and Musa-Okumoto models exhibit high initial failure rates, which gradually decrease over time due to repairs, demonstrating efficient performance.

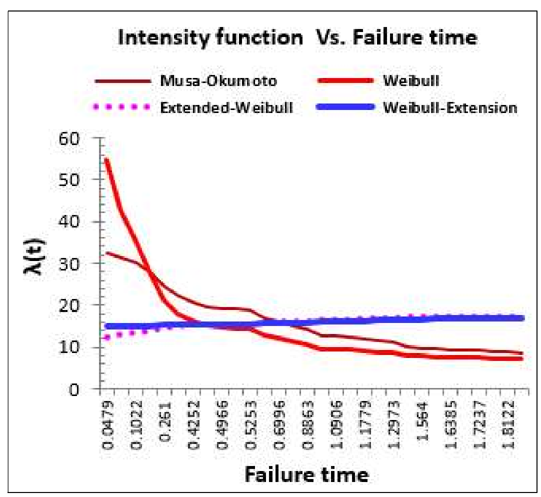

Figure 4: Performance Trend of $\lambda(t)$.

Table 4 shows the performance trend obtained by applying m(t) and λ(t) for each number of failures that occurred during a total of 187.35 failure times.

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Table 4: Trend Analysis of Reliability Performance Attributes.

Failure Number	Reliability Performance Attributes							
	m(t)				λ(t)			
	Musa-Okumoto	Weibull	Extended-Weibull	Weibull-Extension	Musa-Okumoto	Weibull	Extended-Weibull	Weibull-Extension
1	1.6186	5.8708	0.5498	0.7202	32.5397	54.5283	12.5206	15.0601
2	2.4670	7.1456	0.8901	1.1212	31.2699	42.6719	13.0330	15.0875
3	3.3161	8.2247	1.2567	1.5395	30.0487	35.8038	13.4125	15.1160
4	4.9190	9.9725	2.0156	2.3785	27.8719	28.1522	13.9505	15.1734
5	7.6220	12.4818	3.4944	3.9530	24.5521	21.2764	14.6044	15.2809
6	9.8333	14.3285	4.9011	5.4078	22.1327	17.9116	15.0215	15.3803
7	11.3144	15.5087	5.9508	6.4762	20.6469	16.2272	15.2661	15.4532
8	12.5127	16.4422	6.8678	7.4007	19.5182	15.0859	15.4493	15.5164
9	12.7398	16.6175	7.0487	7.5823	19.3113	14.8875	15.4828	15.5288
10	13.0656	16.8683	7.3124	7.8464	19.0184	14.6119	15.5302	15.5468
11	13.2870	17.0382	7.4942	8.0284	18.8219	14.4304	15.5620	15.5592
12	15.5593	18.7663	9.4972	10.0193	16.9185	12.7917	15.8719	15.6952
13	16.3384	19.3547	10.2440	10.7566	16.3113	12.3083	15.9722	15.7456
14	18.1721	20.7377	12.1327	12.6125	14.9666	11.2928	16.1988	15.8723
15	19.1849	21.5027	13.2595	13.7151	14.2721	10.7938	16.3190	15.9476
16	21.7476	23.4512	16.4018	16.7778	12.6553	9.6866	16.6104	16.1568
17	21.9177	23.5815	16.6262	16.9960	12.5547	9.6198	16.6292	16.1717
18	22.2627	23.8461	17.0873	17.4444	12.3531	9.4868	16.6672	16.2024
19	22.9865	24.4034	18.0831	18.4120	11.9407	9.2173	16.7459	16.2684
20	23.8718	25.0891	19.3544	19.6467	11.4549	8.9041	16.8409	16.3528
21	24.3666	25.4745	20.0915	20.3624	11.1920	8.7363	16.8934	16.4017
22	26.7268	27.3372	23.8867	24.0480	10.0189	7.9999	17.1384	16.6534
23	27.1601	27.6840	24.6366	24.7768	9.8172	7.8751	17.1826	16.7032
24	27.4913	27.9502	25.2214	25.3454	9.6658	7.7816	17.2162	16.7420
25	27.8793	28.2632	25.9194	26.0244	9.4915	7.6743	17.2554	16.7884
26	28.4182	28.7002	26.9132	26.9916	9.2545	7.5287	17.3095	16.8544
27	28.6730	28.9078	27.3930	27.4589	9.1446	7.4613	17.3350	16.8863
28	29.0024	29.1771	28.0229	28.0727	9.0043	7.3755	17.3678	16.9283
29	29.4673	29.5590	28.9307	28.9579	8.8101	7.2568	17.4140	16.9887
30	30.0006	29.9997	29.9998	30.0015	8.5923	7.1240	17.4666	17.0600

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3.4.3 Reliability Function ($\widehat{R}(\tau)$)

The reliability function $\widehat{R}(\tau)$ is widely acknowledged as an effective approach for evaluating the reliability of software system models for a specified mission time. To assess reliability performance, future reliability characteristics were examined for mission times (τ) extending beyond the final observed failure point (x_n = 187.35 hours), as detailed in the preceding section. The corresponding equation for calculating $\widehat{R}(\tau)$ is defined as shown in Equation (31) [19].

$$\widehat{R}(\tau|x_n) = exp[-\{m(x_n+\tau) - m(x_n)\}] \quad (31)$$

Figure 5 illustrates the results of the reliability analysis obtained by applying mission times ranging from 1 to 116 hours to the proposed models.

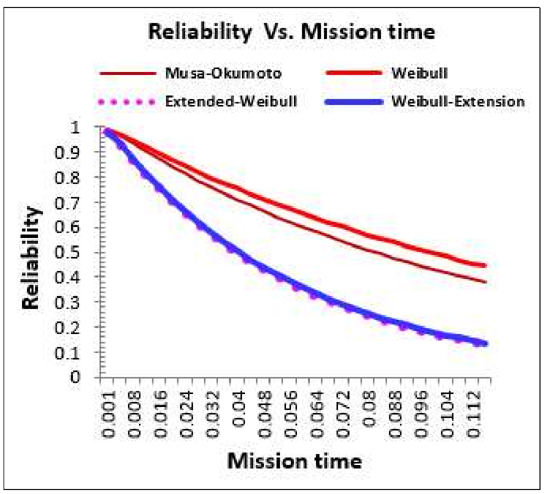

Figure 5: Performance Trend of $\hat{R}(\tau)$.

In general, higher reliability values indicate better model performance. As observed in Figure 5, models that yield consistently high and stable reliability values over time can be considered more reliable. In this regard, the Weibull and Musa-Okumoto models demonstrate relatively strong reliability retention over time, indicating their efficiency. In contrast, the Weibull-Extension and Extended-Weibull models exhibit a steep decline in reliability as mission time increases, suggesting inefficiency.

Table 5 presents a detailed dataset analyzing the trend of reliability values with respect to the applied mission times, to further investigate the properties of the $\widehat{R}(\tau)$ function that determines overall reliability performance.

For analytical convenience, the mission times were scaled down by a factor of 1/1000 (i.e., mission time × 0.001).

Table 5: Trend Analysis of $\hat{R}(\tau)$.

Mission Time (hours)	$\widehat{R}(\tau)$			
	Musa-Okumoto	Weibull	Extended-Weibull	Weibull-Extension
0.001	0.9914	0.9929	0.9827	0.9831
0.004	0.9662	0.9719	0.9325	0.9340
0.008	0.9337	0.9447	0.8696	0.8724
0.012	0.9023	0.9182	0.8109	0.8148
0.016	0.8719	0.8925	0.7561	0.7610
0.02	0.8427	0.8676	0.7050	0.7108
0.024	0.8145	0.8434	0.6574	0.6638
0.028	0.7872	0.8198	0.6130	0.6199
0.032	0.7609	0.7970	0.5716	0.5790
0.036	0.7356	0.7748	0.5329	0.5407
0.04	0.7111	0.7533	0.4969	0.5049
0.044	0.6875	0.7324	0.4633	0.4715
0.048	0.6647	0.7121	0.4320	0.4403
0.052	0.6426	0.6924	0.4028	0.4112
0.056	0.6214	0.6732	0.3755	0.3840
0.06	0.6009	0.6546	0.3501	0.3585
0.064	0.5810	0.6366	0.3264	0.3348
0.068	0.5619	0.6190	0.3043	0.3126
0.072	0.5434	0.6020	0.2837	0.2919
0.076	0.5256	0.5854	0.2645	0.2726
0.08	0.5084	0.5693	0.2466	0.2545
0.084	0.4917	0.5537	0.2299	0.2376
0.088	0.4757	0.5385	0.2143	0.2218
0.092	0.4602	0.5238	0.1998	0.2071
0.096	0.4452	0.5094	0.1863	0.1934
0.1	0.4307	0.4955	0.1736	0.1805
0.104	0.4167	0.4820	0.1619	0.1685
0.108	0.4032	0.4689	0.1509	0.1573
0.112	0.3901	0.4561	0.1407	0.1469
0.116	0.3775	0.4437	0.1311	0.1371

3.5. Step 4: Reliability Performance Evaluation of Proposed Models

The objective of this study is to analyze key efficiency metrics (MSE, R^2) and performance-related attribute functions (m(t), λ(t), $\widehat{R}(\tau)$) that significantly influence the effectiveness of the model.

Accordingly, the proposed model's performance is comparatively evaluated using an efficiency indicator that reflects the model's goodness-of-fit, in conjunction with an analysis of the associated performance attributes.

Table 6 summarizes the results of a comprehensive comparison and assessment of the proposed model's performance using the developed research dataset.

NHPP Model	Model Efficiency		Performance Attributes		
	MSE	R^2	m(t)	λ(t)	$\hat{R}(\tau)$
Musa-Okumoto	Bad	Best	Good	Good	Good
Weibull	Worst	Good	Worst	Good	Best
Extended-Weibull	Good	Best	Best	Worst	Good
Weibull-Extension	Best	Best	Best	Worst	Good

Table 6: Evaluation of Reliability Performance.

These performance results, if effectively applied during the early stages of software development, could not only provide developers with fundamental design parameters but also supply valuable testing data for development planning and preparation.

4. CONCLUSION

If a software developer can design a reliability prediction model in the early stages of testing by utilizing failure time data collected during the development process, the expected timing of future failures can be estimated in advance, thereby improving the overall reliability of the product. Based on this approach, developers may enhance software quality by making informed design and testing decisions. Accordingly, this study applied Weibull-type lifetime distributions suitable for software reliability analysis and testing to an infinite failure NHPP reliability model. Using actual software failure time data, the proposed models were comparatively analyzed, and their performance characteristics were newly identified and evaluated.

The findings of this study are summarized as follows:

First, based on the analysis of reference data (MSE, R^2) used for selecting efficient models, the Weibull-Extension model demonstrated the best goodness-of-fit among the tested models.

Second, through the evaluation of model performance attributes (m(t) and λ(t)), the Weibull-Extension and Extended-Weibull models exhibited the most accurate predictions with the smallest error rates, indicating their high efficiency.

Third**,** the reliability trend analysis revealed that all proposed models showed a decreasing reliability over time. However, the Weibull model maintained the highest reliability trend, demonstrating superior efficiency relative to the others.

In conclusion, among the proposed models, the Weibull-Extension model was identified as the optimal choice. Therefore, this study provides key attribute data that can assist developers during the early stages of software development. Furthermore, the findings suggest the need for continued research to explore optimal distribution models suitable for specific software domains and to conduct a deeper investigation into reliability-related models.

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